Дз №0 Гармонич. осц.

$$\hat{x}_r(t)=\hat{x}_r(0)\cos(\omega t)+\frac{\hat{p}_r(0)}{\omega m}\sin(\omega t)$$

$$\hat{p}_r(t)=\hat{p}_r(0)\cos(\omega t)-m\omega\hat{x}_r(0)\sin(\omega t)$$

$$\langle \hat{x}_r(t)\rangle=\int\psi^*\hat{x}_r(t)\psi\,dx=\frac{1}{\sqrt{\pi}\Delta}\int e^{()}\cdot\left(\hat{x}_r(0)\cos\omega t+\frac{\hat{p}_r(0)}{\omega m}\sin(\omega t)\right)e^{-\frac{(x-x_0)^2}{2\Delta^2}+\frac{i}{\hbar}p_0x}dx=$$

$$=\frac{1}{\sqrt{\pi}\Delta}\cdot\cos(\omega t)\int\hat{x}_r(0)e^{-\frac{(x-x_0)^2}{\Delta^2}}dx+\frac{1}{\sqrt{\pi}\Delta}\frac{\hbar}{i}\cdot\frac{1}{\omega m}\sin(\omega t)\int\left(-\frac{(x-x_0)}{\Delta^2}+\frac{i}{\hbar}p_0\right)e^{()}dx=$$

$$=\frac{1}{\sqrt{\pi}\Delta}\cos(\omega t)\left(\int(x-x_0)e^{-\frac{(x-x_0)^2}{\Delta^2}}dx+x_0\int e^{-\frac{(x-x_0)^2}{\Delta^2}}dx\right)+\frac{\hbar}{\sqrt{\pi}\Delta i m\omega}\sin(\omega t)\left(-\frac{1}{\Delta^2}\int(x-x_0)e^{-\frac{(x-x_0)^2}{2\Delta^2}}dx+\frac{i}{\hbar}p_0\int e^{-\frac{(x-x_0)^2}{\Delta^2}}dx\right)=$$

$$\to 0 \qquad \to 0$$

$$=\frac{1}{\sqrt{\pi}\Delta}\cos(\omega t)\cdot x_0\cdot\sqrt{\pi}\cdot\Delta+\frac{1}{\sqrt{\pi}\Delta}\frac{1}{m\omega}\sin(\omega t)\cdot p_0\sqrt{\pi}\cdot\Delta=x_0\cos(\omega t)+\frac{p_0}{m\omega}\sin\omega t$$

$$\langle\hat{p}_r(t)\rangle=\int\psi^*(\hat{p}_r(0)\cos(\omega t)-m\omega\hat{x}_r(0)\sin\omega t)\psi\,dx=\frac{1}{\sqrt{\pi}\Delta}\left[\frac{\hbar}{i}\cos(\omega t)\cdot\frac{i}{\hbar}p_0\sqrt{\pi}\Delta-m\omega\sin(\omega t)\cdot x_0\sqrt{\pi}\Delta\right]=$$

$$=p_0\cos(\omega t)-m\omega x_0\sin(\omega t)$$

$$\langle x^2(t)\rangle=\langle\hat{x}_r^2(0)\rangle\cos^2\omega t+\frac{\sin(\omega t)\cos(\omega t)}{m\omega}\cdot(\langle x(0)p(0)\rangle+\langle p(0)x(0)\rangle)+\frac{1}{\omega^2m^2}\sin^2(\omega t)\langle\hat{p}_r(0)^2\rangle$$

$$\langle\hat{x}_r^2(0)\rangle=\int_{-\infty}^{+\infty}e^{()}\cdot x^2e^{()}\frac{dx}{\sqrt{\pi}\Delta}=\frac{1}{\sqrt{\pi}\Delta}\left[\int_{-\infty}^{+\infty}(x^2-2xx_0+x_0^2)e^{()}dx+2x_0\int xe^{()}dx-x_0^2\int e^{()}dx\right]=$$

$$=\frac{1}{\sqrt{\pi}\Delta}\left[\int(x-x_0)^2e^{()}dx+2x_0\int(x-x_0)e^{()}dx+(2x_0^2-x_0^2)\int e^{()}dx\right]=\frac{1}{\sqrt{\pi}\Delta}\left[\frac{1}{2}\sqrt{\pi}\Delta^3+x_0^2\sqrt{\pi}\Delta\right]=\frac{1}{2}\Delta^2+x_0^2$$

$$\to 0$$

$$\langle\hat{p}_r^2(0)\rangle=\int_{-\infty}^{+\infty}e^{()}\left(\frac{\hbar}{i}\right)^2\frac{d^2}{dx^2}e^{-\frac{(x-x_0)^2}{2\Delta^2}+\frac{i}{\hbar}p_0x}\frac{dx}{\sqrt{\pi}\Delta}=\frac{1}{\sqrt{\pi}\Delta}\left(\frac{\hbar}{i}\right)^2\int e^{()}\frac{d}{dx}\left(-\frac{(x-x_0)}{\Delta^2}+\frac{i}{\hbar}p_0\right)e^{()}dx=\frac{1}{\sqrt{\pi}\Delta}\left(\frac{\hbar}{i}\right)^2\left[\int e^{()}\left(-\frac{1}{\Delta^2}\right)e^{()}dx+\int e^{()}\left(-\frac{(x-x_0)}{\Delta^2}+\frac{i}{\hbar}p_0\right)^2e^{()}dx\right]=$$

$$=\frac{1}{\sqrt{\pi}\Delta}\left(\frac{\hbar}{i}\right)^2\left[-\frac{1}{\Delta^2}\sqrt{\pi}\Delta+\int\left(\frac{(x-x_0)^2}{\Delta^4}-\frac{2i}{\hbar\Delta^2}(x-x_0)p_0+\left(\frac{i}{\hbar}p_0\right)^2\right)e^{()}dx\right]=\left(\frac{\hbar}{i\Delta}\right)^2+\frac{1}{\sqrt{\pi}\Delta}\left(\frac{\hbar}{i}\right)^2\left[\frac{1}{\Delta^4}\frac{1}{2}\sqrt{\pi}\Delta^3+\left(\frac{i}{\hbar}\right)^2p_0^2\sqrt{\pi}\Delta\right]=$$

$$\to 0$$

$$=-\frac{\hbar^2}{i^2\Delta^2}+\frac{1}{2}\frac{\hbar^2}{\Delta^2 i^2}+p_0^2=p_0^2+\frac{\hbar^2}{\Delta^2}-\frac{1}{2}\frac{\hbar^2}{\Delta^2}=p_0^2+\frac{1}{2}\frac{\hbar^2}{\Delta^2}$$

$$\langle\hat{x}\hat{p}_r(0)\rangle=\int e^{()}x\cdot\left(\frac{\hbar}{i}\right)\frac{d}{dx}e^{()}\frac{dx}{\sqrt{\pi}\Delta}=\frac{1}{\sqrt{\pi}\Delta}\frac{\hbar}{i}\int x\left(\frac{-(x-x_0)}{\Delta^2}+\frac{i}{\hbar}p_0\right)e^{()}dx=\frac{1}{\sqrt{\pi}\Delta}\frac{\hbar}{i}\int\left(-\frac{(x^2-xx_0)}{\Delta^2}+\frac{ip_0x}{\hbar}\right)e^{()}dx=$$

$$=\frac{1}{\sqrt{\pi}\Delta}\frac{\hbar}{i}\left[\int\frac{-x^2}{\Delta^2}e^{()}dx+\int x\left(\frac{x_0}{\Delta^2}+\frac{ip_0}{\hbar}\right)e^{()}dx\right]=\frac{1}{\sqrt{\pi}\Delta}\frac{\hbar}{i}\left[\int-\frac{(x^2-2xx_0+x_0^2)}{\Delta^2}e^{()}dx-\frac{2x_0}{\Delta^2}\int xe^{()}dx+\frac{x_0^2}{\Delta^2}\int e^{()}dx+\right.$$

$$\left.+\left(\frac{x_0}{\Delta^2}+\frac{ip_0}{\hbar}\right)\int xe^{()}dx\right]=\frac{1}{\sqrt{\pi}\Delta}\frac{\hbar}{i}\left[-\int\frac{(x-x_0)^2}{\Delta^2}e^{()}dx-\frac{2x_0}{\Delta^2}\int(x-x_0)e^{()}dx-\frac{2x_0^2}{\Delta^2}\int e^{()}dx+\frac{x_0^2}{\Delta^2}\int e^{()}dx+\left(\frac{x_0}{\Delta^2}+\frac{ip_0}{\hbar}\right)\int(x-x_0)e^{()}dx\right.$$

$$\to 0 \qquad \to 0$$

$$\left.+\left(\frac{x_0}{\Delta^2}+\frac{ip_0}{\hbar}\right)x_0\int e^{()}dx\right]=\frac{1}{\sqrt{\pi}\Delta}\frac{\hbar}{i}\left[-\frac{1}{\Delta^2}\frac{\sqrt{\pi}\Delta^3}{2}-\frac{x_0^2}{\Delta^2}\sqrt{\pi}\Delta+\left(\frac{x_0}{\Delta^2}+\frac{ip_0}{\hbar}\right)x_0\sqrt{\pi}\Delta\right]=\frac{\hbar}{i}\left(-\frac{1}{2}-\frac{x_0^2}{\Delta^2}+\frac{x_0^2}{\Delta^2}+\frac{ip_0x_0}{\hbar}\right)=p_0x_0+\frac{i\hbar}{2}$$

$$\langle[x,p]\rangle=\langle xp-px\rangle=\langle xp-px\rangle=i\hbar\Rightarrow\langle px\rangle=\langle xp\rangle-i\hbar=p_0x_0+\frac{i\hbar}{2}-i\hbar=p_0x_0-\frac{i\hbar}{2}$$

$$\langle x^2(t)\rangle=\left(\frac{1}{2}\Delta^2+x_0^2\right)\cos^2(\omega t)+\frac{\sin\omega t\cos\omega t}{m\omega}2p_0x_0+\frac{1}{\omega^2m^2}\sin^2(\omega t)\left(p_0^2+\frac{1}{2}\frac{\hbar^2}{\Delta^2}\right)$$

$$\langle x(t)\rangle^2=x_0^2\cos^2\omega t+\frac{2x_0p_0}{m\omega}\cos\omega t\sin\omega t+\frac{p_0^2}{m^2\omega^2}\sin^2\omega t$$

$$D_x=\langle x^2\rangle-\langle x\rangle^2=\frac{1}{2}\Delta^2\cos^2(\omega t)+\frac{1}{2}\frac{\hbar^2}{\Delta^2}\frac{1}{\omega^2m^2}\sin^2(\omega t)$$

$$\langle p_r(t)^2\rangle = \langle p(0)^2\rangle\cos^2\omega t - (\langle p(0)x(0)\rangle + \langle x(0)p(0)\rangle)\cos(\omega t)\cdot\sin(\omega t)\cdot m\omega + \langle x(0)^2\rangle m^2\omega^2\sin^2(\omega t) =$$

$$= \left(p_0^2 + \frac{1}{2}\frac{\hbar^2}{\Delta^2}\right)\cos^2\omega t - (p_0x_0 \quad)\cdot 2\cos(\omega t)\sin(\omega t)\cdot m\omega + \left(x_0^2 + \frac{1}{2}\Delta^2\right)m^2\omega^2\sin^2(\omega t)$$

$$\langle p_r(t)\rangle^2 = p_0^2\cos^2(\omega t) - p_0x_0\cdot 2\cos(\omega t)\sin(\omega t)\cdot m\omega + x_0^2 m^2\omega^2\sin^2(\omega t)$$

$$D_p = \langle p_r(t)^2\rangle - \langle p_r(t)\rangle^2 = +\frac{1}{2}\frac{\hbar^2}{\Delta^2}\cos^2(\omega t) \qquad + \frac{1}{2}\Delta^2 m^2\omega^2\sin^2(\omega t)$$

Свободн. част.

$$\hat{p}_r(t) = \hat{p}_r(0)$$

$$\hat{x}_r(t) = \frac{\hat{p}_r(0)}{m}t + \hat{x}_r(0)$$

$$\langle x_r(t)\rangle = \frac{t}{m}\langle p_r(0)\rangle + \langle\hat{x}_r(0)\rangle = \frac{p_0}{m}t + x_0$$

$$\langle x_r(t)\rangle^2 = \frac{p_0^2}{m^2}t^2 + \frac{2p_0x_0}{m}t + x_0^2$$

$$\langle x_r(t)^2\rangle = \langle p_r(0)^2\rangle\frac{t^2}{m^2} + (\langle p(0)x(0)\rangle + \langle x(0)p(0)\rangle)\frac{t}{m} + \langle x(0)^2\rangle = \left(p_0^2 + \frac{1}{2}\frac{\hbar^2}{\Delta^2}\right)\frac{t^2}{m^2} + (p_0x_0 \quad)\frac{2t}{m} + \left(x_0^2 + \frac{\Delta^2}{2}\right)$$

$$D_x = \langle x_r(t)^2\rangle - \langle x_r(t)\rangle^2 = +\frac{1}{2}\frac{\hbar^2}{\Delta^2}\frac{t^2}{m^2} \qquad + \frac{\Delta^2}{2}$$

$$\langle p_r(t)\rangle = \langle\hat{p}_r(0)\rangle = p_0$$

$$\langle p_r(t)\rangle^2 = p_0^2$$

$$\langle p_r(t)^2\rangle = \langle\hat{p}_r(0)^2\rangle = p_0^2 + \frac{1}{2}\frac{\hbar^2}{\Delta^2}$$

$$D_p = \langle p_r(t)^2\rangle - \langle p_r(t)\rangle^2 = +\frac{1}{2}\frac{\hbar^2}{\Delta^2}$$

№1 $H = \frac{p^2}{2m} - \frac{kx^2}{2} = \frac{p^2}{2m} + \frac{k(ix)^2}{2}$

$$\frac{d}{dt}\hat{p}_r(t) = 0 + \frac{i}{\hbar}\left[\frac{p^2}{2m} + \frac{k(ix)^2}{2}; p\right] = \frac{i}{\hbar}\left[\frac{p^2}{2m}; p\right]^{\nearrow 0} + \frac{i}{\hbar}\left[\frac{k(ix)^2}{2}, p\right] = \frac{i}{\hbar}\frac{k}{2}\cdot i^2\cdot 2i\hbar x = kx$$

$$\frac{d}{dt}\hat{x}_r(t) = 0 + \frac{i}{\hbar}\left[\frac{p^2}{2m} + \frac{k(ix)^2}{2}; x\right] = \frac{i}{\hbar}\frac{1}{2m}[p^2; x] = \frac{i}{\hbar}\frac{1}{2m}\cdot 2(-i\hbar p) = \frac{p}{m}$$

$$\left.\begin{aligned}\frac{d}{dt}\hat{p}_r(t) &= kx\\ \frac{d}{dt}\hat{x}_r(t) &= \frac{p}{m}\end{aligned}\right\} \Rightarrow \begin{aligned} p &= m\frac{d\hat{x}_r(t)}{dt}\\ \frac{d}{dt}&\left(m\frac{d\hat{x}_r(t)}{dt}\right) - k\hat{x}_r = 0\end{aligned} \Rightarrow \frac{d^2}{dt^2}\hat{x}_r(t) - \frac{k}{m}\hat{x}_r(t) = 0 \qquad \omega = \sqrt{\frac{k}{m}}$$

$$\frac{d^2}{dt^2}\hat{x}_r(t) + (i\omega)^2\hat{x}_r(t) = 0$$

$$x_r(t) = C_1\cos(i\omega t) + C_2\sin(i\omega t) = C_1\,\mathrm{ch}(\omega t) + C_2\,\mathrm{sh}(\omega t)$$

$$\hat{x}_r(0) = C_1 =$$

$$\frac{d}{dt}x_r = x(0)\cdot\omega\,\mathrm{sh}(\omega t) + C_2\cdot\omega\cdot\mathrm{ch}(\omega t)$$

$$\left.\frac{d}{dt}x_r(t)\right|_{t=0} = C_2\cdot\omega = \frac{p_r(0)}{m} \Rightarrow C_2 = \frac{p_r(0)}{m\omega} \Rightarrow x_r(t) = \hat{x}_r(0)\,\mathrm{ch}(\omega t) + \frac{p_r(0)}{m\omega}\mathrm{sh}(\omega t)$$

$$\hat{p}_r(t) = \hat{p}_r(0)\,\mathrm{ch}(\omega t) + \hat{x}_r(0)m\omega\,\mathrm{sh}(\omega t)$$

$$\langle \hat{x}_r(t)\rangle = \langle \hat{x}_r(0)\rangle ch(\omega t) + \frac{\langle \hat{p}_r(0)\rangle}{m\omega} sh(\omega t) = x_0\, ch(\omega t) + \frac{p_0}{m\omega} sh(\omega t) \qquad \omega^2 = \frac{k}{m}$$

$$\langle \hat{x}_r(t)\rangle^2 = x_0^2 ch^2(\omega t) + \left(\frac{p_0}{m\omega}\right)^2 sh^2(\omega t) + \frac{2p_0x_0}{m\omega} ch(\omega t) sh(\omega t)$$

$$\langle x_r(t)^2\rangle = \langle \hat{x}_r(0)^2\rangle ch^2(\omega t) + \left(\langle x(0)p(0)\rangle + \langle p(0)x(0)\rangle\right) \frac{ch(\omega t)\cdot sh(\omega t)}{m\omega} + \frac{\langle \hat{p}_r(0)^2\rangle}{m^2\omega^2} sh^2(\omega t) =$$

$$= \left(x_0^2 + \frac{\Delta^2}{2}\right) ch^2(\omega t) + (p_0x_0 \quad)\cdot \frac{2ch(\omega t)sh(\omega t)}{m\omega} + \left(p_0^2 + \frac{1}{2}\frac{\hbar^2}{\Delta^2}\right)\frac{1}{m^2\omega^2} sh^2(\omega t)$$

$$D_x = \langle x_r(t)^2\rangle - \langle x_r(t)\rangle^2 = \frac{\Delta^2}{2} ch^2(\omega t) + \qquad + \frac{1}{2}\frac{\hbar^2}{\Delta^2}\frac{1}{m^2\omega^2} sh^2(\omega t)$$

$$\langle \hat{p}_r(t)\rangle = \langle \hat{p}_r(0)\rangle ch(\omega t) + \langle \hat{x}_r(0)\rangle m\omega\, sh(\omega t) = p_0\, ch(\omega t) + x_0 m\omega\, sh(\omega t)$$

$$\langle \hat{p}_r(t)\rangle^2 = p_0^2 ch^2(\omega t) + 2p_0x_0 m\omega\, ch(\omega t) sh(\omega t) + x_0^2 m^2\omega^2 sh^2(\omega t)$$

$$\langle \hat{p}_r^2(t)\rangle = \langle \hat{p}_r^2(0)\rangle ch^2(\omega t) + \left(\langle p(0)x(0)\rangle + \langle x(0)p(0)\rangle\right) ch(\omega t) sh(\omega t) + \langle \hat{x}^2(0)\rangle m^2\omega^2 sh^2(\omega t) =$$

$$= \left(p_0^2 + \frac{\hbar^2}{2\Delta^2}\right) ch^2(\omega t) + 2m\omega(p_0x_0 \quad) ch(\omega t) sh(\omega t) + \left(x_0^2 + \frac{\Delta^2}{2}\right) m^2\omega^2 sh^2(\omega t)$$

$$D_p = \langle \hat{p}_r^2(t)\rangle - \langle \hat{p}_r(t)\rangle^2 = \frac{\hbar^2}{2\Delta^2} ch^2(\omega t) \qquad + \frac{\Delta^2}{2} m^2\omega^2 sh^2(\omega t)$$

$$\langle E\rangle = \langle H\rangle = \left\langle \frac{\hat{p}_r^2(t)}{2m} - \frac{k\hat{x}_r^2(t)}{2}\right\rangle = \frac{1}{2m}\langle \hat{p}_r^2(t)\rangle - \frac{k}{2}\langle \hat{x}_r^2(t)\rangle =$$

$$= \frac{1}{2m}\left[\left(p_0^2 + \frac{\hbar^2}{2\Delta^2}\right) ch^2(\omega t) + 2m\omega(p_0x_0 \quad) ch(\omega t) sh(\omega t) + \left(x_0^2 + \frac{\Delta^2}{2}\right) m^2\omega^2 sh^2(\omega t)\right] -$$

$$- \frac{k}{2}\left[\left(p_0^2 + \frac{\hbar^2}{2\Delta^2}\right)\frac{1}{m^2\omega^2} sh^2(\omega t) + \frac{2}{m\omega}(p_0x_0 \quad) ch(\omega t) sh(\omega t) + \left(x_0^2 + \frac{\Delta^2}{2}\right) ch^2(\omega t)\right] =$$

$$= \frac{1}{2}\left[\left(p_0^2 + \frac{\hbar^2}{2\Delta^2}\right)\frac{1}{m}\underbrace{\left(ch^2(\omega t) - sh^2(\omega t)\right)}_{1} + \qquad - \omega^2 m\left(x_0^2 + \frac{\Delta^2}{2}\right)\underbrace{\left(ch^2(\omega t) - sh^2(\omega t)\right)}_{1}\right] =$$

$$= \frac{1}{2m}\left(p_0^2 + \frac{\hbar^2}{2\Delta^2}\right) \qquad - \frac{\omega^2 m}{2}\left(x_0^2 + \frac{\Delta^2}{2}\right)$$

№2 $\langle x(t)\cdot x(0)\rangle$ для Г.О. — корреляционная функция x

$$\langle x(t)\cdot x(0)\rangle = \left\langle \left(x(0)\cdot \cos\omega t + \frac{p(0)}{m\omega}\sin(\omega t)\right) x(0)\right\rangle = \langle x^2(0)\rangle \cos\omega t + \langle p(0)\cdot x(0)\rangle \frac{1}{m\omega}\sin(\omega t)$$

$$\langle x(0)^2\rangle = \frac{1}{\sqrt{\pi}\Delta}\left[\int e^{()}\cdot(x^2 - 2xx_0 + x_0^2)dx + 2x_0\left[\int (x-x_0)e^{()}dx + \int x_0 e^{()}dx\right] - x_0^2\int e^{()}dx\right] = x_0^2 + \frac{1}{2}\Delta^2$$

$$\langle p(0)\cdot x(0)\rangle = p_0x_0 - \frac{i\hbar}{2}$$

$$\langle x(t)\cdot x(0)\rangle = \left(x_0^2 + \frac{1}{2}\Delta^2\right)\cos(\omega t) + \left(p_0x_0 - \frac{i\hbar}{2}\right)\frac{1}{m\omega}\sin(\omega t)$$

№3 П.О.

$$[\hat{x}(t), \hat{p}(\tau)] = \hat{x}(t)\cdot\hat{p}(\tau) - \hat{p}(\tau)\cdot\hat{x}(t) = \left(\hat{x}_r(0)\cos(\omega t) + \frac{\hat{p}(0)}{m\omega}\sin(\omega t)\right)\left(\hat{p}_r(0)\cos(\omega\tau) - \hat{x}_r(0)m\omega\sin(\omega\tau)\right) -$$

$$- \left(\hat{p}_r(0)\cos(\omega\tau) - \hat{x}_r(0)m\omega\sin(\omega\tau)\right)\left(\hat{x}_r(0)\cos(\omega t) + \frac{\hat{p}_r(0)}{m\omega}\sin(\omega t)\right) = x(0)p(0)\cos(\omega t)\cos(\omega\tau) - \hat{x}(0)^2 m\omega\cos(\omega t)\sin(\omega\tau) + \frac{\hat{p}(0)^2}{m\omega}\sin(\omega t)\cos(\omega\tau) -$$

$$- \frac{p(0)x(0)}{m}\sin(\omega t)\sin(\omega\tau) - \left[p(0)x(0)\cos(\omega t)\cos(\omega\tau) + \frac{p^2(0)}{m\omega}\cos(\omega\tau)\sin(\omega t) - x^2(0)m\omega\sin(\omega\tau)\cos(\omega t) - x(0)p(0)\sin(\omega\tau)\sin(\omega t)\right] = (x(0)p(0) - p(0)x(0))\cos(\omega t)\cos(\omega\tau) +$$

$$+ (x(0)p(0) - p(0)x(0))\sin(\omega t)\sin(\omega\tau) = i\hbar\left[\cos(\omega t)\cos(\omega\tau) + \sin(\omega t)\sin(\omega\tau)\right] = i\hbar\cos(\omega t - \omega\tau)$$

№4?

$[A,B] = iC$ $\quad (AB)^+ = B^+A^+$

$$\left.\begin{array}{l}[A,B] = AB - BA = iC \\ \cancel{A^*B^* - B^*A^* = iC^*} \\ B^+A^+ - A^+B^+ = iC^+\end{array}\right\} \Rightarrow \cancel{CC^* = (AB-BA)(A^*B^*-B^*A^*) = ABA^*B^* - AB\,B^*A^* - BA\,A^*B^* + BA\,B^*A^*}$$

$$\cancel{= AB\,A^*B^* + BA\,B^*A^* - 2}$$

$$\Rightarrow C^+C = (AB - BA)(BA^+ - A^+B^+) = ABBA^+ - ABA^+B^+ - BABA^+ + BAA^+B^+ =$$

№5 $\quad |L\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\quad$ а) $H = \begin{pmatrix}0 & \Delta\\ \Delta & 0\end{pmatrix}$ $\quad$ а') $|\psi(0)\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\quad P_R = ?$

$|R\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $\quad$ б) $H = \begin{pmatrix}E_L & \Delta\\ \Delta & E_R\end{pmatrix}$ $\quad$ б') $|\psi(0)\rangle = a|L\rangle + b|R\rangle = \begin{pmatrix}a\\b\end{pmatrix}$ $\quad P_R = ?$

аа') — сделано: $|\psi(t)\rangle = a(t)|L\rangle + b(t)|R\rangle = \begin{pmatrix}a(t)\\b(t)\end{pmatrix}$

$i\hbar\frac{d}{dt}\begin{pmatrix}a(t)\\b(t)\end{pmatrix} = \begin{pmatrix}0 & \Delta\\ \Delta & 0\end{pmatrix}\begin{pmatrix}a(t)\\b(t)\end{pmatrix}$ $\quad \begin{array}{l}a(0)=1\\b(0)=0\end{array}$

$$\left.\begin{array}{l}i\hbar\dot{a} = \Delta b\\ i\hbar\dot{b} = \Delta a\end{array}\right\} \Rightarrow a = \frac{i\hbar\dot{b}}{\Delta} \Rightarrow \Delta b = -\frac{\hbar^2}{\Delta}\ddot{b} \Rightarrow \ddot{b} + \frac{\Delta^2}{\hbar^2}b = 0$$

$b(t) = C\cdot\sin\frac{\Delta}{\hbar}t$, т.к. $b(0)=0$

$a(0) = 1 = \frac{i\hbar}{\Delta}\cdot\dot{b}\big|_{t=0} = \frac{i\hbar}{\Delta}\cdot C\cdot\frac{\Delta}{\hbar} = iC \Rightarrow C = -i$

$\psi(t) = \begin{pmatrix}\cos\frac{\Delta}{\hbar}t\\ -i\sin\frac{\Delta}{\hbar}t\end{pmatrix}$; $\quad P_{|R\rangle} = |\langle R|\psi(t)\rangle|^2 = \sin^2\frac{\Delta}{\hbar}t$

аб')

$\begin{array}{l}a(0)=a_0\\b(0)=b_0\end{array}$

$b(t) = C_1\sin\frac{\Delta}{\hbar}t + C_2\cos\frac{\Delta}{\hbar}t$

$b(0) = C_1\cdot 0 + C_2\cdot 1 = C_2 = b_0$

$i\hbar\dot{b} = \Delta a$

$\frac{i\hbar}{\Delta}\dot{b}(t)\big|_{t=0} = a(0) \Rightarrow \frac{i\hbar}{\Delta}\left(C_1\cdot\frac{\Delta}{\hbar}\cos\frac{\Delta}{\hbar}t - b_0\frac{\Delta}{\hbar}\sin\frac{\Delta}{\hbar}t\right)\Big|_{t=0} = iC_1 - 0 = a_0 \Rightarrow C_1 = \frac{a_0}{i} = -ia_0$

$b(t) = b_0\cos\frac{\Delta}{\hbar}t - ia_0\sin\frac{\Delta}{\hbar}t$

$a(t) = C_1'\cos\frac{\Delta}{\hbar}t + C_2\sin\frac{\Delta}{\hbar}t$

$a(0) = a_0 = C_1'$

$\frac{i\hbar}{\Delta}\dot{a}(t)\big|_{t=0} = \frac{i\hbar}{\Delta}\left(-\frac{\Delta}{\hbar}a_0\sin\frac{\Delta}{\hbar}t + C_2\frac{\Delta}{\hbar}\cos\frac{\Delta}{\hbar}t\right)\Big|_{t=0} = iC_2 = b_0 \Rightarrow C_2 = \frac{b_0}{i} = -ib_0$

$a(t) = a_0\cos\frac{\Delta}{\hbar} - ib_0\sin\frac{\Delta}{\hbar}t \Rightarrow \psi(t) = \begin{pmatrix}a_0\cos\frac{\Delta}{\hbar} - ib_0\sin\frac{\Delta}{\hbar}t\\ b_0\cos\frac{\Delta}{\hbar} - ia_0\sin\frac{\Delta}{\hbar}t\end{pmatrix} \Rightarrow P_{|R\rangle} = |\langle R|\psi(t)\rangle|^2$

$$P_{|R>} = \left|\langle (0\ 1)|\psi(t)\rangle\right|^2 = \left|b_0\cos\frac{\alpha}{\hbar} - i a_0 \sin\frac{\alpha}{\hbar}t\right|^2$$

б a') $H = \begin{pmatrix} E_L & \alpha \\ \alpha & E_R \end{pmatrix}$ $\quad |\psi(t)\rangle = a(t)|L\rangle + b(t)|R\rangle = \begin{pmatrix} a(t) \\ b(t)\end{pmatrix}$ $\quad a(0)=1$, $b(0)=0$

$$i\hbar\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t)\end{pmatrix} = \begin{pmatrix} E_L & \alpha \\ \alpha & E_R \end{pmatrix}\begin{pmatrix} a(t) \\ b(t)\end{pmatrix}$$

$2\times 2 \quad 2\times 1 = 2\times 1$

$$\begin{cases} i\hbar\dot a(t) = E_L a(t) + \alpha b(t) \\ i\hbar \dot b(t) = \alpha a(t) + E_R b(t) \end{cases} \Rightarrow a(t) = \frac{i\hbar}{\alpha}\dot b(t) - \frac{E_R}{\alpha}b(t) \Rightarrow i\hbar\left(\frac{i\hbar}{\alpha}\dot b(t) - \frac{E_R}{\alpha}b(t)\right)' = E_L\left(\frac{i\hbar}{\alpha}\dot b(t) - \frac{E_R}{\alpha}b(t)\right) + \alpha b(t)$$

$$-\frac{\hbar^2}{\alpha}\ddot b(t) - \frac{i\hbar E_R}{\alpha}\dot b(t) - \frac{i\hbar}{\alpha}E_L\dot b(t) + \frac{E_R E_L}{\alpha}b(t) - \alpha b(t) = 0$$

$$-\frac{\hbar^2}{\alpha}\ddot b(t) - \frac{i\hbar}{\alpha}(E_R+E_L)\dot b(t) + \left(\frac{E_R E_L}{\alpha} - \alpha\right)b(t) = 0$$

$$-\frac{\hbar^2}{\alpha}\lambda^2 - \frac{i\hbar}{\alpha}(E_R+E_L)\lambda + \left(\frac{E_R E_L}{\alpha} - \alpha\right) = 0 \Rightarrow -\frac{\hbar^2}{\alpha^2}\lambda^2 - \frac{i\hbar}{\alpha^2}(E_R+E_L)\lambda + \left(\frac{E_R E_L}{\alpha^2} - 1\right) = 0$$

$$\lambda^2 + \frac{i}{\hbar}(E_R+E_L)\lambda - \left(\frac{E_R E_L}{\hbar^2} - \frac{\alpha^2}{\hbar^2}\right) = 0$$

$$D = -\frac{(E_R+E_L)^2}{\hbar^2} + 4\left(\frac{E_R E_L}{\hbar^2} - \frac{\alpha^2}{\hbar^2}\right) = -\frac{(E_R^2 + 2E_R E_L + E_L^2)}{\hbar^2} + \frac{4E_R E_L}{\hbar^2} - \frac{4\alpha^2}{\hbar^2} = \frac{-(E_R-E_L)^2}{\hbar^2} - \frac{4\alpha^2}{\hbar^2} = \left(\frac{i}{\hbar}\right)^2\left[(E_R-E_L)^2 + 4\alpha^2\right]$$

$$\lambda = -\frac{i}{2\hbar}(E_R+E_L) \pm \frac{i}{2\hbar}\sqrt{(E_R+E_L)^2 + (2\alpha)^2}$$

$$b(t) = C_1\exp\left(\frac{it}{2\hbar}\left(-(E_R+E_L) + \sqrt{(E_R+E_L)^2 + (2\alpha)^2}\right)\right) + C_2\exp\left(\frac{-it}{2\hbar}\left((E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right)\right)$$

$b(0) = C_1 + C_2 = 0 \Rightarrow C_1 = -C_2$
$C_2 = -C_1$

$$a(0) = 1 = \frac{i\hbar}{\alpha}C_1\cdot\frac{i}{2\hbar}\left(-(E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) + \frac{i\hbar}{\alpha}\cdot\frac{-i}{2\hbar}C_2\cdot\left((E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) - \frac{E_R}{\alpha}C_1 - \frac{E_R}{\alpha}C_2 =$$

$$= \frac{-C_1}{2\alpha}\left(-(E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) - \frac{C_1}{2\alpha}\left((E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) - \cancel{\frac{E_R}{\alpha}C_1} + \cancel{\frac{E_R}{\alpha}C_1} =$$

$$= -\frac{C_1}{\alpha}\sqrt{(E_R+E_L)^2+(2\alpha)^2} = 1 \Rightarrow C_1 = \frac{-\alpha}{\sqrt{(E_R+E_L)^2+(2\alpha)^2}}\ ; \ C_2 = -C_1 = \frac{\alpha}{\sqrt{(E_R+E_L)^2+(2\alpha)^2}}$$

$$b(t) = \frac{\alpha}{\sqrt{(E_R+E_L)^2+(2\alpha)^2}}\left[-\exp\left(\frac{it}{2\hbar}\left(-(E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right)\right) + \exp\left(-\frac{it}{2\hbar}\left((E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right)\right)\right]$$

$$a(t) = C_3\exp\left(\frac{it}{2\hbar}\left(-(E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right)\right) + C_4\exp\left(\frac{-it}{2\hbar}\left(+(E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right)\right)$$

$a(0) = 1 \Rightarrow C_3 + C_4 = 1 \Rightarrow C_4 = 1 - C_3$

$$b(0) = 0 \Rightarrow \left(\frac{i\hbar}{\alpha}\dot a(t) - \frac{E_L}{\alpha}a(t)\right)\Big|_{t=0} = -\frac{C_3}{2\alpha}\left(-(E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) + \frac{1-C_3}{2\alpha}\left((E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) - \frac{E_L}{\alpha}C_3 - \frac{E_L}{\alpha}(1-C_3) =$$

$$= -\frac{C_3}{\alpha}\sqrt{(E_R+E_L)^2+(2\alpha)^2} + \frac{1}{2\alpha}\left((E_R+E_L) + \sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) - \frac{E_L}{\alpha} = 0 \Rightarrow C_3 = \frac{+1}{2}\left(\frac{(E_R-E_L)}{\sqrt{(E_R+E_L)^2+(2\alpha)^2}} + 1\right)$$

$$C_4 = 1 - C_3 = \frac{1}{2}\left(1 - \frac{(E_R-E_L)}{\sqrt{(E_R+E_L)^2+(2\alpha)^2}}\right)$$

$D_{\sigma_1} = \langle\sigma_1^2\rangle - \langle\sigma_1\rangle^2 = \langle 1\rangle - \langle\sigma_1\rangle^2 = 1-\sin^2(2\omega t) = \cos^2(2\omega t)$

$$\Psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \Rightarrow P_{|R\rangle} = \left|\langle R|\Psi(t)\rangle\right|^2 = \left|\langle (0\ 1)|\Psi(t)\rangle\right|^2 = |b(t)|^2 =$$

$$= \left|\frac{\alpha^2}{(E_R+E_L)^2+(2\alpha)^2}\left[\exp\left(\frac{-it}{\hbar}\left(E_R+E_L+\sqrt{(E_R+E_L)^2+(2\alpha)^2}\right)\right) - 2\exp\left(\frac{-it}{\hbar}(E_R+E_L)\right) + \exp\left(\frac{it}{\hbar}\left(-(E_R+E_L)+\sqrt{(E_R+E_L)^2+(2\alpha)^2}\right)\right)\right]\right|$$

бб')

$$b(t) = C_1\exp\left(\frac{it}{2\hbar}\left(-(E_R+E_L)+\sqrt{(E_R+E_L)^2+(2\alpha)^2}\right)\right) + C_2\exp\left(\frac{-it}{2\hbar}\left((E_R+E_L)+\sqrt{(E_R+E_L)^2+(2\alpha)^2}\right)\right)$$

$b(0) = b_0 \Rightarrow C_1 + C_2 = b_0 \Rightarrow C_2 = b_0 - C_1$

$$a(0) = a_0 = \frac{i\hbar}{\alpha}\dot b(0) - \frac{E_R}{\alpha}b(0) = \frac{-C_1}{2\alpha}\left(-(E_R+E_L)+\sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) + \frac{(b_0-C_1)}{2\alpha}\left(E_R+E_L+\sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) - \frac{E_R}{\alpha}C_1 - \frac{E_R}{\alpha}(b_0-C_1) =$$

$$= \frac{-C_1}{\alpha}\sqrt{(E_R+E_L)^2+(2\alpha)^2} + \frac{b_0}{2\alpha}\left((E_R+E_L)+\sqrt{(E_R+E_L)^2+(2\alpha)^2}\right) - \frac{E_R}{\alpha}b_0 = a_0$$

$$C_1 = \frac{b_0}{2}\left(\frac{(E_L-E_R)}{\sqrt{(E_R+E_L)^2+(2\alpha)^2}}+1\right) - \frac{a_0\alpha}{\sqrt{(E_R+E_L)^2+(2\alpha)^2}}; \quad C_2 = b_0 - C_1 = \frac{b_0}{2}\left(1-\frac{(E_L-E_R)}{\sqrt{(E_R+E_L)^2+(2\alpha)^2}}\right) - \frac{a\alpha}{\sqrt{(E_R+E_L)^2+(2\alpha)^2}}$$

Для $a(t)$ аналогично, но $\begin{cases} E_L \to E_R \\ E_R \to E_L \\ a_0 \to b_0 \\ b \to a_0 \end{cases}$

$$\Psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \Rightarrow P_{|R\rangle} = \left|\langle R|\Psi(t)\rangle\right|^2 = \left|\langle (0\ 1)|\Psi(t)\rangle\right|^2 = |b(t)|^2$$

№6 $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $H = \hbar\omega\sigma_2$ $\quad |\psi(t)\rangle - ?$ $\langle\sigma_1\rangle - ?$ $\langle\sigma_3\rangle - ?$ $D\sigma_1 - ?$ $D\sigma_3 - ?$

$|\psi(t)\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$ $\begin{cases} a(0)=1 \\ b(0)=0 \end{cases}$

$$i\hbar\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \hbar\omega\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$\begin{cases} i\hbar\dot a(t) = \hbar\omega(-ib(t)) \\ i\hbar\dot b(t) = \hbar\omega(ia(t)) \end{cases} \Rightarrow \frac{\dot b(t)}{\omega} = a(t) \Rightarrow \frac{\ddot b(t)}{\omega} = -\omega b(t) \Rightarrow \ddot b(t) + \omega^2 b(t) = 0$$

$b(t) = C_1\cos(\omega t) + C_2\sin(\omega t)$

$b(0) = 0 \Rightarrow C_1 = 0$ $\qquad \Rightarrow b(t) = \sin(\omega t)$

$\frac{1}{\omega}\dot b(t)\big|_{t=0} = a(t)\big|_{t=0} = 1 = \frac{1}{\omega}\cdot C_2\cdot\omega\cdot\cos(\omega t)\big|_{t=0} \Rightarrow C_2 = 1$

$a(t) = C_3\cos(\omega t) + C_4\sin(\omega t)$

$a(0) = 1 \Rightarrow C_3 = 1$

$-\frac{1}{\omega}\dot a(t)\big|_{t=0} = b(t)\big|_{t=0} = 0 = \frac{-1}{\omega}\cdot\frac{-1}{\omega}\cdot\sin(\omega t) - \frac{1}{\omega}C_4\,\omega\cos(\omega t)\big|_{t=0} = -C_4 \cdot 1 = 0 \Rightarrow C_4 = 0 \Rightarrow a(t) = \cos(\omega t)$

$|\psi(t)\rangle = \begin{pmatrix} \cos(\omega t) \\ \sin(\omega t) \end{pmatrix}$

$$\langle\sigma_1\rangle = \langle\psi(t)|\sigma_1|\psi(t)\rangle = \left\langle \cos(\omega t)\ \sin(\omega t)\right|\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}\left|\begin{matrix} \cos(\omega t) \\ \sin(\omega t) \end{matrix}\right\rangle =$$

$$= \left\langle \cos\omega t\ \sin\omega t\right|\left.\begin{matrix} \sin(\omega t) \\ \cos(\omega t) \end{matrix}\right\rangle = 2\sin(\omega t)\cos(\omega t) = \sin(2\omega t)$$

$$\mathcal{D}\sigma_1 = \langle \underbrace{\sigma_1^2}_{\mathbb{1}} \rangle - \langle \sigma_1 \rangle^2 = \langle \mathbb{1} \rangle - \langle \sigma_1^* \rangle^2 = 1 - \sin^2(2\omega t) = \cos^2(2\omega t)$$

$$\langle \sigma_3 \rangle = \langle \psi(t) | \sigma_3 | \psi(t) \rangle = \langle \cos(\omega t)\ \sin(\omega t) | \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\omega t) \\ \sin(\omega t) \end{pmatrix} \rangle = \langle \cos(\omega t)\ \sin(\omega t) | \begin{pmatrix} -\cos(\omega t) \\ \sin(\omega t) \end{pmatrix} \rangle =$$

$$= -\cos^2(\omega t) + \sin^2(\omega t) = -\cos(2\omega t)$$

$$\mathcal{D}\sigma_3 = \langle \underbrace{\sigma_3^2}_{\mathbb{1}} \rangle - \langle \sigma_1 \rangle^2 = \langle \mathbb{1} \rangle - \langle \sigma_3 \rangle^2 = 1 - (-\cos 2\omega t)^2 = 1 - \cos^2(2\omega t) = \sin^2(2\omega t)$$

№ 7 $\quad H = \begin{pmatrix} a & b \\ b^* & c \end{pmatrix} = U \underbrace{\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}}_{h} U^+ \qquad U = e^{i\alpha} \begin{pmatrix} \cos\theta\, e^{i\varphi} & -\sin\theta\, e^{i\xi} \\ \sin\theta\, e^{-i\xi} & \cos\theta\, e^{-i\varphi} \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$P_{12} = |\langle 1 | e^{\frac{i}{\hbar} H t} | 2 \rangle|^2 - ?$$

$$P_{21} = |\langle 2 | e^{\frac{i}{\hbar} H t} | 1 \rangle|^2 - ?$$

$$H \cdot H = U h \underbrace{U^+ U}_{\mathbb{1}} h U^+ = U h^2 U^+$$

$$H^n = U h^n U^+$$

$$e^{\frac{i}{\hbar} H t} = U\left(1 + \frac{\left(\frac{i}{\hbar} h t\right)^1}{1!} + \frac{\left(\frac{i}{\hbar} h t\right)^2}{2!} + \dots\right) U^+ \to \begin{pmatrix} e^{\frac{i}{\hbar} E_1 t} & 0 \\ 0 & e^{\frac{i}{\hbar} E_2 t} \end{pmatrix}, \quad h^n = \begin{pmatrix} E_1^n & 0 \\ 0 & E_2^n \end{pmatrix}$$

$$\langle 1 | U \begin{pmatrix} e^{\frac{i}{\hbar} E_1 t} & 0 \\ 0 & e^{\frac{i}{\hbar} E_2 t} \end{pmatrix} U^+ | 2 \rangle$$

$$U^+ |2\rangle = e^{-i\alpha} \underbrace{\begin{pmatrix} \cos\theta\, e^{-i\varphi} & +\sin\theta\, e^{i\xi} \\ -\sin\theta\, e^{-i\xi} & \cos\theta\, e^{i\varphi} \end{pmatrix}}_{2\times 2} \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{2\times 1} = e^{-i\alpha} \begin{pmatrix} +\sin\theta\, e^{i\xi} \\ \cos\theta\, e^{i\varphi} \end{pmatrix}$$

$$\langle 2 | U = e^{i\alpha} \left(\sin\theta\, e^{-i\xi}\ \ \cos\theta\, e^{-i\varphi} \right)$$

$$U^+ |1\rangle = e^{-i\alpha} \begin{pmatrix} \cos\theta\, e^{-i\varphi} & \sin\theta\, e^{i\xi} \\ -\sin\theta\, e^{-i\xi} & \cos\theta\, e^{i\varphi} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = e^{-i\alpha} \begin{pmatrix} \cos\theta\, e^{-i\varphi} \\ -\sin\theta\, e^{-i\xi} \end{pmatrix}$$

$$\langle 1 | U = e^{i\alpha} \left(\cos\theta\, e^{+i\varphi}\ \ -\sin\theta\, e^{i\xi} \right)$$

$$\begin{pmatrix} e^{\frac{i}{\hbar} E_1 t} & 0 \\ 0 & e^{\frac{i}{\hbar} E_2 t} \end{pmatrix} \cdot e^{-i\alpha} \begin{pmatrix} \sin\theta\, e^{i\xi} \\ \cos\theta\, e^{i\varphi} \end{pmatrix} = e^{-i\alpha} \begin{pmatrix} \sin\theta\, e^{i\xi + \frac{i}{\hbar} E_1 t} \\ \cos\theta\, e^{i\varphi + \frac{i}{\hbar} E_2 t} \end{pmatrix}$$

$$e^{i\alpha} \left(\cos\theta\, e^{i\varphi}\ \ -\sin\theta\, e^{i\xi} \right) e^{-i\alpha} \begin{pmatrix} \sin\theta\, e^{i\xi + \frac{i}{\hbar} E_1 t} \\ \cos\theta\, e^{i\varphi + \frac{i}{\hbar} E_2 t} \end{pmatrix} = \cos\theta \sin\theta\, e^{i\varphi + i\xi + \frac{i}{\hbar} E_1 t} - \cos\theta \sin\theta\, e^{i\varphi + i\xi + \frac{i}{\hbar} E_2 t} =$$

$$= \cos\theta \sin\theta\, e^{i(\varphi+\xi)} \left(e^{\frac{i}{\hbar} E_1 t} - e^{\frac{i}{\hbar} E_2 t} \right) \Rightarrow P_{12} = \cos^2\theta \sin^2\theta\, e^{2i(\varphi+\xi)} \left(e^{\frac{i}{\hbar} E_1 t} - e^{\frac{i}{\hbar} E_2 t} \right)^2$$

$$\langle 2 | U \begin{pmatrix} e^{\frac{i}{\hbar} E_1 t} & 0 \\ 0 & e^{\frac{i}{\hbar} E_2 t} \end{pmatrix} U^+ | 1 \rangle$$

$$\begin{pmatrix} e^{\frac{i}{\hbar} E_1 t} & 0 \\ 0 & e^{\frac{i}{\hbar} E_2 t} \end{pmatrix} e^{-i\alpha} \begin{pmatrix} \cos\theta\, e^{-i\varphi} \\ -\sin\theta\, e^{-i\xi} \end{pmatrix} = e^{-i\alpha} \begin{pmatrix} \cos\theta\, e^{-i\varphi + \frac{i}{\hbar} E_1 t} \\ -\sin\theta\, e^{-i\xi + \frac{i}{\hbar} E_2 t} \end{pmatrix}$$

$$e^{i\alpha} \left(\sin\theta\, e^{-i\xi} \cdot \cos\theta\, e^{-i\varphi} \right) e^{-i\alpha} \begin{pmatrix} \cos\theta\, e^{-i\varphi + \frac{i}{\hbar} E_1 t} \\ -\sin\theta\, e^{-i\xi + \frac{i}{\hbar} E_2 t} \end{pmatrix} = \sin\theta \cos\theta\, e^{-i(\xi+\varphi) + \frac{i}{\hbar} E_1 t} - \sin\theta \cos\theta\, e^{-i(\xi+\varphi) + \frac{i}{\hbar} E_2 t} = \cos\theta \sin\theta\, e^{-i(\xi+\varphi)} \left(e^{\frac{iE_1 t}{\hbar}} - e^{\frac{iE_2 t}{\hbar}} \right)$$

$$P_{21} = \cos^2\theta \sin^2\theta\, e^{-2i(\xi+\varphi)} \left(e^{\frac{iE_1 t}{\hbar}} - e^{\frac{iE_2 t}{\hbar}} \right)^2$$

Сравнить?

$$\frac{P_{12}}{P_{21}} = e^{4i(\xi+\varphi)}; \quad P_{12} - P_{21} = \sin^2\theta \cos^2\theta \left(e^{\frac{iE_1 t}{\hbar}} - e^{\frac{iE_2 t}{\hbar}} \right)^2 \left[e^{2i(\xi+\varphi)} - e^{-2i(\xi+\varphi)} \right] =$$

$$= \frac{1}{8} \sin^2(2\theta) \left(e^{\frac{iE_1 t}{\hbar}} - e^{\frac{iE_2 t}{\hbar}} \right)^2 \cdot \mathrm{Sh}(2i(\xi+\varphi)) =$$

$$= -\frac{i}{8} \sin^2(2\theta) \left(e^{\frac{iE_1 t}{\hbar}} - e^{\frac{iE_2 t}{\hbar}} \right)^2 \sin(2(\xi+\varphi)) =$$

Дз 7.04.14

№1 3^x уровневая система.

$$|1\rangle = \frac{1}{\sqrt3}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad |2\rangle = \frac{1}{2}\begin{pmatrix}1\\i\\-1-i\end{pmatrix} \qquad H = \begin{pmatrix}E_1 & & 0\\ & E_2 & \\ 0 & & E_3\end{pmatrix}$$

$P_{12}(t)$ - ?
$P_{21}(t)$ - ?
$P_{12}-P_{21}$ - ?

$$e^{\frac{iHt}{\hbar}} = \begin{pmatrix} e^{\frac{iE_1t}{\hbar}} & 0 & 0\\ 0 & e^{\frac{iE_2t}{\hbar}} & 0\\ 0 & 0 & e^{\frac{iE_3t}{\hbar}}\end{pmatrix}$$

$$P_{12} = |\langle 1| e^{\frac{i}{\hbar}Ht}|2\rangle|^2$$

$\frac{E_1t}{\hbar}=\omega_1$
$\frac{E_2t}{\hbar}=\omega_2$
$\frac{E_3t}{\hbar}=\omega_3$

$$e^{\frac{iHt}{\hbar}}|2\rangle = \frac12 \underset{3\times3}{\begin{pmatrix} e^{\frac{iE_1t}{\hbar}} & 0 & 0\\ 0 & e^{\frac{iE_2t}{\hbar}} & 0\\ 0 & 0 & e^{\frac{iE_3t}{\hbar}}\end{pmatrix}} \underset{3\times1}{\begin{pmatrix}1\\i\\-1-i\end{pmatrix}} = \frac12\begin{pmatrix} e^{\frac{iE_1t}{\hbar}}\\ ie^{\frac{iE_2t}{\hbar}}\\ (-1-i)e^{\frac{iE_3t}{\hbar}}\end{pmatrix}$$

$$\langle 1| e^{\frac{iHt}{\hbar}}|2\rangle = \frac{1}{2\sqrt3}\underset{1\times3}{\begin{pmatrix}1&1&1\end{pmatrix}} \underset{3\times1}{\begin{pmatrix} e^{\frac{iE_1t}{\hbar}}\\ ie^{\frac{iE_2t}{\hbar}}\\ (-1-i)e^{\frac{iE_3t}{\hbar}}\end{pmatrix}} = \frac{1}{2\sqrt3}\left[e^{\frac{iE_1t}{\hbar}} - e^{\frac{iE_3t}{\hbar}} + i\left(e^{\frac{iE_2t}{\hbar}} - e^{\frac{iE_3t}{\hbar}}\right)\right]$$

$$P_{12} = |\langle1|e^{\frac{iHt}{\hbar}}|2\rangle|^2 = \frac1{12}\left[\left(e^{\frac{iE_1t}{\hbar}} - e^{\frac{iE_3t}{\hbar}}\right) + i\left(e^{\frac{iE_2t}{\hbar}} - e^{\frac{iE_3t}{\hbar}}\right)\right]\left[\left(e^{\frac{-iE_1t}{\hbar}} - e^{\frac{-iE_3t}{\hbar}}\right) - i\left(e^{-\frac{iE_2t}{\hbar}} - e^{\frac{-iE_3t}{\hbar}}\right)\right] =$$

$$= \frac1{12}\left[\left(e^{\frac{iE_1t}{\hbar}} - e^{\frac{iE_3t}{\hbar}}\right)\left(e^{\frac{-iE_1t}{\hbar}} - e^{\frac{-iE_3t}{\hbar}}\right) - i\left(e^{\frac{iE_1t}{\hbar}} - e^{\frac{iE_3t}{\hbar}}\right)\left(e^{-\frac{iE_2t}{\hbar}} - e^{-\frac{iE_3t}{\hbar}}\right) + i\left(e^{\frac{iE_2t}{\hbar}} - e^{\frac{+iE_3t}{\hbar}}\right)\left(e^{-\frac{iE_1t}{\hbar}} - e^{-\frac{iE_3t}{\hbar}}\right) + \left(e^{\frac{iE_2t}{\hbar}} - e^{\frac{iE_3t}{\hbar}}\right)\left(e^{-\frac{iE_2t}{\hbar}} - e^{-\frac{iE_3t}{\hbar}}\right)\right] =$$

$$= \frac1{12}\left[\left(2 - e^{i(\omega_1-\omega_3)} - e^{i(\omega_3-\omega_1)}\right) - i\left(1 + e^{i(\omega_1-\omega_2)} - e^{i(\omega_1-\omega_3)} - e^{i(\omega_3-\omega_2)}\right) + i\left(1 + e^{i(\omega_2-\omega_1)} - e^{i(\omega_2-\omega_3)} - e^{i(\omega_3-\omega_1)}\right) + \left(2 - e^{i(\omega_2-\omega_3)} - e^{i(\omega_3-\omega_2)}\right)\right] =$$

$$= \frac1{12}\left[(2-2\cos(\omega_1-\omega_3)) + (2-2\cos(\omega_2-\omega_3)) + i\left(e^{i(\omega_2-\omega_1)} - e^{-i(\omega_2-\omega_1)}\right) + i\left(e^{i(\omega_1-\omega_3)} - e^{-i(\omega_1-\omega_3)}\right) + i\left(e^{i(\omega_3-\omega_2)} - e^{-i(\omega_3-\omega_2)}\right)\right] =$$

$$= \frac1{12}\left[\sin^2\left(\frac{\omega_1-\omega_3}{2}\right) + \sin^2\left(\frac{\omega_2-\omega_3}{2}\right) - 2\sin(\omega_2-\omega_1) - 2\sin(\omega_1-\omega_3) - 2\sin(\omega_3-\omega_2)\right]$$

$$e^{\frac{iHt}{\hbar}}|1\rangle = \frac1{\sqrt3}\begin{pmatrix} e^{i\omega_1} & 0 & 0\\ 0 & e^{i\omega_2} & 0\\ 0 & 0 & e^{i\omega_3}\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = \frac1{\sqrt3}\begin{pmatrix}e^{i\omega_1}\\e^{i\omega_2}\\e^{i\omega_3}\end{pmatrix}$$

$$\langle2|e^{\frac{iHt}{\hbar}}|1\rangle = \frac1{2\sqrt3}\begin{pmatrix}1 & -i & -1+i\end{pmatrix}\begin{pmatrix}e^{i\omega_1}\\e^{i\omega_2}\\e^{i\omega_3}\end{pmatrix} = \frac1{2\sqrt3}\left(\left(e^{i\omega_1} - e^{i\omega_3}\right) + i\left(e^{i\omega_3} - e^{i\omega_2}\right)\right)$$

$$P_{21} = |\langle2|e^{\frac{iHt}{\hbar}}|1\rangle|^2 = \frac1{12}\left[\left(e^{i\omega_1} - e^{i\omega_3}\right) + i\left(e^{i\omega_3} - e^{i\omega_2}\right)\right]\left[\left(e^{-i\omega_1} - e^{-i\omega_3}\right) - i\left(e^{-i\omega_3} - e^{-i\omega_2}\right)\right] =$$

$$= \frac1{12}\left[\left(e^{i\omega_1} - e^{i\omega_3}\right)\left(e^{-i\omega_1} - e^{-i\omega_3}\right) - i\left(e^{i\omega_1} - e^{i\omega_3}\right)\left(e^{-i\omega_3} - e^{-i\omega_2}\right) + i\left(e^{i\omega_3} - e^{i\omega_2}\right)\left(e^{-i\omega_1} - e^{-i\omega_3}\right) + \left(e^{i\omega_3} - e^{i\omega_2}\right)\left(e^{-i\omega_3} - e^{-i\omega_2}\right)\right] =$$

$$= \frac1{12}\left[\sin^2\left(\frac{\omega_1-\omega_3}{2}\right) + \sin^2\left(\frac{\omega_3-\omega_2}{2}\right) - i\left(e^{i(\omega_1-\omega_3)} - e^{i(\omega_1-\omega_2)} + e^{i(\omega_3-\omega_2)} - 1\right) + i\left(e^{i(\omega_3-\omega_1)} - e^{i(\omega_2-\omega_1)} + e^{i(\omega_3-\omega_3)} - 1\right)\right] =$$

$$= \frac1{12}\left[\sin^2\left(\frac{\omega_1-\omega_3}{2}\right) + \sin^2\left(\frac{\omega_3-\omega_2}{2}\right) + i\left(e^{i(\omega_3-\omega_1)} - e^{-i(\omega_3-\omega_1)}\right) + i\left(e^{i(\omega_1-\omega_2)} - e^{-i(\omega_1-\omega_2)}\right) + i\left(e^{i(\omega_2-\omega_3)} - e^{-i(\omega_2-\omega_3)}\right)\right] =$$

$$= \frac1{12}\left[\sin^2\left(\frac{\omega_1-\omega_3}{2}\right) + \sin^2\left(\frac{\omega_3-\omega_2}{2}\right) - 2\sin(\omega_3-\omega_1) - 2\sin(\omega_1-\omega_2) - 2\sin(\omega_2-\omega_3)\right]$$

$$P_{12}-P_{21} = \frac1{12}\left[-2\sin(\omega_2-\omega_1) - 2\sin(\omega_1-\omega_3) - 2\sin(\omega_3-\omega_2) + 2\sin(\omega_3-\omega_1) + 2\sin(\omega_1-\omega_2) + 2\sin(\omega_2-\omega_3)\right] =$$

$$= \frac1{12}\left[4\sin(\omega_1-\omega_2) + 4\sin(\omega_3-\omega_1) + 4\sin(\omega_2-\omega_3)\right] =$$

$$= \frac13\left[\sin(\omega_1-\omega_2) + \sin(\omega_3-\omega_1) + \sin(\omega_2-\omega_3)\right]$$

№2 $i\hbar \frac{d}{dt}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi(x,t) + V(x,t)\Psi(x,t)$, $V(x,t)=0$

$$i\hbar \frac{d}{dt} G(x,x',t) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} G(t,x,x')$$

$$G(0,x,x') = \delta(x-x')$$

$$G(t,x,x') = \int dk \frac{e^{ik(x-x')}}{2\pi} f(t,k) \quad , f(0,k)=1$$

$$\int dk \frac{e^{ik(x-x')}}{2\pi} i\hbar \frac{d}{dt} f(k,t) = \int dk \frac{e^{ik(x-x')}}{2\pi}\left(\frac{-\hbar^2}{2m}\right)(-k^2)\cdot f(t,k) \Rightarrow i\hbar\frac{d}{dt}f(t,k) = \frac{+\hbar^2 k^2}{2m} f(t,k) \Rightarrow$$

$$\Rightarrow \frac{df(k,t)}{f(k,t)} = \frac{-\hbar k^2 \cdot i}{2m} dt \Rightarrow \ln f(k,t) = \frac{-\hbar k^2}{2m} it \Rightarrow f(k,t) = 1\exp\left(\frac{-i\hbar k^2 t}{2m}\right)$$

$$G(t,x,x') = \int dk \frac{e^{ik(x-x')}}{2\pi} e^{\frac{-i\hbar k^2}{2m}t}$$

$$\left(ik(x-x') - \frac{i\hbar k^2}{2m}t\right) = -i\left(k^2\cdot\frac{\hbar}{2m}t - 2\cdot\sqrt{\frac{\hbar}{2m}t}\cdot k\cdot(x-x')\cdot\sqrt{\frac{2m}{\hbar t}}\cdot\frac{1}{2} + \frac{(x-x')^2}{4}\left(\frac{2m}{\hbar t}\right)\right) + i\frac{(x-x')^2}{4}\left(\frac{2m}{\hbar t}\right) =$$

$$= -i\left(\sqrt{\frac{\hbar t}{2m}}\cdot k - \frac{(x-x')}{2}\sqrt{\frac{2m}{\hbar t}}\right)^2 + i\frac{(x-x')^2}{4}\frac{2m}{\hbar t}$$

$$G(t,x,x') = \int dk \frac{1}{2\pi}\exp\left(-i\left(\sqrt{\frac{\hbar t}{2m}}k - \frac{(x-x')}{2}\sqrt{\frac{2m}{\hbar t}}\right)^2\right)\exp\left(i\frac{(x-x')^2}{4}\frac{2m}{\hbar t}\right) = \left[\sqrt{\frac{\hbar t}{2m}}k = y;\ dy = \sqrt{\frac{\hbar t}{2m}}dk \Rightarrow dk = \sqrt{\frac{2m}{\hbar t}}dy\right]$$

$$= \frac{1}{2\pi}\exp\left(\frac{i(x-x')^2}{4}\frac{2m}{\hbar t}\right)\cdot\sqrt{\frac{2m}{\hbar t}}\int dy \exp(-i(y-y_0)^2) = \frac{1}{2\pi}\exp\left(\frac{i(x-x')^2}{4}\frac{2m}{\hbar t}\right)\cdot\sqrt{\frac{2m}{\hbar t}}\frac{\sqrt{\pi}}{\sqrt{i}}$$

№3 Г.О.

$|\Psi(t)\rangle = ?$

$\langle x\rangle = \langle\Psi(t)|x|\Psi(t)\rangle = ?$

$$|\Psi(0)\rangle = \sqrt{\tfrac{1}{3}}|\Psi_1\rangle + i\sqrt{\tfrac{2}{3}}|\Psi_2\rangle$$

$E_1 = \hbar\omega(1+1/2)$ $\quad E_2 = \hbar\omega(2+1/2)$

$H_1 = 2x$

$H_2 = 4x^2 - 2$

$$|\Psi(t)\rangle = \sqrt{\tfrac{1}{3}}e^{-\frac{i}{\hbar}E_1 t}|\Psi_1\rangle + i\sqrt{\tfrac{2}{3}}e^{-\frac{i}{\hbar}E_2 t}|\Psi_2\rangle$$

$$\Psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)H_n\left(x\sqrt{\frac{m\omega}{\hbar}}\right)$$

$$\Psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2}}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\cdot 2x\sqrt{\frac{m\omega}{\hbar}} = \sqrt{2}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\cdot x\sqrt{\frac{m\omega}{\hbar}}$$

$$\Psi_2(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{4\cdot 2}}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\cdot\left(4x^2\frac{m\omega}{\hbar} - 2\right) = \frac{1}{\sqrt{2}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\left(2x^2\frac{m\omega}{\hbar} - 1\right)$$

$$|\Psi(t)\rangle = \sqrt{\tfrac{1}{3}}e^{-\frac{i}{\hbar}E_1 t}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\sqrt{2}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)x\sqrt{\frac{m\omega}{\hbar}} + i\sqrt{\tfrac{2}{3}}e^{-\frac{iE_2 t}{\hbar}}\frac{1}{\sqrt{2}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\left(2x^2\frac{m\omega}{\hbar} - 1\right) =$$

$$= \sqrt{\tfrac{1}{3}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\left[\sqrt{2}e^{-\frac{iE_1 t}{\hbar}}\sqrt{\frac{m\omega}{\hbar}}x + ie^{-\frac{iE_2 t}{\hbar}}\left(2x^2\frac{m\omega}{\hbar} - 1\right)\right]$$

$$\langle\Psi(t)| = \sqrt{\tfrac{1}{3}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\left[\sqrt{2}e^{\frac{iE_1 t}{\hbar}}\sqrt{\frac{m\omega}{\hbar}}x - ie^{\frac{iE_2 t}{\hbar}}\left(2x^2\frac{m\omega}{\hbar} - 1\right)\right]$$

$\langle\Psi(t)|x|\Psi(t)\rangle =$ ^ω^

Другой вариант

$H=\frac{1}{2m}p^2+\frac{1}{2}m\omega^2q^2$, $[q,p]=i\hbar E$; $p_0=\sqrt{\hbar m\omega}$, $q_0=\sqrt{\frac{\hbar}{m\omega}}$, $p_0q_0=\hbar$

$a=\frac{1}{\sqrt2}\left(\frac{q}{q_0}+i\frac{p}{p_0}\right)$; $a^+=\frac{1}{\sqrt2}\left(\frac{q}{q_0}-i\frac{p}{p_0}\right)$; $[a,a^+]=E \Rightarrow q=\frac{q_0}{\sqrt2}(a+a^+)$, $p=\frac{p_0}{i\sqrt2}(a-a^+)$

$H=\hbar\omega\left(N+\frac{1}{2}E\right)$, где $N=a^+a$

$|\psi_n\rangle=(a^+)^n|\psi_0\rangle\frac{1}{\sqrt{n!}}$ $\Rightarrow |\psi_1\rangle=a^+|\psi_0\rangle$ $\qquad a\psi_0=0$

$|\psi_2\rangle=(a^+)^2|\psi_0\rangle\frac{1}{\sqrt2}$ $\qquad \langle\psi_0|\psi_0\rangle=1$

$$\langle\psi(t)|x|\psi(t)\rangle=\frac{1}{3}\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\exp\left(-\frac{m\omega}{\hbar}x^2\right)\int_{-\infty}^{+\infty}\left[\sqrt2 e^{-i\omega_1t}\sqrt{\frac{m\omega}{\hbar}}x+ie^{-i\omega_2t}\left(2x^2\frac{m\omega}{\hbar}-1\right)\right]\cdot x\cdot\left[\sqrt2 e^{i\omega_1t}\sqrt{\frac{m\omega}{\hbar}}x-ie^{i\omega_2t}\left(2x^2\frac{m\omega}{\hbar}-1\right)\right]dx=$$

$$=\frac{1}{3}\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{+\infty}x\exp\left(-\frac{m\omega}{\hbar}x^2\right)\left[2x^2\left(\frac{m\omega}{\hbar}\right)+\left(2x^2\frac{m\omega}{\hbar}-1\right)^2-i\sqrt2\sqrt{\frac{m\omega}{\hbar}}e^{i(\omega_2-\omega_1)t}\left(2x^3\frac{m\omega}{\hbar}-x\right)+i\sqrt2\sqrt{\frac{m\omega}{\hbar}}e^{i(\omega_1-\omega_2)}\left(2x^3\frac{m\omega}{\hbar}-x\right)\right]dx=$$

$$=\frac{1}{3}\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{+\infty}x\cdot\exp\left(-\frac{m\omega}{\hbar}x^2\right)\left[2x^2\left(\frac{m\omega}{\hbar}\right)+\left(4x^4\left(\frac{m\omega}{\hbar}\right)^2-4x^2\frac{m\omega}{\hbar}+1\right)-i\sqrt2\sqrt{\frac{m\omega}{\hbar}}\left(2x^3\frac{m\omega}{\hbar}-x\right)\left(e^{i(\omega_2-\omega_1)t}-e^{-i(\omega_2-\omega_1)t}\right)\right]dx=$$

$$=\frac{1}{3}\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{+\infty}\exp\left(-\frac{m\omega}{\hbar}x^2\right)\left[2x^3\frac{m\omega}{\hbar}+\left(4x^5\left(\frac{m\omega}{\hbar}\right)^2-4x^3\frac{m\omega}{\hbar}+x\right)+2\sqrt2\sqrt{\frac{m\omega}{\hbar}}\left(2x^4\frac{m\omega}{\hbar}-x^2\right)\cdot\sin((\omega_2-\omega_1)t)\right]dx=$$

$$=\frac{1}{3}\sqrt{\frac{m\omega}{\pi\hbar}}\cdot2\sqrt2\sqrt{\frac{m\omega}{\hbar}}\cdot\sin[(\omega_2-\omega_1)t]\cdot\int_{-\infty}^{+\infty}\left(2x^4\frac{m\omega}{\hbar}-x^2\right)\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx=$$

$$=\frac{2\sqrt2}{3}\frac{m\omega}{\hbar\sqrt\pi}\sin((\omega_2-\omega_1)t)\cdot\left[-\frac{1}{2}\sqrt\pi\cdot\left(\frac{m\omega}{\hbar}\right)^{-3/2}+2\frac{m\omega}{\hbar}\cdot\frac{3}{4}\sqrt\pi\cdot\left(\frac{m\omega}{\hbar}\right)^{-5/2}\right]=$$

$$=\frac{2\sqrt2}{3}\sin((\omega_2-\omega_1)t)\left[-\frac{1}{2}\cdot\sqrt{\frac{\hbar}{m\omega}}+2\cdot\frac{3}{4}\sqrt{\frac{\hbar}{m\omega}}\right]=\frac{2\sqrt2}{3}\sin((\omega_2-\omega_1)t)\left[\sqrt{\frac{\hbar}{m\omega}}\right]$$

$$=\frac{2\sqrt2}{3}\sin\left[\frac{E_2t}{\hbar}-\frac{E_1t}{\hbar}\right]\cdot\sqrt{\frac{\hbar}{m\omega}}$$

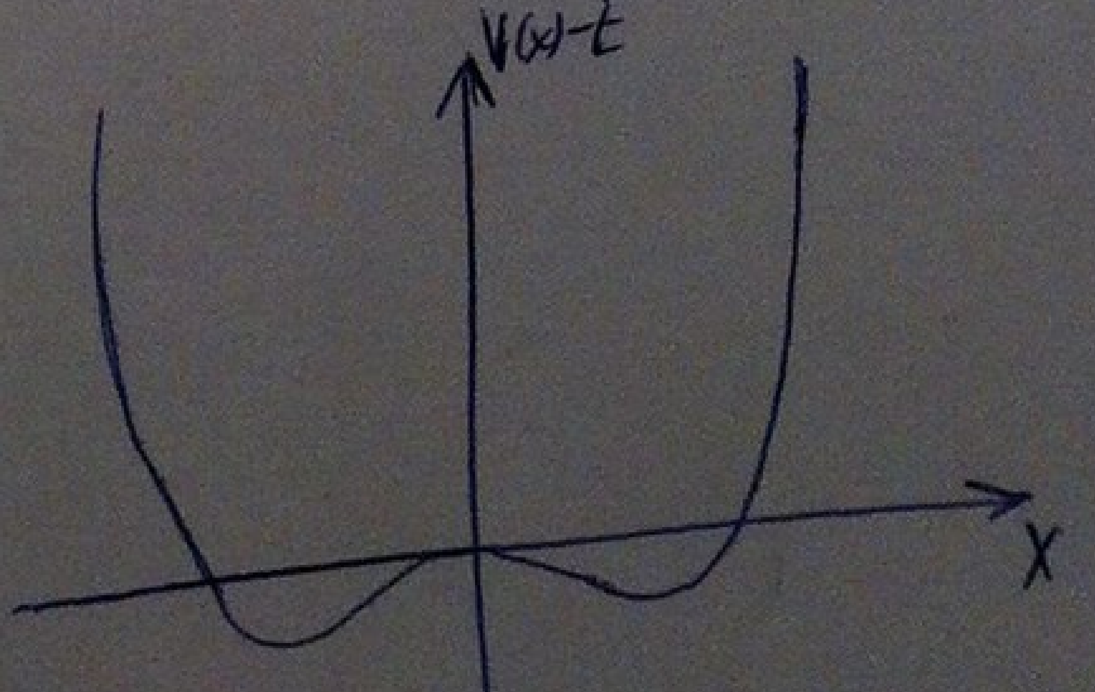
№ 4 ВФ осн. сост. $\psi(x)=Ne^{-\frac{x^4}{4a^4}}$ $\qquad V(x)=?$

$\psi''+\frac{2m}{\hbar^2}(E-V(x))\psi=0$ $\qquad \psi'=N\left(-\frac{x^3}{a^4}\right)\exp\left(-\frac{x^4}{4a^4}\right)$

$$\psi''=N\exp\left(-\frac{x^4}{4a^4}\right)\cdot\left(-\frac{3x^2}{a^4}\right)+N\left(\frac{x^6}{a^8}\right)\exp\left(-\frac{x^4}{4a^4}\right)=\psi(x)\cdot\left[\frac{x^6}{a^8}-\frac{3x^2}{a^4}\right]$$

$$\psi(x)\left[\frac{x^6}{a^8}-\frac{3x^2}{a^4}\right]+\frac{2m}{\hbar^2}(E-V(x))\psi(x)=0$$

$$E-V(x)=\left(\frac{3x^2}{a^4}-\frac{x^6}{a^8}\right)\frac{\hbar^2}{2m}\Rightarrow V(x)-E=\frac{\hbar^2}{2m}\left(\frac{x^6}{a^8}-\frac{3x^2}{a^4}\right)$$



№5 $\Delta z = ?$
$E_0 = ?$

$$\Delta z \cdot \Delta p = \frac{\hbar}{2} \qquad \frac{mv^2}{2} = mg\Delta z \Rightarrow v = \sqrt{2g\Delta z} \Rightarrow \Delta z \cdot m\sqrt{2g\Delta z} = \frac{\hbar}{2} \Rightarrow \Delta z^3 = \frac{\hbar^2}{8gm^2}$$

$$\Delta p = m \cdot v$$

$$\Delta z = \sqrt[3]{\frac{\hbar^2}{8gm^2}} = \sqrt[3]{\frac{\hbar^2 c^4}{8g(mc^2)^2}}$$

$$\Delta z = \sqrt[3]{\frac{(200\,МэВ\cdot фм)^2 \cdot (3\cdot10^8\,\frac{м}{с})^2}{8\cdot 9{,}8\,\frac{м}{с^2}\cdot(939{,}6\,МэВ)^2}} = \sqrt[3]{\frac{200^2\cdot 9\cdot 10^{-14}}{8\cdot 9{,}8\cdot 939{,}6^2}} = \sqrt[3]{\frac{200^2\cdot 0{,}09\cdot}{8\cdot 9{,}8\cdot 939{,}6^2}}\cdot 10^{-4} = 0{,}037327\cdot10^{-4}\,м =$$
$$= 3{,}7\cdot мкм$$

$$p = \sqrt{2mE} = \sqrt{2m^2 g\Delta z} \Rightarrow E_0 = mg\Delta z = \frac{mc^2 g\Delta z}{c^2} = \frac{939{,}6\,МэВ\cdot 9{,}8\cdot 3{,}7\cdot10^{-6}}{(3\cdot10^8)^2} = 378{,}6\cdot10^{-15}\,эВ$$
$$\approx 0{,}38\cdot10^{-12}\,эВ$$

**

$$\begin{cases} \frac{\hbar^2}{2m}\frac{d^2\psi(z)}{dz^2} + (mgz)\psi(z) = E\psi(z) \\ \psi(z) = 0 \\ \psi(z) \underset{z\to\infty}{\to} 0 \end{cases} \Rightarrow \quad R = \left(\frac{\hbar^2}{2m^2 g}\right)^{1/3}, \quad \xi = \frac{z}{R}, \quad \varepsilon = \frac{E}{mgR}$$

$$\left(-\frac{\hbar^2}{2m}\frac{1}{R^2}\frac{\partial^2}{\partial\xi^2} + mgR\xi\right)\psi(\xi) = mgR\varepsilon\psi(\xi)$$

$$\left(-\frac{\hbar^2}{2m}\frac{1}{R^2}\frac{\partial^2}{\partial\xi^2} + mgR(\xi-\varepsilon)\right)\psi(\xi) = 0$$

$$\left(-\frac{\partial^2}{\partial\xi^2} + (\xi-\varepsilon)\right)\psi(\xi) = 0$$

$$\psi''(\xi) - (\xi-\varepsilon)\psi(\xi) = 0$$

A_i, B_i – функции Эйри

$$\begin{cases} \psi = C_1 A_i(\xi-\varepsilon) + C_2 B_i(\xi-\varepsilon) \\ \psi(\xi) = 0 \quad \xi \le 0 \\ \psi(\xi) \to 0 \quad \xi \to +\infty \end{cases}$$

т.к. $\lim\limits_{\xi\to+\infty} B_i(\xi-\varepsilon) = +\infty \Rightarrow C_2 = 0$

$$A_i(-\varepsilon) = 0$$

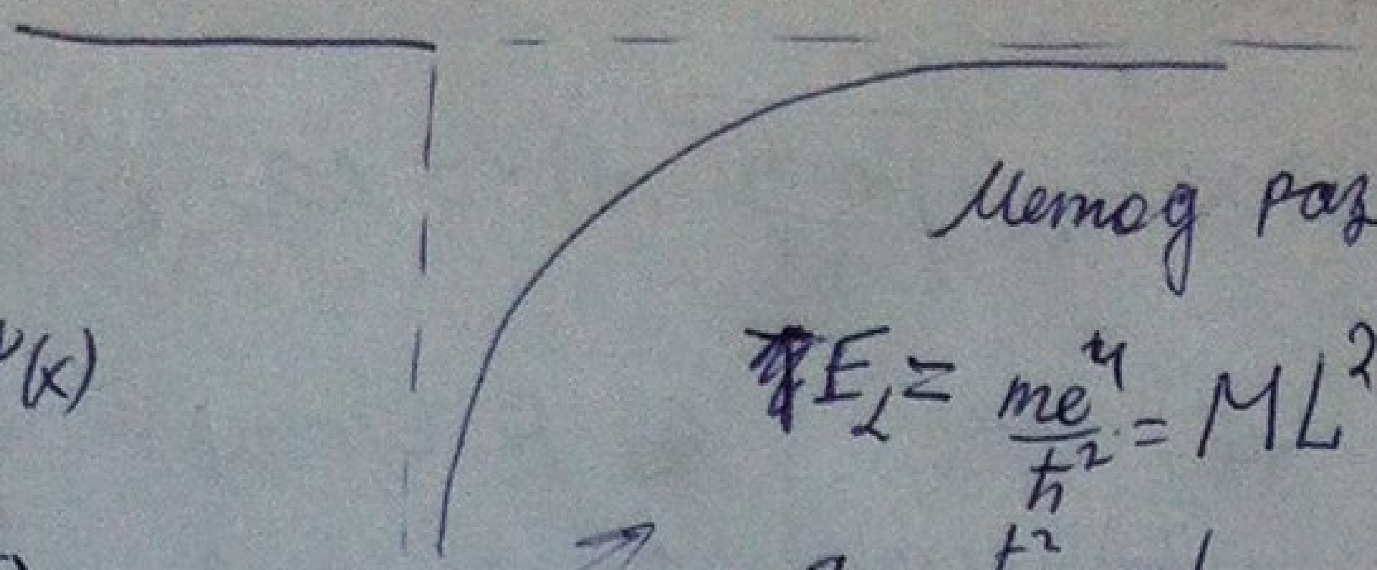
т.к. $\varepsilon = \frac{E}{mgR}$ – спектр дискретный

$$\psi(\xi) = C\cdot A_i(\xi-\varepsilon)$$

C – нормализующая константа

ДЗ на 14.04.14 (1)

$$V(x)=\begin{cases} x<0, & 0\\ x>0, & -\frac{\alpha}{\sqrt{x}}\end{cases}$$



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - \frac{\alpha}{\sqrt{x}}\psi(x)=E\psi(x)$$

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \left(\frac{\alpha}{\sqrt{x}}+E\right)\psi(x)=0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E+\frac{\alpha}{\sqrt{x}}\right)\psi(x)=0$$

Метод размерностей

$$E_\alpha = \frac{me^4}{\hbar^2} = ML^2T^{-2}$$

$$a_0 = \frac{\hbar^2}{me^2} = L$$

размерност x — м(см)

поэтому $\sqrt{x}\sim L^{1/2}$

$$\frac{\alpha}{\sqrt{x}}=E \Rightarrow \alpha = E\cdot\sqrt{x}$$

$$[\alpha]=[E]\cdot L^{1/2} = ML^{5/2}T^{-2}$$

$$ML^2T^{-2} = [E] = m^x\alpha^y\hbar^z = M^x(ML^{5/2}T^{-2})^y(ML^2T^{-1})^z =$$

$$\boxed{\begin{aligned} e&=M^{1/2}L^{3/2}T^{-1}\\ \hbar&=ML^2T^{-1}\\ c&=LT^{-1}\end{aligned}}$$

$$= M^x\cdot M^y\cdot M^z L^{5/2y}L^{2z}T^{-2y}T^{-z} = M^{x+y+z}L^{5/2y+2z}T^{-2y-z}$$

Приравниваем коэф-ты

$$\begin{cases} x+y+z=1\\ 5/2y+2z=2\\ -2y-z=-2\end{cases} \Rightarrow \begin{cases} x+y+z=1\\ 5y+4z=4\\ 2y+z=2\end{cases} \Rightarrow 3y=4 \Rightarrow y=4/3$$

$$z=2-8/3=-2/3$$

$$x=1-4/3+2/3=1/3$$

$$[E]=m^{1/3}\cdot\alpha^{4/3}\cdot\hbar^{-2/3} = \left(\frac{m\alpha^4}{\hbar^2}\right)^{1/3}$$

---

(2) Может ли быть связ сост с $E=0$ и $V(x\to+\infty)=0$?

$$-\frac{\hbar^2}{2m}\psi''(x) + (V(x)-E)\psi(x)=0$$ при быстром $V(x)\xrightarrow[x\to\pm\infty]{}0$ и $E=0$ УШ:

$$-\frac{\hbar^2}{2m}\psi''(x)=0 \Rightarrow \psi''(x)=0\Rightarrow \psi=C_1x+C_2,\ x\to\pm\infty$$ решение возрастает при $x\to\pm\infty$

При $E_n=|E_n|$

$$\psi''+\frac{2m|E_n|}{\hbar^2}\psi=0$$

$$\psi = C_1\exp\left(\sqrt{\frac{2m|E_n|}{\hbar^2}}|x|\right)+C_2\exp\left(-\sqrt{\frac{2m|E_n|}{\hbar^2}}|x|\right)$$, $C_1=0$, т.к. $\psi \xrightarrow[x\to\pm\infty]{} 0 \Rightarrow \psi\sim\exp\left(-\sqrt{\frac{2m|E_n|}{\hbar^2}}|x|\right)$

При уменьшении глубины уровни смещаются вверх, $|E_n|\downarrow$

a – выход на константу

Если $E_n=0$ при $x\to\pm\infty$, то $\psi_{E_n=0}(x) \underset{x\to\pm\infty}{\approx} const$

т.к. В.Ф. четная $(|x|)$, то $\psi_n \approx \exp\left(-\sqrt{\frac{2m|E_n|}{\hbar^2}}|x|\right) \approx \cos\left(\sqrt{\frac{2m|E_n|}{\hbar^2}}x+\varphi\right)$

При $E=0$

$$\psi_{E=0}(x)=\begin{cases} A, & x<0 \\ B\cos\left(\sqrt{\frac{2mU_0}{\hbar^2}}x+\varphi\right) \\ C, & x>2a \end{cases}$$

$$\left.\begin{aligned} \psi_1(0)&=\psi_2(0) \\ \psi_2(2a)&=\psi_3(2a) \\ \psi_1'(0)&=\psi_2'(0) \\ \psi_2'(2a)&=\psi_3'(2a) \end{aligned}\right\}$$

$A=B\cos\varphi \Rightarrow \varphi=0$, $A=B$

$B\cos(2æa)=C$

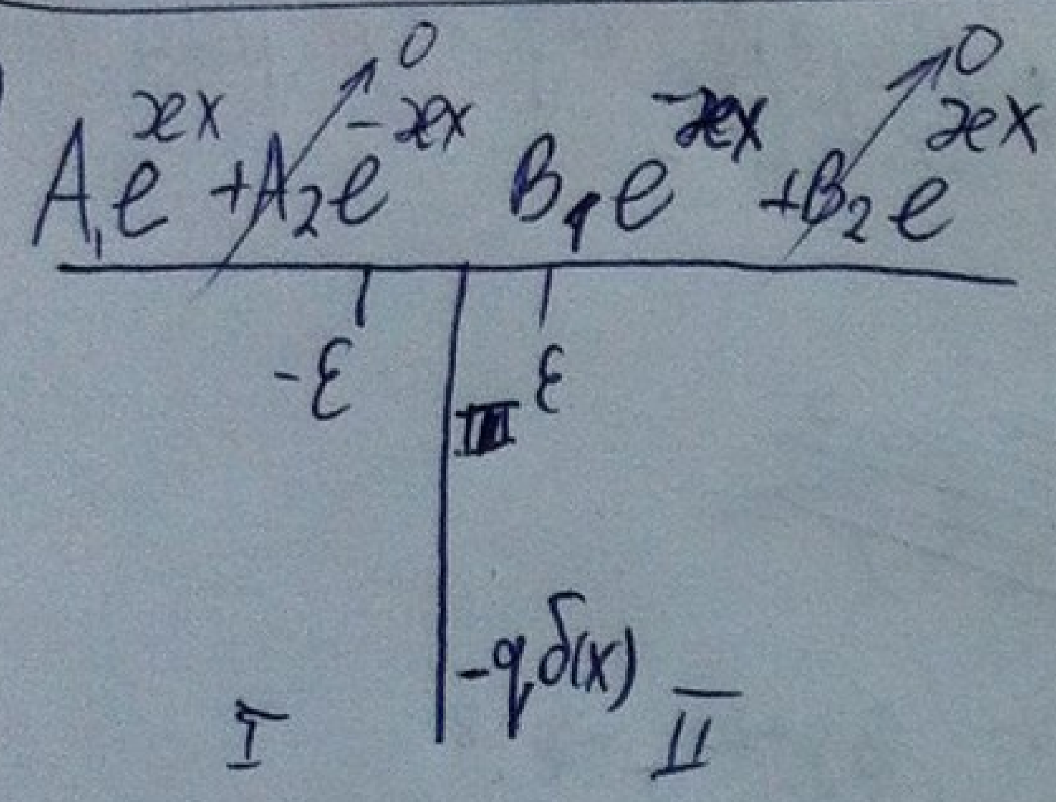
$B\sin(2æa)=0 \Rightarrow \sin(2æa)=0$

$2æa=\pi n$ $\quad n=1,2,3\ldots$

$2a\sqrt{\frac{2mU_0}{\hbar^2}}=\pi n$

---

③



$A_1=B_1$

$\psi_{II}(\varepsilon)=\psi_I(-\varepsilon) \Rightarrow A_1=B_1=A$

$\frac{d\psi_1(-\varepsilon)}{dx}=Aæe^{-æ\varepsilon}$

$\frac{d\psi_2(\varepsilon)}{dx}=-Aæe^{-æ\varepsilon}$

$E=-\frac{\hbar^2}{2m}æ^2$

$-\varepsilon<x<\varepsilon$

$-\frac{\hbar^2}{2m}\psi''(x)-q\delta(x)\psi=-|E|\psi$

$\psi''(x)+\frac{2m}{\hbar^2}q\delta(x)\psi-\underbrace{\frac{|E|\cdot 2m}{\hbar^2}}_{æ^2}\psi=0$

$\int_{-\varepsilon}^{\varepsilon}\left(\psi''+\frac{2m}{\hbar^2}q\delta(x)\psi-æ^2\psi\right)dx \;⊜$

$\int_{-\varepsilon}^{\varepsilon}\psi''dx=\left.\frac{d\psi}{dx}\right|_{-\varepsilon}^{\varepsilon}=\frac{d\psi(\varepsilon)}{dx}-\frac{d\psi(-\varepsilon)}{dx}=-Aæe^{-æ\varepsilon}-Aæe^{-æ\varepsilon}=$
$=-2Aæe^{-æ\varepsilon}$
$\underset{\varepsilon\to0}{=}-2Aæ$

$\int_{-\varepsilon}^{\varepsilon}\frac{2m}{\hbar^2}q\delta(x)\psi(x)dx=\frac{2m}{\hbar^2}q\psi(0)\approx\frac{2m}{\hbar^2}qA$

$\int_{-\varepsilon}^{+\varepsilon}æ^2\psi(x)dx\approx-æ^2\cdot2\varepsilon\psi(0)\underset{\varepsilon\to0}{\to}0$

$⊜\;-2Aæ+\frac{2m}{\hbar^2}qA=0 \Rightarrow æ_0=\frac{mq}{\hbar^2}$

$E=-\frac{\hbar^2}{2m}æ^2=-\frac{q^2m}{2\hbar^2}$

Условие нормировки $\int_{-\infty}^{+\infty}\psi^*(x)\psi^*dx=1$

$\psi_I=Ae^{æx}$, $x<0$; $\psi_{II}=Ae^{-æx}$, $x>0$ $\Rightarrow \psi=Ae^{-æ|x|}$

~~$\int_{-\infty}^{-\varepsilon}A^2e^{2æx}dx+\int_{\varepsilon}^{+\infty}A^2e^{-2æx}dx+\int_{-\varepsilon}^{\varepsilon}A^2$~~

$\int_{-\infty}^{+\infty}A^2e^{-2æ_0|x|}dx=1 \Rightarrow \frac{1}{A^2}=\int_{-\infty}^{0}e^{2æx}dx+\int_{0}^{+\infty}e^{-2æx}dx=\frac{1}{2æ_0}+\frac{1}{2æ_0}=\frac{1}{æ_0}\Rightarrow A=\sqrt{æ_0}$

$\psi=\sqrt{æ_0}\,e^{-æ_0|x|}$

$$\langle x\rangle=\int_{-\infty}^{+\infty}\psi^* x\,\psi dx=\varkappa_0\int_{-\infty}^{+\infty}x e^{-2\varkappa|x|}dx=\varkappa_0\left[\int_{-\infty}^{0}xe^{2\varkappa x}dx+\int_{0}^{\infty}xe^{-2\varkappa x}dx\right]=\varkappa_0[0+0]=0$$

$$\langle x^2\rangle=\int_{-\infty}^{+\infty}\psi^* x^2\psi dx=\varkappa_0\int_{-\infty}^{+\infty}x^2e^{-2\varkappa|x|}dx=\varkappa_0\left[\int_{-\infty}^{0}x^2e^{2\varkappa x}dx+\int_{0}^{\infty}x^2e^{-2\varkappa x}dx\right]=$$

$$=\varkappa_0\left[\underbrace{\frac{x^2e^{+2\varkappa x}}{2\varkappa_0}\Big|_{-\infty}^{0}}_{=0}-\int_{-\infty}^{0}\frac{e^{2\varkappa x}}{2\varkappa_0}2x\,dx+\underbrace{\frac{x^2e^{-2\varkappa x}}{-2\varkappa_0}\Big|_{0}^{\infty}}_{=0}-\int_{0}^{\infty}\frac{e^{-2\varkappa x}}{-2\varkappa_0}2x\,dx\right]=-\frac{\varkappa_0}{\varkappa_0}\left[\int_{-\infty}^{0}xe^{2\varkappa x}dx-\int_{0}^{\infty}xe^{-2\varkappa x}dx\right]=$$

$$=-\left[\underbrace{\frac{xe^{2\varkappa x}}{2\varkappa_0}\Big|_{-\infty}^{0}}_{=0}-\int_{-\infty}^{0}\frac{e^{2\varkappa x}}{2\varkappa_0}dx-\underbrace{\frac{xe^{-2\varkappa x}}{-2\varkappa_0}\Big|_{0}^{\infty}}_{=0}+\int_{0}^{\infty}\frac{e^{-2\varkappa x}}{-2\varkappa_0}dx\right]=+\left[\frac{1}{2\varkappa_0}\cdot\frac{1}{2\varkappa_0}+\frac{1}{2\varkappa_0}\left(\frac{-1}{-2\varkappa_0}\right)\right]=\frac{1}{2\varkappa_0^2}$$

$$D_x=\langle x^2\rangle-\langle x\rangle^2=\frac{1}{2\varkappa_0^2}=\frac{1}{2}\frac{\hbar^4}{m^2q^2}$$

$$\langle p\rangle=\int_{-\infty}^{+\infty}\psi^*\frac{\hbar}{i}\frac{d}{dx}\psi dx=\frac{\hbar\varkappa_0}{i}\int_{-\infty}^{+\infty}e^{-\varkappa_0|x|}\frac{d}{dx}e^{-\varkappa_0|x|}dx=\frac{\hbar}{i}\varkappa_0\left[\int_{-\infty}^{0}\varkappa_0e^{2\varkappa_0x}dx+\int_{0}^{+\infty}(-\varkappa_0)e^{-2\varkappa_0x}dx\right]=0$$

$$\langle p^2\rangle=\int_{-\infty}^{+\infty}\psi^*\left(\frac{\hbar}{i}\right)^2\frac{d^2}{dx^2}\psi dx=\left(\frac{\hbar}{i}\right)^2\varkappa_0\left[\int_{-\infty}^{0}\varkappa_0^2e^{2\varkappa_0x}dx+\int_{0}^{+\infty}(-\varkappa_0)^2e^{-2\varkappa_0x}dx\right]=\left(\frac{\hbar}{i}\right)^2\varkappa_0^3\cdot\frac{1}{\varkappa_0}=-\hbar^2\varkappa_0^2$$

$$D_p=\langle p^2\rangle-\langle p\rangle^2=-\hbar^2\varkappa_0^2=-\hbar^2\cdot\frac{m^2q^2}{\hbar^4}=\frac{-m^2q^2}{\hbar^2}$$

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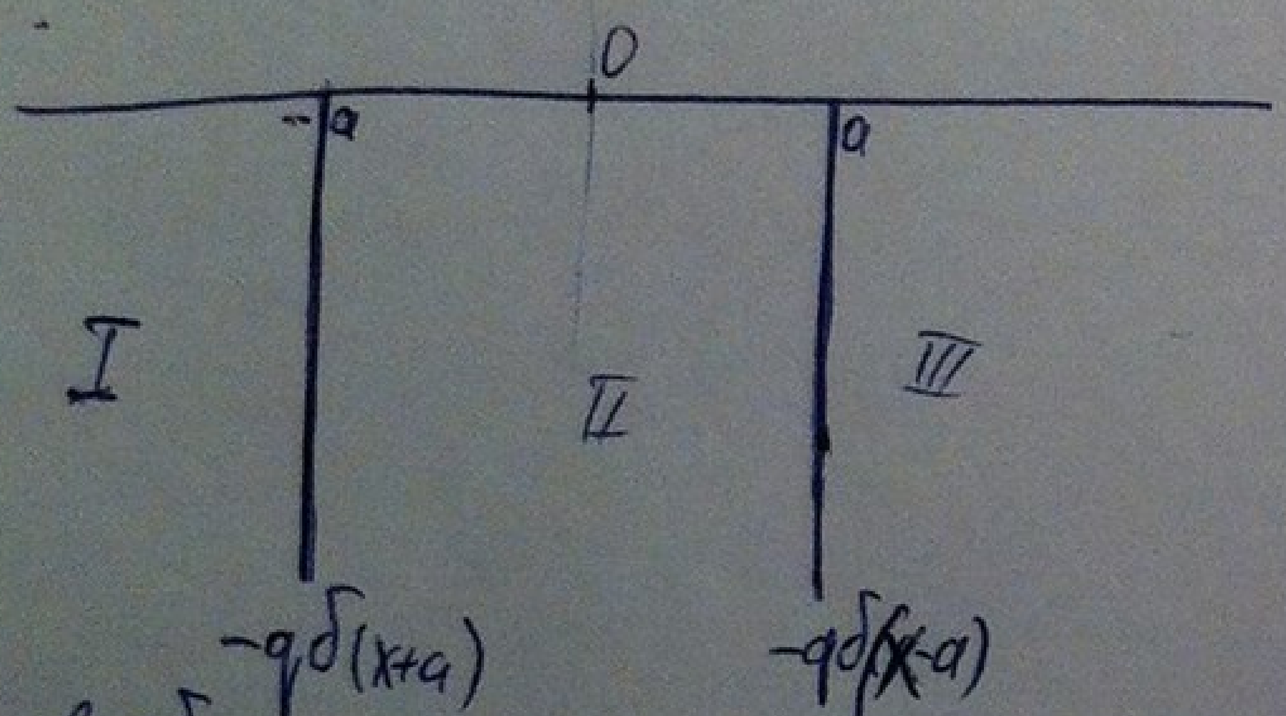
(4)



$\psi_1=Ae^{\varkappa x}$

$\psi_2=B\,ch\,\varkappa x$

$\psi_3=Ae^{-\varkappa x}$

$\varkappa_0=\frac{mq}{\hbar^2}$

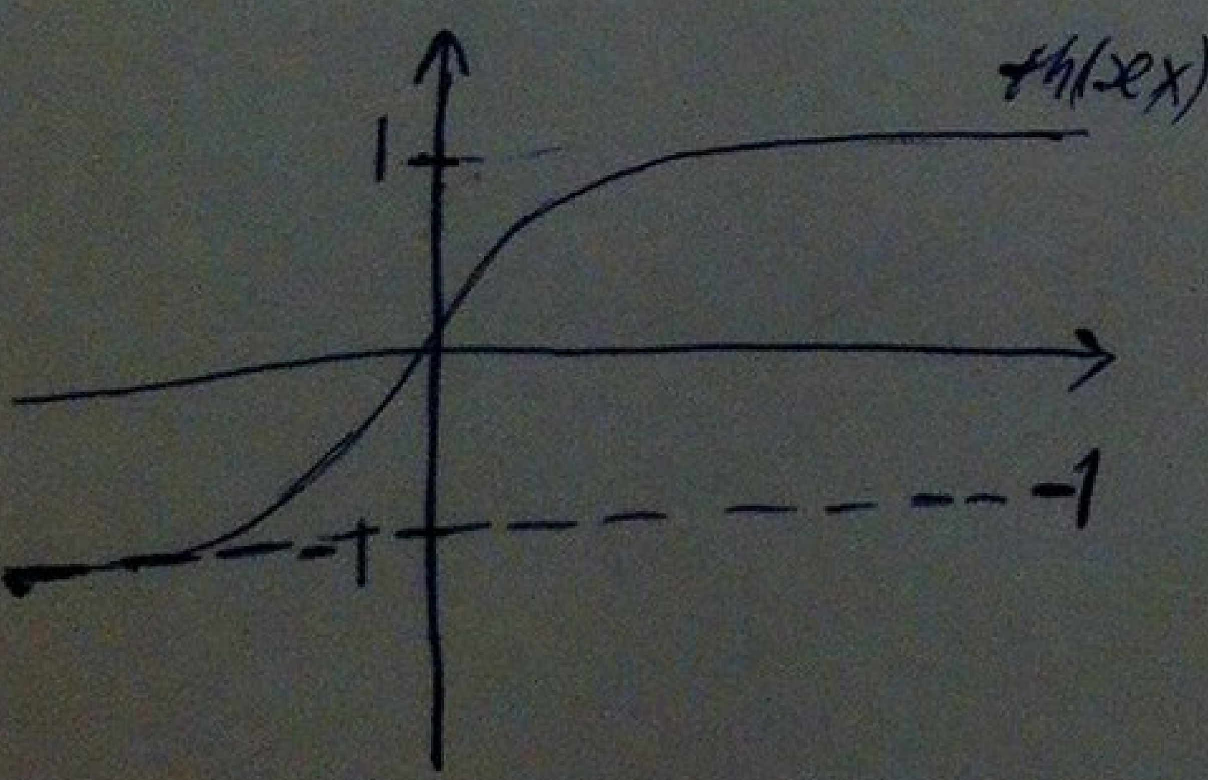
$$\begin{cases}\psi_1(-a+0)=\psi_2(a-0)\\ \psi_2(a+0)=\psi_3(a-0)\\ \psi_1'(-a+0)=\psi_2'(-a-0)\\ \psi_2'(a+0)=\psi_3'(a-0)\end{cases}$$

Для связ. сост.

$$\begin{cases}Ae^{-\varkappa a}=B\,ch(\varkappa a)=B\,ch\,\varkappa a\\ \varkappa Ae^{-\varkappa a}=B\varkappa\,sh(-\varkappa a)=-B\varkappa\,sh(\varkappa a)\end{cases}$$

$\varkappa=-\varkappa\,th(\varkappa a)$

$th(\varkappa a)=-1$



Для возб. сост.

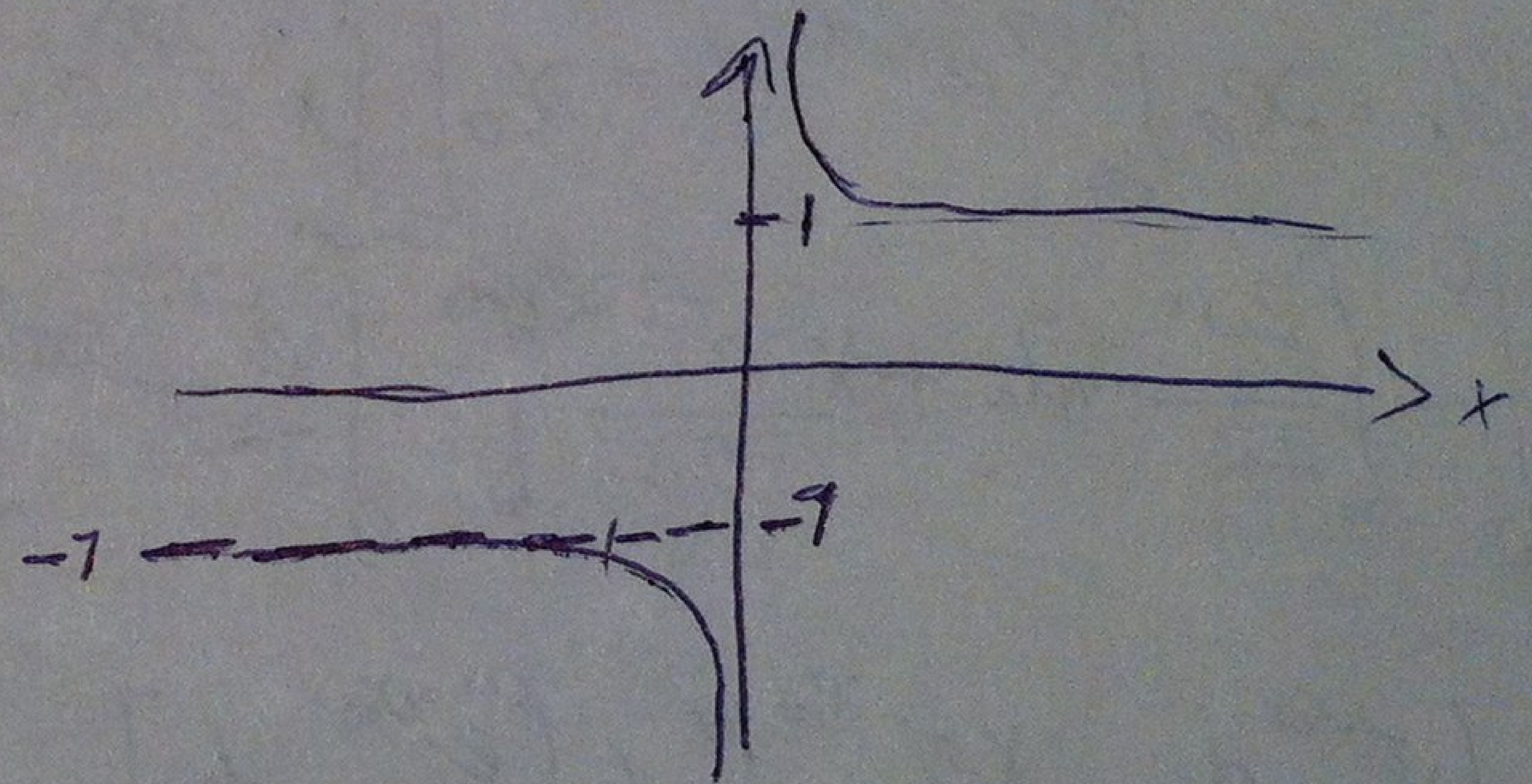
$\Psi_3 = +Ae^{-\varkappa x}$

$\Psi_1 = -Ae^{\varkappa x}$

$\Psi_2 = B\,sh(\varkappa x)$

$$\begin{cases} \Psi_1(-a) = \Psi_2(-a) \\ \Psi_1'(-a) = \Psi_2'(-a) \end{cases} \quad \begin{cases} -Ae^{-\varkappa a} = B\,sh(-\varkappa a) = -B\,sh(\varkappa a) \\ -\varkappa A e^{-\varkappa a} = B\varkappa\,ch(-\varkappa a) = B\varkappa\,ch(\varkappa a) \end{cases} \quad \begin{cases} Ae^{-\varkappa a} = B\,sh(\varkappa a) \\ -Ae^{-\varkappa a} = B\,ch(\varkappa a) \end{cases}$$

$$-1 = cth(\varkappa a)$$



№7 $\langle m|x^4|n\rangle = ?$

$$\langle m|x^4|n\rangle = \frac{x_0^4}{2^2}\langle m|(a+a^+)^4|n\rangle = \frac{x_0^4}{4}\langle m|(a+a^+)(a+a^+)(a+a^+)(a+a^+)|n\rangle =$$

$$= \frac{x_0^4}{4}\langle m|a^4 + a^3a^+ + a^2a^+a + a^2(a^+)^2 + aa^+a^2 + aa^+aa^+ + a(a^+)^2a + a(a^+)^3 +$$

$$+ a^+a^3 + a^+a^2a^+ + a^+aa^+a + a^+a(a^+)^2 + (a^+)^2a^2 + (a^+)^2aa^+ + (a^+)^3a + (a^+)^4|n\rangle =$$

$$= \frac{x_0^4}{4}\langle m|\Big(\sqrt{n(n-1)(n-2)(n-3)}\,|n-4\rangle + \sqrt{(n-1)^2 n(n-1)}\,|n-2\rangle + \sqrt{n^3(n-1)}\,|n-2\rangle + \sqrt{(n+1)^2(n+2)^2}\,|n\rangle +$$

$$+ \sqrt{n(n-1)^3}\,|n-2\rangle + (n+1)^2|n\rangle + n(n+1)|n\rangle + \sqrt{(n+1)(n+2)(n+3)^2}\,|n+2\rangle +$$

$$+ \sqrt{n(n-1)(n-2)^2}\,|n-2\rangle + (n+1)n|n\rangle + n^2|n\rangle + \sqrt{(n+1)(n+2)^3}\,|n+2\rangle +$$

$$+ \sqrt{n^2(n-1)^2}\,|n\rangle + (n+1)\sqrt{(n+1)(n+2)}\,|n+2\rangle + n\sqrt{(n+1)(n+2)}\,|n+2\rangle + \sqrt{(n+1)(n+2)(n+3)(n+4)}\,|n+4\rangle\Big) =$$

$$= \frac{x_0^4}{4}\langle m|\Big(\sqrt{n(n-1)(n-2)(n-3)}\,|n-4\rangle + \sqrt{n(n-1)}\big((n+1)+n+(n-1)+(n-2)\big)|n-2\rangle + \sqrt{(n+1)(n+2)(n+3)(n+4)}\,|n+4\rangle$$

$$+ \sqrt{(n+1)(n+2)}\big((n+3)+(n+2)+(n+1)+n\big)|n+2\rangle + \big((n+1)(n+2)+(n+1)^2+n(n+1)+(n+1)n+n^2+n(n-1)\big)|n\rangle\Big) =$$

$$= \frac{x_0^4}{4}\cdot\langle m|\sqrt{n(n-1)(n-2)(n-3)}\,|n-4\rangle + \sqrt{n(n-1)}(4n-2)\,|n-2\rangle + \sqrt{(n+1)(n+2)}(4n+6)\,|n+2\rangle +$$

$$+ (n^2+3n+2+n^2+2n+1+n^2+n+n^2+n+n^2+n^2-n)|n\rangle =$$

$$= \frac{x_0^4}{4}\Big(\delta_{m,n-4}\sqrt{n(n-1)(n-2)(n-3)} + \delta_{m,n-2}\sqrt{n(n-1)}(4n-2) + \delta_{m,n+2}\sqrt{(n+1)(n+2)}(4n+6) + \delta_{m,n}(6n^2+6n+3) + \delta_{m,n+4}\sqrt{(n+1)(n+2)(n+3)(n+4)}\Big)$$

№8 $\langle 0|x^{2014}|2015\rangle = \langle 0|(a+a^+)^{2014}|2015\rangle = 0$

т.к. min: $m = n-k$ для $\langle m|x^k|n\rangle$, т.к. $0 = 2015-2014 = 1 \Rightarrow \langle 0|x^{2014}|2015\rangle = 0$

$\langle 0|x^{2015}|2014\rangle = \langle 0|(a+a^+)^{2015}|2014\rangle$

min $m = n-k$: $0 = 2014-2015 = -1$ подходит, но как можно заметить, индексы у $\delta_{m,n}$ чередуются через 2: $\delta_{m,n-2}, \delta_{m,n+2}, \delta_{m,n}, \delta_{m,n+4}, \delta_{m,n-4} \ldots \Rightarrow$

$\Rightarrow$ будут $\delta_{0,-1}; \delta_{0,1}; \delta_{0,3}$, а $\delta_{0,0}$ не будет $\Rightarrow \langle 0|x^{2015}|2014\rangle = 0$

$\langle 0|x^{2015}|2015\rangle = \langle 0|(a+a^+)^{2015}|2015\rangle$

min $m = n-k$: $0 = 2015-2015 = 0$; $\delta_{0,0}$ будет;

$\langle 0|x^{2015}|2015\rangle = \sqrt{2015!}$

⑨ $\langle 0|p^2x^2|0\rangle = \frac{x_0^2}{2}\cdot\frac{p_0^2}{2(i)^2}\langle 0|(a-a^+)^2(a+a^+)^2|0\rangle =$ $\qquad n=0$

$$= -\frac{x_0^2p_0^2}{4}\langle 0|(a-a^+)(a-a^+)(a+a^+)(a+a^+)|0\rangle =$$

$$= -\frac{x_0^2p_0^2}{4}\langle 0|a^4 + a^3a^+ + a^2a^+a + a^2(a^+)^2 - (aa^+a^2 + aa^+aa^+ + a(a^+)^2a + a(a^+)^3) -$$

$$-(a^+a^3 + a^+a^2a^+ + a^+aa^+a + a^+a(a^+)^2) + (a^+)^2a^2 + (a^+)^2aa^+ + (a^+)^3a + (a^+)^4|0\rangle =$$

$$= -\frac{x_0^2p_0^2}{4}\Big(\langle 0|\sqrt{n+1}\,(n+2)\sqrt{n+1}|0\rangle - (\langle 0|(n+1)^2|0\rangle + \langle 0|n\cdot(n+1)|0\rangle) - ((n+1)n|0\rangle + \langle 0|n^2|0\rangle) +$$

$$+\langle 0|\sqrt{n}\,(n-1)\sqrt{n}|0\rangle\Big) = -\frac{x_0^2p_0^2}{4}\big(1\cdot 2 - (1^2+0\cdot 1) - ((0+1)\cdot 0 + 0^2) + 0\cdot(-1)\big) = -\frac{x_0^2p_0^2}{4}$$

$$\langle 0|p^2x^2|0\rangle = -\frac{x_0^2p_0^2}{4}$$

---

⑩ $\langle n|x|n\rangle = 0$

$\langle n|p|n\rangle = 0$

$$\langle n|x^2|n\rangle = \langle n|(a+a^+)^2|n\rangle\frac{x_0^2}{2} = \frac{x_0^2}{2}\langle n|a^2 + aa^+ + a^+a + (a^+)^2|n\rangle =$$

$$= \frac{x_0^2}{2}\Big(\langle n|\sqrt{n(n-1)}|n-2\rangle + \langle n|(n+1)|n\rangle + \langle n|n|n\rangle + \langle n|\sqrt{(n+1)(n+2)}|n+2\rangle\Big) =$$

$$= \frac{x_0^2}{2}(n+1+n) = \frac{x_0^2}{2}(2n+1)$$

$$\langle n|p^2|n\rangle = \frac{p_0^2}{2(i)^2}\langle n|(a-a^+)^2|n\rangle = \frac{-p_0^2}{2}\langle n|a^2 - aa^+ - a^+a + (a^+)^2|n\rangle =$$

$$= \frac{-p_0^2}{2}\Big(\langle n|\sqrt{n(n-1)}|n-2\rangle - \langle n|(n+1)|n\rangle - \langle n|n|n\rangle + \langle n|\sqrt{(n+1)(n+2)}|n+2\rangle\Big) =$$

$$= +\frac{p_0^2}{2}(n+1+n) = \frac{p_0^2}{2}(2n+1)$$

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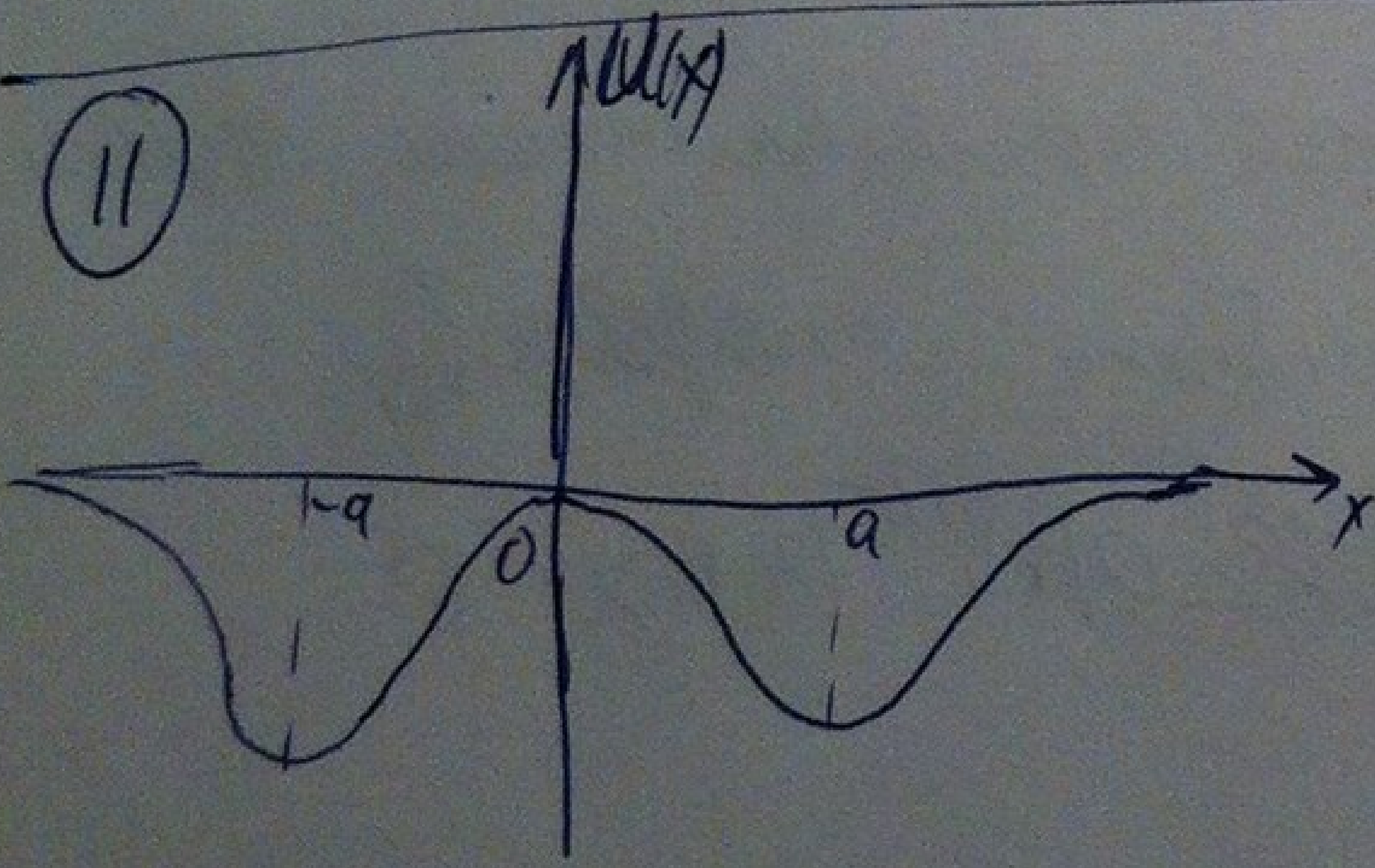
⑪



$F = -\nabla U = -\frac{\partial U}{\partial x}$

$\psi_n'(0) = 0$ для чётн. в.ф. $\psi^{(+)}$

$\psi_n(0) = 0$ для нечётн. в.ф. $\psi^{(-)}$

$$\overline{F}_{nn} = -\int_{-\infty}^{+\infty}\psi_n^*\left(\frac{dU}{dx}\right)\psi_n\,dx =$$

$$= -\int_{-\infty}^{+\infty}\frac{d}{dx}\left(U|\psi_n|^2\right)dx + \int_{-\infty}^{+\infty}\left(\psi_n^*U\psi_n' + (\psi_n^*)'U\psi_n\right)dx =$$

$$= E_n\int\frac{d}{dx}|\psi_n|^2dx - \frac{1}{2m}\int\{(\hat{p}^2\psi_n^*)\psi_n' + (\psi_n^*)'\hat{p}^2\psi_n\}dx =$$

$$= E_n\int\frac{d}{dx}|\psi_n|^2dx + \frac{\hbar^2}{2m}\int\frac{d}{dx}\left((\psi_n^*)'\psi_n'\right)dx$$

$$(F_{np})_{nn} = \begin{cases} E_n|\psi_n^{(+)}(0)|^2 < 0 \\ \frac{\hbar^2}{2m}|(\psi_n^{(-)})'|^2 > 0 \end{cases}$$

③ $E = -\frac{mq^2}{2\hbar^2}$ $\langle \hat{U} \rangle - E = \frac{\langle p^2 \rangle}{2m} - q\langle \delta(x) \rangle$

$$\langle \delta(x) \rangle = \int_{-\infty}^{+\infty} \psi^*(x)\delta(x)\psi(x)dx = \psi^2(0) = \varkappa_0$$

$$\langle p^2 \rangle = (E + q\varkappa_0)2m = (\varkappa q - \frac{mq^2}{2\hbar^2})\cdot 2m = \frac{q^2 m}{2\hbar^2}\cdot 2m = \frac{q^2 m^2}{\hbar^2}$$

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⑪ $\psi_2 = \frac{1}{\sqrt{2}}\psi_0(x-L) - \frac{1}{\sqrt{2}}\psi_0(x+L)$

$$\left.\begin{aligned} &\psi_0'' + \frac{2m}{\hbar^2}(E_0 - V(x))\psi_0 = 0 \\ &\psi_2'' + \frac{2m}{\hbar^2}(E_2 - V(x))\psi_2 = 0 \end{aligned}\right\} \begin{matrix}\psi_2\\ \psi_0\end{matrix}$$

$$\psi_2\psi_0'' - \psi_0\psi_2'' + \frac{2m}{\hbar^2}(E_0 - E_2)\psi_0\psi_2 = 0 \ ; x>0$$

$$\underbrace{\psi_0\psi_2'' - \psi_2\psi_0''}_{\frac{d}{dx}(\psi_0\psi_2' - \psi_2\psi_0')} + \frac{2m}{\hbar^2}(E_2 - E_0)\psi_0\psi_2 = 0$$

$$(\psi_0\psi_2' - \psi_2\psi_0')\Big|_0^{+\infty \to 0} + \frac{2m}{\hbar^2}(E_2 - E_0)\int_0^\infty \psi_0\psi_2 dx = 0$$

$\psi_2(0) = 0$

$\psi_0(0) = Ce^{\varkappa(x-L)}\Big|_0 = Ce^{-\varkappa L}$

$\psi_2'(0) = \frac{1}{\sqrt{2}}\psi_0'(0) = \frac{1}{\sqrt{2}}\varkappa C e^{\varkappa(x-L)}\Big|_0$

$$\frac{2m}{\hbar^2}(E_2 - E_0)\cdot\frac{1}{2}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\varkappa C^2 e^{-2\varkappa L}$$

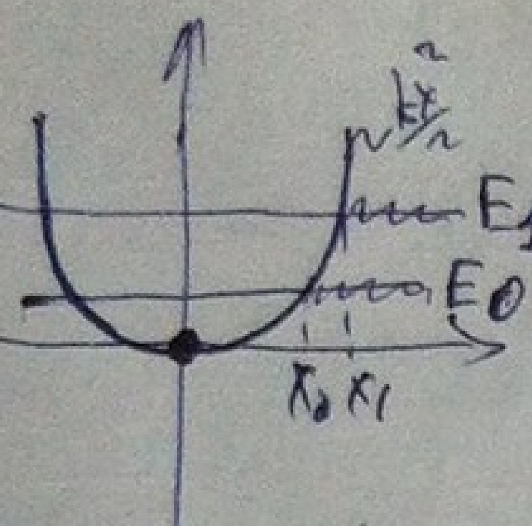
$$E_2 - E_0 = +\tilde{C}e^{-2\varkappa L}$$

$$\int_0^\infty \psi_0\psi_2 = \frac{+1}{\sqrt{2}}\cdot\frac{1}{2}$$

$$\psi_0\psi_2'\Big|_0^\infty = \psi_0(0)\psi_2'(0) = 0$$

Dz на 21.04.2014 $\qquad \omega=\sqrt{\frac{k}{m}}$

① $P_{з,0} = 2\int\limits_{x_0}^{\infty} dx\,|\psi_0(x)|^2 = ?$



$P_{з,1} = 2\int\limits_{x_1}^{\infty} dx\,|\psi_1(x)|^2 = ?$

$E_n = \hbar\omega(n+1/2)$

$E_0 = \frac{1}{2}\hbar\omega$

$E_1 = \frac{3}{2}\hbar\omega$

$U = \frac{kx^2}{2}$

$E_0 = U \Rightarrow \frac{1}{2}\hbar\omega = \frac{kx_0^2}{2} \Rightarrow x_0 = \sqrt{\frac{\hbar\omega}{k}}$

$E_1 = U \Rightarrow \frac{3}{2}\hbar\omega = \frac{kx_1^2}{2} \Rightarrow x_1 = \sqrt{\frac{3\hbar\omega}{k}}$

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)H_n\left(x\sqrt{\frac{m\omega}{\hbar}}\right)$$

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\cdot 1$$

$$\psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2}}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\cdot 2x\sqrt{\frac{m\omega}{\hbar}} = \sqrt{2}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\cdot x\sqrt{\frac{m\omega}{\hbar}}$$

$$P_{з,0} = 2\int\limits_{x_0}^{\infty}\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx = \frac{2}{\sqrt{\pi}}\int \exp\left(-\left(\sqrt{\frac{m\omega}{\hbar}}x\right)^2\right)d\left(\sqrt{\frac{m\omega}{\hbar}}x\right) = \left[\begin{array}{l}\sqrt{\frac{m\omega}{\hbar}}x = y \\ x_0\sqrt{\frac{m\omega}{\hbar}} = y_0 = \sqrt{\frac{\hbar\omega}{k}}\sqrt{\frac{m\omega}{\hbar}} = 1\end{array}\right] =$$

$$= \frac{2}{\sqrt{\pi}}\int\limits_1^{\infty}\exp(-y^2)dy = 1 - \frac{2}{\sqrt{\pi}}\int\limits_0^1 \exp(-y^2)dy = 0{,}157$$

$$P_{з,1} = 2\int\limits_{x_1}^{\infty}\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\cdot 2\cdot\exp\left(-\frac{m\omega}{\hbar}x^2\right)\cdot x^2\left(\sqrt{\frac{m\omega}{\hbar}}\right)^2 dx = \frac{4}{\sqrt{\pi}}\int\limits_{x_1}^{\infty}\exp\left(-\left(\sqrt{\frac{m\omega}{\hbar}}x\right)^2\right)\left(\sqrt{\frac{m\omega}{\hbar}}x\right)^2 d\left(\sqrt{\frac{m\omega}{\hbar}}x\right) =$$

$$= \frac{4}{\sqrt{\pi}}\int\limits_{y_1}^{\infty} y^2\exp(-y^2)dy = \left[x_1\cdot\sqrt{\frac{m\omega}{\hbar}} = y_1 = \sqrt{3}\right] = \frac{4}{\sqrt{\pi}}\int\limits_{\sqrt{3}}^{\infty} y^2\exp(-y^2)dy =$$

$$= \frac{4}{\sqrt{\pi}}\int\limits_{\sqrt{3}}^{\infty}\frac{1}{2}y\exp(-y^2)d(y^2) = \left[\begin{array}{l} v = y \\ dv = dy \\ du = \exp(-y^2)dy^2 \\ u = -\exp(-y^2)\end{array}\right] = \frac{4}{\sqrt{\pi}}\cdot\frac{1}{2}\left[-y\exp(-y^2)\Big|_{\sqrt{3}}^{\infty} + \int\limits_{\sqrt{3}}^{\infty}\exp(-y^2)dy\right] =$$

$$= \frac{2}{\sqrt{\pi}}\left[\sqrt{3}\exp(-3) + \int\limits_{\sqrt{3}}^{\infty}\exp(-y^2)dy\right] = \frac{2\sqrt{3}}{\sqrt{\pi}}\exp(-3) + \frac{2}{\sqrt{\pi}}\int\limits_{\sqrt{3}}^{\infty}\exp(-y^2)dy = \frac{2\sqrt{3}}{\sqrt{\pi}}\exp(-3) + \mathrm{erfc}(\sqrt{3}) =$$

$$= \frac{2\sqrt{3}}{\sqrt{\pi}}\exp(-3) + 1 - \mathrm{erf}(\sqrt{3}) = 0{,}1116$$

---

② $\psi(x) = N\exp\left[\left(-\frac{x^2}{2a^2}\right) - \eta|x|\right]$ $\qquad x<0,\ x>0,\ x=0 \qquad U(x) - ?$

$$\psi''(x) + \frac{2m}{\hbar^2}(E - U(x))\psi(x) = 0$$

a) $x<0 \Rightarrow \psi(x) = N\exp\left[-\frac{x^2}{2a^2} + \eta x\right]$

$$\psi'(x) = N\left(-\frac{x}{a^2} + \eta\right)\exp\left(-\frac{x^2}{2a^2} + \eta x\right)$$

$$\psi''(x) = N\left[-\frac{1}{a^2}\exp\left(-\frac{x^2}{2a^2} + \eta x\right) + \left(-\frac{x}{a^2} + \eta\right)^2\exp\left(-\frac{x^2}{2a^2} + \eta x\right)\right] = \psi(x)\cdot\left[-\frac{1}{a^2} + \left(-\frac{x}{a^2} + \eta\right)^2\right]$$

$$\psi(x)\cdot\left[-\frac{1}{a^2}+\left(\frac{x}{a^2}+\eta\right)^2\right]+\frac{2m}{\hbar^2}(E-U(x))\psi(x)=0$$

$$U(x)-E=\frac{\hbar^2}{2m}\left[-\frac{1}{a^2}+\left(\frac{x}{a^2}+\eta\right)^2\right]=\frac{\hbar^2}{2m}\left[-\frac{1}{a^2}+\frac{x^2}{a^4}-\frac{2x}{a^2}\eta+\eta^2\right]$$ ← парабола

$$U(x)-E=\frac{\hbar^2}{2m}\frac{1}{a^2}\left[-1+\frac{1}{a^2}(x-\eta a^2)^2\right]$$

2) $x>0 \Rightarrow \psi(x)=N\exp\left[\left(-\frac{x^2}{2a^2}\right)-\eta x\right]$

$$\psi'(x)=N\left(-\frac{x}{a^2}-\eta\right)\exp\left[\left(-\frac{x^2}{2a^2}\right)-\eta x\right]$$

$$\psi''(x)=N\left[-\frac{1}{a^2}\exp\left[-\frac{x^2}{2a}-\eta x\right]+\left(-\frac{x}{a^2}-\eta\right)^2\exp\left(-\frac{x^2}{2a}-\eta x\right)\right]=\psi(x)\cdot\left[-\frac{1}{a^2}+\left(\frac{x}{a^2}+\eta\right)^2\right]$$

$$\psi(x)\left[-\frac{1}{a^2}+\left(\frac{x}{a^2}+\eta\right)^2\right]+\frac{2m}{\hbar^2}(E-U(x))\psi(x)=0$$

$$U(x)-E=\frac{\hbar^2}{2m}\left[-\frac{1}{a^2}+\left(\frac{x}{a^2}+\eta\right)^2\right]=\frac{\hbar^2}{2m}\frac{1}{a^2}\left[-1+\frac{1}{a^2}(x+\eta a^2)^2\right]$$ ← парабола

3) $x=0 \Rightarrow \psi(x)=N\exp[0-0]=N$

сшивка производных

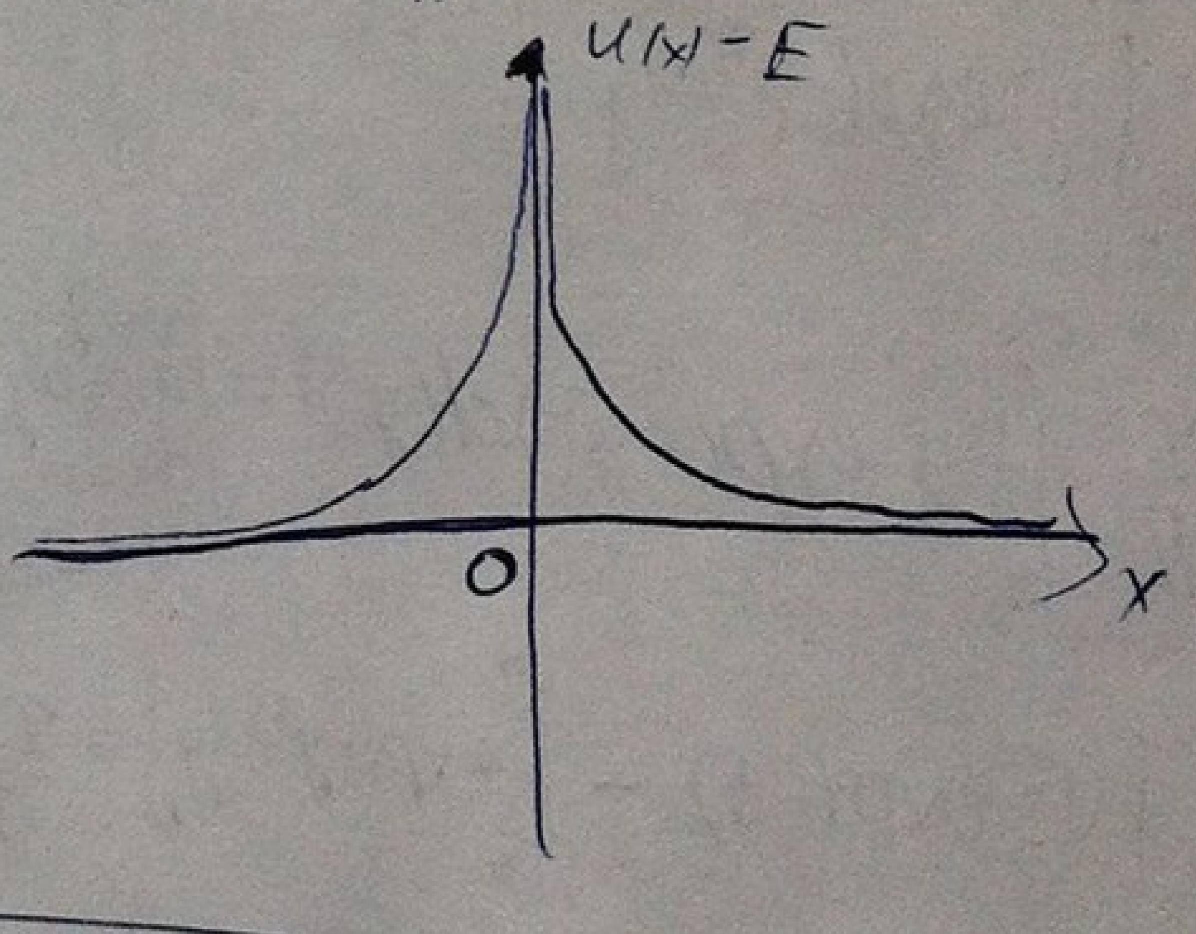
$$\psi'(0+0)=\psi'(0-0)+\frac{2m}{\hbar^2}q\,\psi(0)$$

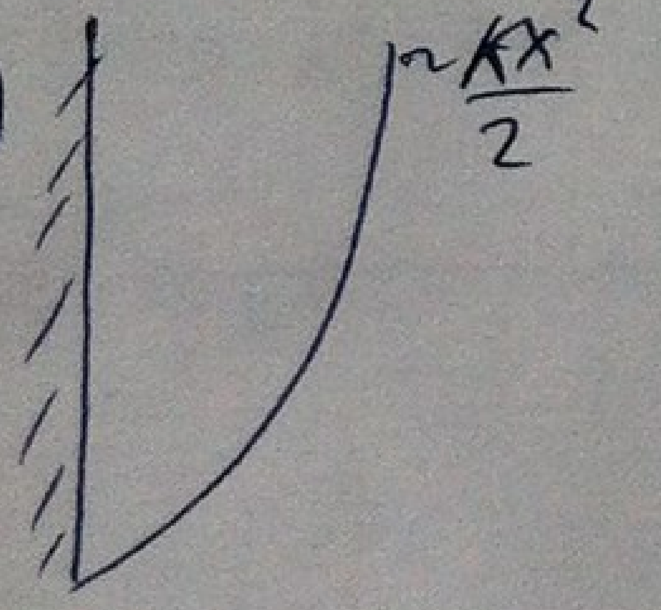
$$N\left(-\frac{x}{a^2}-\eta\right)\exp\left(\left(-\frac{x^2}{2a^2}-\eta x\right)\right)\Big|_{x=0}=N\left(-\frac{x}{a^2}+\eta\right)\exp\left[\left(-\frac{x^2}{2a^2}+\eta x\right)\right]\Big|_{x=0}+\frac{2m}{\hbar^2}qN\exp\left(-\frac{x^2}{2a^2}-\eta|x|\right)\Big|_{x=0}$$

$$-\eta=+\eta+\frac{2m}{\hbar^2}q \Rightarrow q=-2\eta\frac{\hbar^2}{2m}=\frac{\eta\hbar^2}{m}$$

$\cancel{\psi \leq q\delta(x)} =$

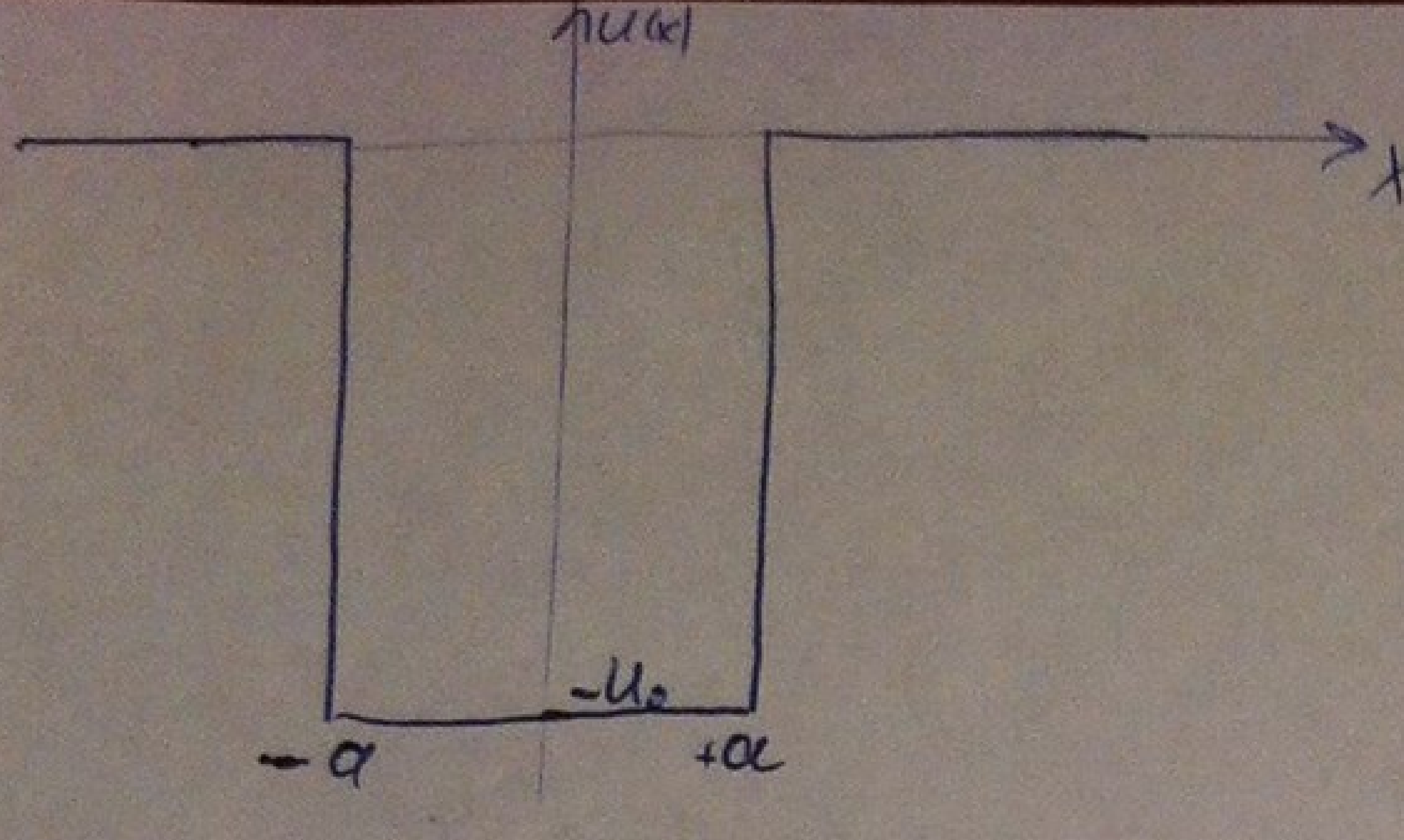
$$U(x)-E=\frac{\hbar^2}{2m}\left[-\frac{1}{a^2}+\left(\frac{|x|}{a^2}+\eta\right)^2\right]-\frac{\eta\hbar^2}{m}\delta(x)$$



③ 

ВФ = ? $\psi_1\cdot\sqrt{2}$; $\psi_3\cdot\sqrt{2}$; $\psi_5\cdot\sqrt{2}$ …

4



$a = 3\cdot10^{-8}$ см $= 3\cdot10^{-10}$ м $= 3\cdot10^{5}$ фм

$m = m_e \Rightarrow m_e c^2 = 0{,}511$ МэВ

$U_0 =$ а) $0{,}1$ эВ

б) 10 эВ

Не решая УШ оценить E_0

в) решить УШ для а, б

а) $\ddot\psi + \underbrace{\frac{2ma^2}{\hbar^2}U_0}_{B}\Big(\underbrace{\frac{E}{U_0}}_{-\varepsilon} + f(y)\Big)\psi(y) = 0$

$\ddot\psi + B(-\varepsilon + f(y))\psi(y) = 0$

$y = \frac{x}{a}$

$B_1 = \frac{2m_e a^2 U_0}{\hbar^2} = \frac{2m_e c^2 a^2 U_0}{\hbar^2 c^2} =$

$= \frac{2\cdot 0{,}511\cdot10^6\,\text{эВ}\cdot 0{,}1\,\text{эВ}\cdot 9\cdot10^{10}\,\text{фм}^2}{200^2\cdot10^{12}\,\text{эВ}^2\cdot\text{фм}^2} = 0{,}22995$

$B \ll 1 \Rightarrow \varepsilon \simeq B - 2B^2 \simeq 0{,}124195995$

$\Rightarrow E = U_0\cdot\varepsilon = 0{,}012$ эВ

Приближение δ ямы: $\varepsilon \simeq B = 0{,}23$

$E = B\cdot U_0 = 0{,}023$ эВ

б) $U_0 = 10$ эВ

$B_2 = \frac{2m_e c^2 a^2 U_0}{\hbar^2 c^2} = \frac{2\cdot 0{,}511\cdot10^6\,\text{эВ}\cdot 10\,\text{эВ}\cdot 9\cdot10^{10}\,\text{фм}^2}{200^2\cdot10^{12}\,\text{эВ}^2\cdot\text{фм}^2} = 22{,}995 \Rightarrow B \gg 1$, приближение δ ямы не работает

$\Delta x\cdot\Delta p = \hbar$; $\Delta x = 2a \Rightarrow \Delta p = \frac{\hbar}{2a}$; $E_0 = \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{4a^2 m} = \frac{\hbar^2 c^2}{4a^2 m_e c^2}$

$E_0 = \frac{200^2\cdot10^{12}\,\text{эВ}^2\cdot\text{фм}^2}{4\cdot 9\cdot10^{10}\,\text{фм}^2\cdot 0{,}511\cdot10^6\,\text{эВ}} = 0{,}21743$ эВ

в) $\ddot\psi + B(1-\varepsilon)\psi = 0$ внутри ямы $\ddot\psi \; B\varepsilon\psi = 0$ вне ямы.

$\psi_0 = A_1\exp(-\sqrt{B\varepsilon}\,y) + \cancel{A_2}\exp(+\sqrt{B\varepsilon}\,y)$ т.к. $\psi \xrightarrow[y\to\infty]{} 0$

Четн: $\psi_ч = A_3\cos(\sqrt{B(1-\varepsilon)}\,y)$

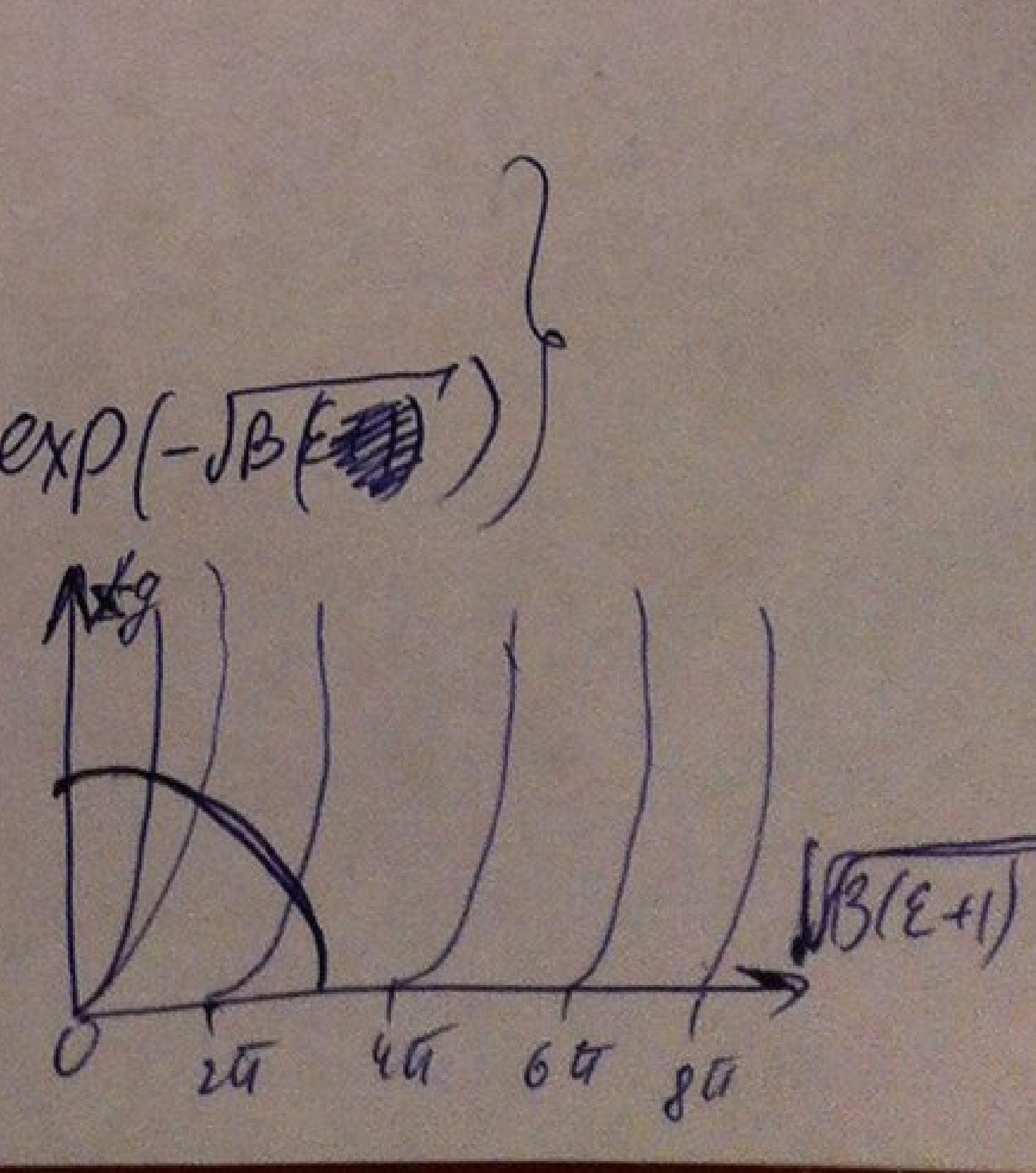
Нечётн: $\psi_н = A_4\sin(\sqrt{B(1-\varepsilon)}\,y)$

Чётн: $\psi_ч(1) = \psi_0(1) \Rightarrow A_3\cos\sqrt{B(1-\varepsilon)} = A_1\exp(-\sqrt{B\varepsilon})$

$\psi'_ч(1) = \psi'_0(1) \Rightarrow -A_3\sqrt{B(1-\varepsilon)}\sin\sqrt{B(1-\varepsilon)} = -A_1\sqrt{B\varepsilon}\exp(-\sqrt{B\varepsilon})$

$-\sqrt{B(1-\varepsilon)}\,tg\sqrt{B(1-\varepsilon)} = -\sqrt{B\varepsilon}$

$tg\sqrt{B(1-\varepsilon)} = \sqrt{\frac{B\varepsilon}{B(1-\varepsilon)}} = \sqrt{\frac{\varepsilon}{1-\varepsilon}}$

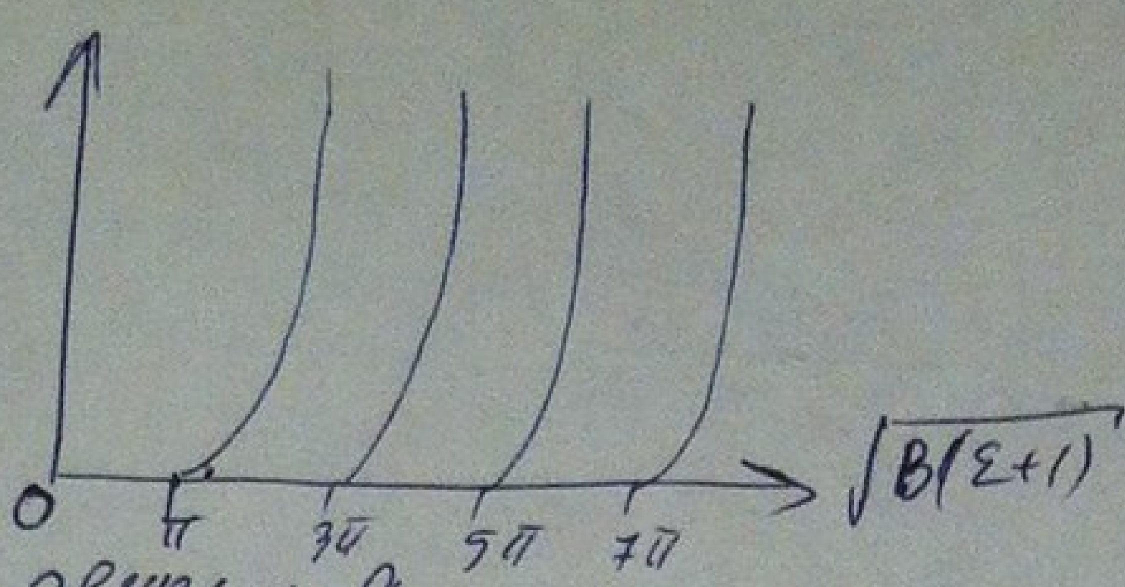


Нечётн: $\Psi_н(1)=\Psi_0(1) \Rightarrow A_н \sin(\sqrt{B(1-\varepsilon)}) = A_1 \exp(-\sqrt{B(\varepsilon)})$

$\Psi'_н(1)=\Psi'_0(1) \Rightarrow \sqrt{B(1-\varepsilon)}\, A_н \cos(\sqrt{B(\ )}) = -\sqrt{B(\varepsilon)}\, A_1 \exp(-\sqrt{B(\varepsilon)})$

$$\frac{1}{\sqrt{B(\varepsilon+1)}}\, tg\sqrt{B(1-\varepsilon)} = \frac{-1}{\sqrt{B(\varepsilon)}}$$

$$tg\sqrt{B(1-\varepsilon)} = -\sqrt{\frac{1-\varepsilon}{\varepsilon}}$$



для $U_0=0{,}1$ эВ яма мелкая, используем чётн. решения

$tg\sqrt{B(1-\varepsilon)} = \sqrt{\frac{\varepsilon}{1-\varepsilon}} \Rightarrow tg\sqrt{B(1-\varepsilon)} \approx \sqrt{B(1-\varepsilon)} \Rightarrow \sqrt{B(1-\varepsilon)} = \sqrt{\frac{\varepsilon}{1-\varepsilon}} \Rightarrow B(1-\varepsilon)^2=\varepsilon$

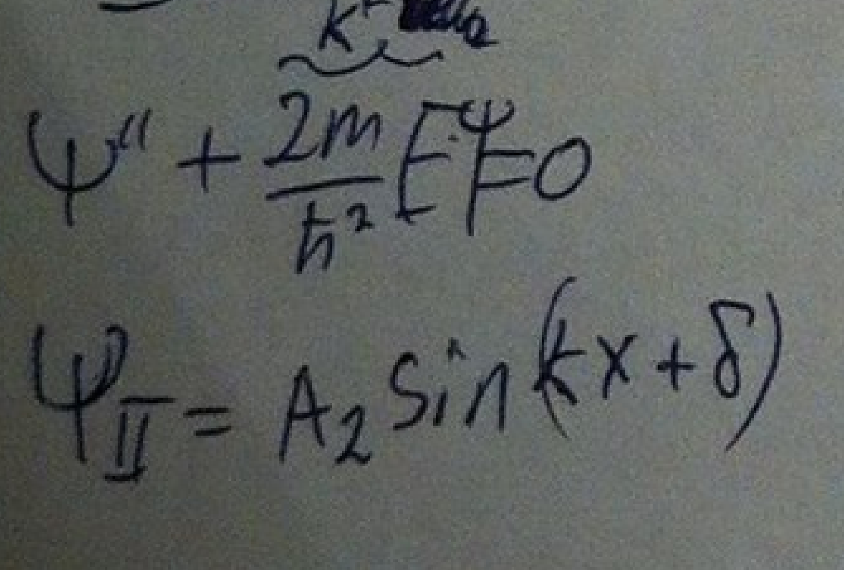
$B\varepsilon^2 - 2B\varepsilon + B - \varepsilon = 0 \Rightarrow B\varepsilon^2 - \varepsilon(2B+1)+B=0 \Rightarrow \varepsilon^2 - \varepsilon\left(\frac{2B+1}{B}\right)+1=0$

$$\varepsilon = \frac{\frac{1+2B}{B} \pm \sqrt{\left(\frac{2B+1}{B}\right)^2-4}}{2} = \frac{\frac{1}{B}+2 \pm\sqrt{\frac{1}{B^2}+\frac{4}{B}}}{2} =$$

$$E = U_0\cdot\varepsilon = \frac{U_0}{2}\left[\frac1B - 2 + \sqrt{\frac{1}{B^2}+\frac4B}\right] = \frac{0{,}1}{2}\left[\frac{1}{0{,}23}-2+\sqrt{\frac{1}{0{,}23^2}+\frac{4}{0{,}23}}\right] = 0{,}18 \text{ эВ}$$

для $U_0=10$ эВ. яма глубокая. используем нечётн. реш.

5



а) E_n = ?
б) ? E_1
в) $U_1\to\infty$
г) $U_1\to U_2$

I $(x<0)$ $\Psi'' + \frac{2m}{\hbar^2}\overbrace{(E-U_1(x))}^{-\varkappa_1^2}\Psi(x)=0$

$\Psi_I = A_1 e^{-\varkappa_1 x} + B_1 e^{\varkappa_1 x}$, т.к. $\Psi_I \xrightarrow[x\to-\infty]{} 0$

III $x>a$ $\Psi'' + \frac{2m}{\hbar^2}\overbrace{(E-U_2)}^{-\varkappa_3^2}\Psi(x)=0$

$\Psi_{III} = A_3 e^{-\varkappa_3 x} + B_3 e^{\varkappa_3 x}$; т.к. $\Psi_{III} \xrightarrow[x\to+\infty]{} 0$

II $0<x<a$

$\Psi'' + \underbrace{\frac{2m}{\hbar^2}E}_{k^2}\Psi=0$

$\Psi_{II} = A_2\sin(kx+\delta)$

$\Gamma(0): \quad \psi_I(0) = \psi_{II}(0), \quad \psi_I'(0) = \psi_{II}'(0)$

$$\left.\begin{aligned} B_1 &= A_2 \sin\delta \\ æ_1 B_1 &= A_2 k \cos(k\overset{0}{x}+\delta) \end{aligned}\right\} \quad \begin{aligned} &tg\,\delta = \frac{k}{æ_1} \Rightarrow ctg\,\delta = \frac{æ_1}{k} = \sqrt{\frac{U_1}{E}-1} \\ &k \cdot ctg\,\delta = \sqrt{\frac{2m}{\hbar^2}U_1 - k^2} \end{aligned}$$

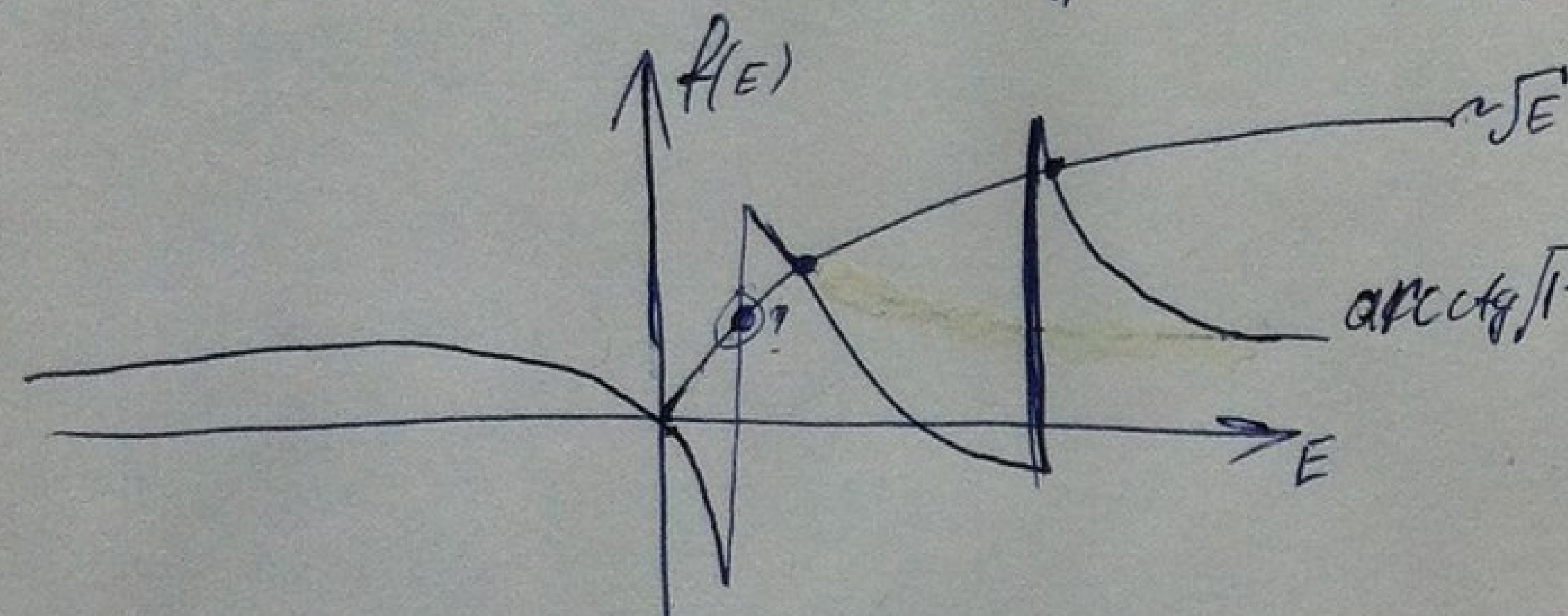
$\Gamma(a): \quad \psi_{II}(a) = \psi_3(a), \quad \psi_2'(a) = \psi_3'(a)$

$$\left.\begin{aligned} A_3 e^{-æ_3 a} &= A_2 \sin(ka+\delta) \\ -æ_3 A_3 e^{-æ_3 a} &= A_2 k \cos(ka+\delta) \end{aligned}\right\} \quad \begin{aligned} &tg(ka+\delta) = -\frac{k}{æ_3} \\ &k\,ctg(ka+\delta) = \sqrt{\frac{2m}{\hbar^2}U_2 - k^2} \end{aligned}$$

$$\left.\begin{aligned} \delta &= arcctg\sqrt{\frac{U_1}{E}-1} \\ ka+\delta &= -arcctg\sqrt{\frac{U_2}{E}-1} \end{aligned}\right\} \quad ka + arcctg\sqrt{\frac{U_1}{E}-1} = -arcctg\sqrt{\frac{U_2}{E}-1}$$

$$\sqrt{\frac{2m}{\hbar^2}E}\,a = -\left(arcctg\sqrt{\frac{U_1}{E}-1} + arcctg\sqrt{\frac{U_2}{E}-1}\right)$$

$$\sqrt{\frac{2m}{\hbar^2}E}\,a = arcctg\sqrt{1-\frac{U_1}{E}} + arcctg\sqrt{1-\frac{U_2}{E}}$$

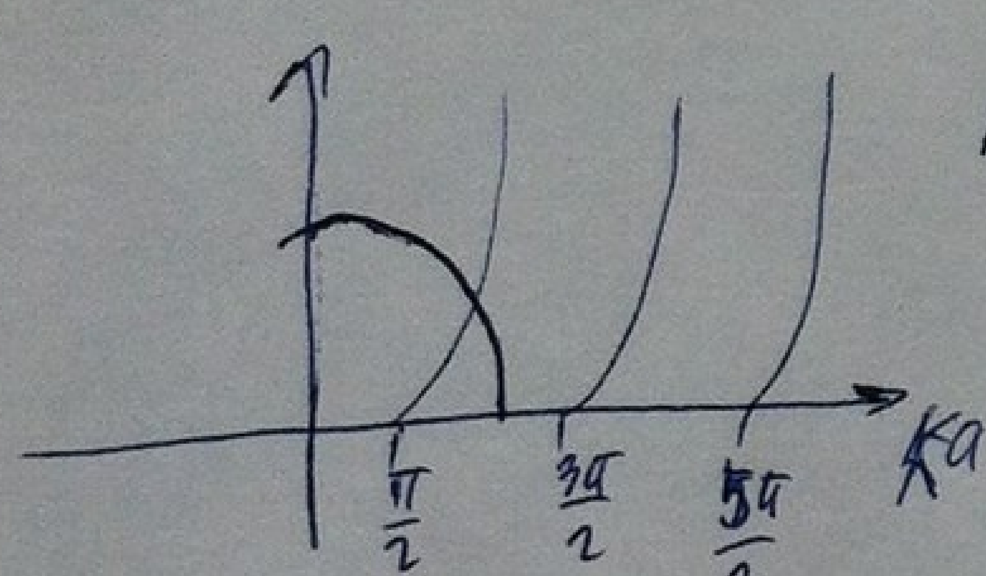


1) При $U_1 \to \infty$ $\quad \sqrt{\frac{2mE}{\hbar^2}}\,a = -arcctg\sqrt{1-\frac{U_2}{E}}$

$ctg\,ka = -\sqrt{1-\frac{U_2}{E}}$

$\Rightarrow k \cdot ctg(ka) = -\sqrt{\frac{2m}{\hbar^2}U_1 - k^2}$

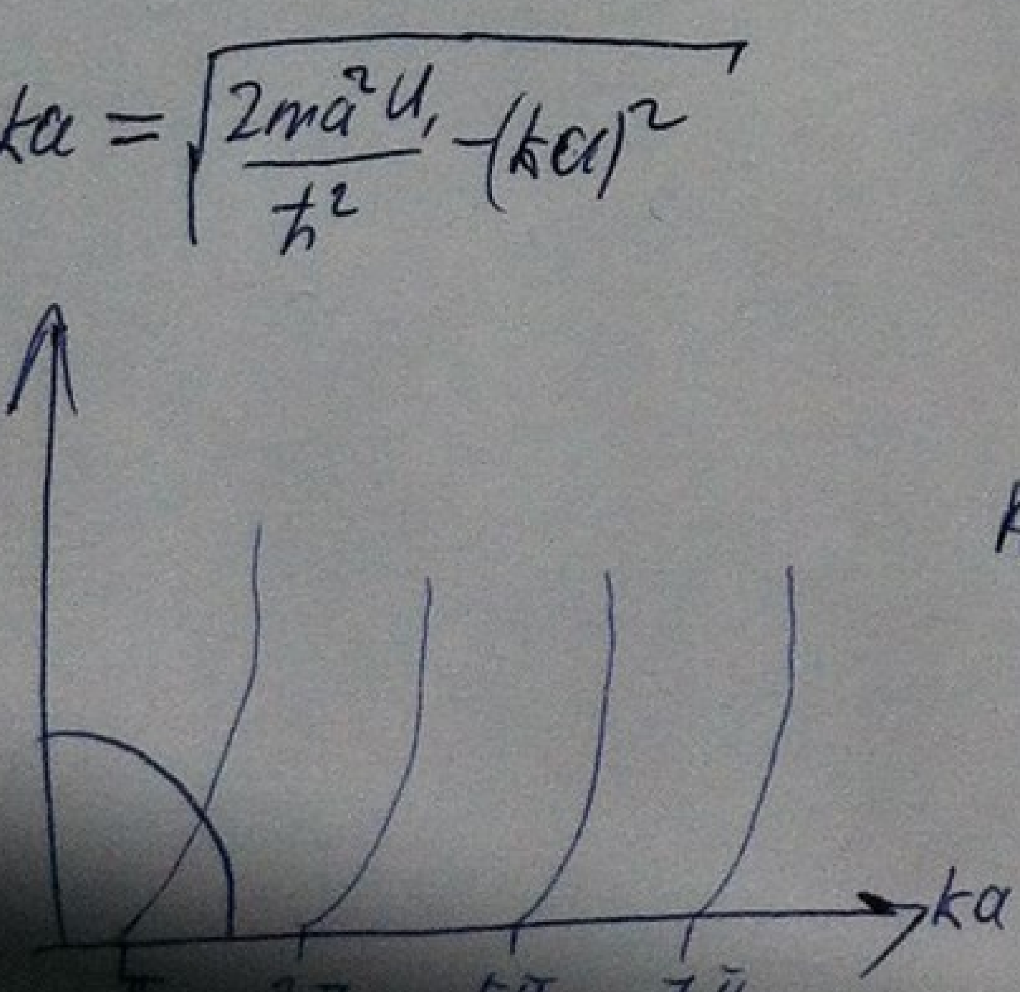
$-ka \cdot ctg(ka) = \sqrt{\frac{2ma^2}{\hbar^2}U_1 - (ka)^2}$



первый корень при $\frac{2ma^2}{\hbar^2}U_1 = \frac{\pi}{2}$

2) При $U_2 \to U_1$ $\quad \sqrt{\frac{2m}{\hbar^2}E}\,a = 2arcctg\sqrt{1-\frac{U_1}{E}} \Rightarrow \frac{1}{2}\sqrt{\frac{2m}{\hbar^2}E}\,a = arcctg\sqrt{1-\frac{U_1}{E}} \Rightarrow \sqrt{1-\frac{U_1}{E}} = ctg\,\frac{1}{2}\sqrt{\frac{2m}{\hbar^2}E}\,a$

$-ka \cdot ctg\,\frac{1}{2}ka = \sqrt{\frac{2ma^2U_1}{\hbar^2} - (ka)^2}$



первый корень при $\frac{2ma^2U_1}{\hbar^2} = \pi$

⑥ $V(x) = -U_0 e^{-x^2/2a^2}$, промоделировать E_0, E_1

а) $\sqcap$ ($-a$, a, $-U_0$), $S_U = S_{U^*}$

б) Г.О. $-U_0 e^{-x^2/2a} \to U_0\left(1-\frac{x^2}{2a^2}\right)$

в) δ-яма $B = 10;\ 3;\ 1;\ 0{,}1$ $E_0 = ?$ $E_1 = ?$ уб E_1 ...

а) $\int\limits_{-\infty}^{+\infty} -U_0 e^{-x^2/2a^2} dx = -U_0 \int\limits_{-\infty}^{+\infty} e^{-\frac{x^2}{2a^2}} dx = -U_0 \cdot \sqrt{\pi}\sqrt{2a^2} = -U^* \cdot 2a$

$(S_U = S_{U^*})$

$\Rightarrow U^* = \dfrac{U_0\sqrt{\pi}\sqrt{2a^2}}{2a} = \dfrac{U_0\sqrt{\pi}}{\sqrt{2}} \Rightarrow U_0 = U^*\sqrt{\frac{2}{\pi}}$

$\varepsilon = -\dfrac{E^*}{U^*} \Rightarrow E^* = -\varepsilon U^*$

$E_n^0 = -\varepsilon U_0\sqrt{\frac{\pi}{2}}$

Решение для U^*: чётн: $\operatorname{tg}\sqrt{B(1-\varepsilon)} = \sqrt{\frac{\varepsilon}{1-\varepsilon}}$ – для 0 уровня

нечётн: $\operatorname{tg}\sqrt{B(1-\varepsilon)} = -\sqrt{\frac{1-\varepsilon}{\varepsilon}}$ – для 1ого уровня.

B	10	3	1	0,1
ε_0	0,86	0,68	0,45	0,089
ε_1	0,46	0,2	—	—
E_0^0	$-1{,}078U_0$	$-0{,}854U_0$	$-0{,}564U_0$	$-0{,}11U_0$
E_1^0	$-0{,}576U_0$	$-0{,}25\,U_0$	—	—

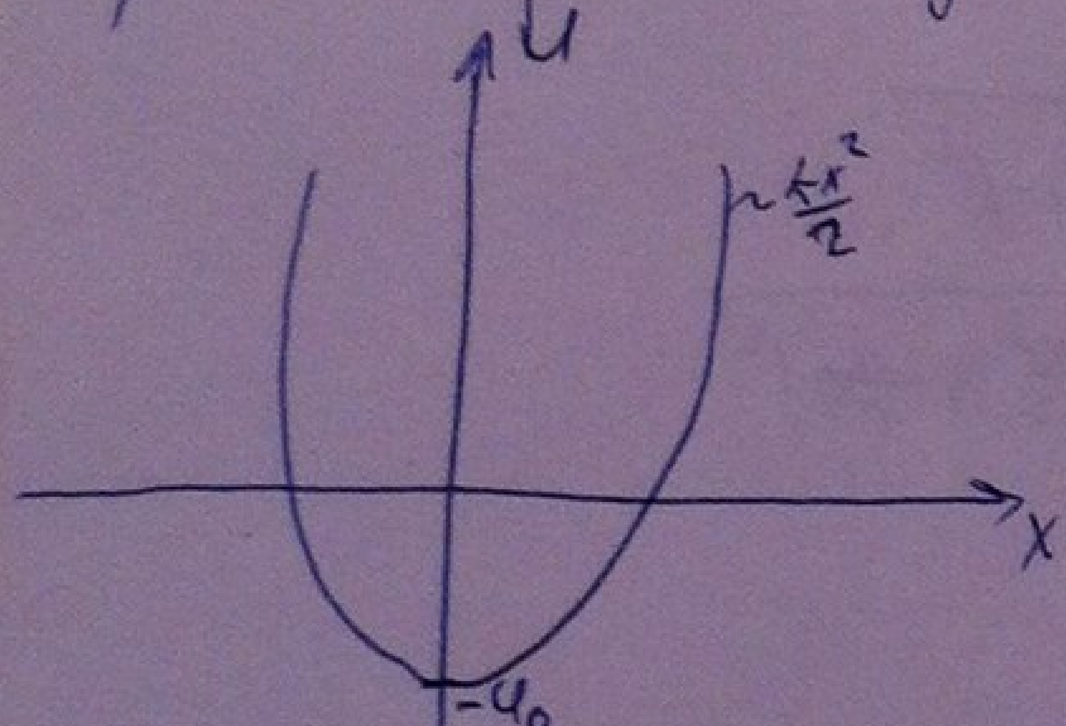
б) ~~...~~ в ряд Тейлора $V(x)$: $V(x) = -U_0\left(1-\frac{x^2}{2a^2}\right) = -U_0 + U_0\frac{x^2}{2a^2}$

$= -U_0 + \frac{kx^2}{2}$

$k = \frac{U_0}{a^2}$

$\omega = \sqrt{\frac{k}{m}}$

$\omega = \sqrt{\frac{U_0}{a^2 m}}$



$E_n^* = \hbar\omega(n+1/2)$

$E_n^0 = \hbar\omega(n+1/2) - U_0$

$E_n^0 = \hbar\sqrt{\frac{U_0}{a^2m}}(n+1/2) - U_0$

$\dfrac{E_n^0}{U_0} = \sqrt{\dfrac{\hbar^2}{U_0 a^2 m}}(n+1/2) - 1 = \sqrt{\dfrac{2}{B}} \cdot (n+1/2) - 1$

$\varepsilon = 1 - \sqrt{\frac{2}{B}}(n+1/2)$

B	10	3	1	0,1
ε_0	0,776	0,592	0,293	−1,23
ε_1	0,329	−0,225	−1,121	−5,708

в) $E = -\dfrac{q^2 m}{2\hbar^2}$

$U^* = -q\delta(x)$; $\int U_0 = -U_0\sqrt{\pi}\sqrt{2a^2}$

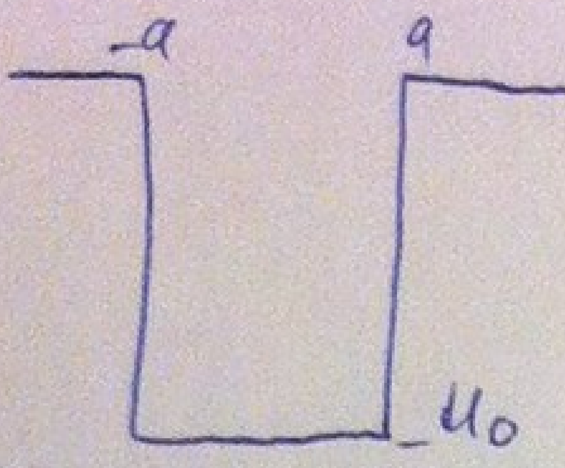
$\int U^* = \int -q\delta(x)dx = -q$

$\int U^* = \int U_0$

$E_0 = \dfrac{-m}{2\hbar^2}\,U_0^2\pi\cdot 2a^2 = -U_0\pi\,\dfrac{U_0 a^2 m}{\hbar^2}$

$-\dfrac{E_0}{U_0} = \varepsilon = +\pi B$

(7)



$\varepsilon_0 = -1$

$\varepsilon_1 = -0{,}7$

$tg\sqrt{B(1-\varepsilon)} = \sqrt{\frac{\varepsilon}{1-\varepsilon}}$ для 0

$tg\sqrt{B(1-\varepsilon)} = -\sqrt{\frac{1-\varepsilon}{\varepsilon}}$ для 1

$$tg\sqrt{2B} = \sqrt{\frac{-1}{2}} \Rightarrow \sqrt{2B} = arctg\sqrt{\frac{1}{2}}$$

$$tg\sqrt{1{,}7B} = -\sqrt{\frac{1{,}7}{-0{,}7}} \Rightarrow \sqrt{1{,}7B} = arctg\sqrt{\frac{1{,}7}{0{,}7}}$$

$$tg\sqrt{2B} = 0{,}707107i$$

$$tg\sqrt{1{,}7B} = -1{,}55839i$$

$$B = -0{,}5((-\pi i n + 0{,}881374))^2$$

$$B = -0{,}588235((\pi i n + (0{,}76104 - 1{,}5708i)))^2$$

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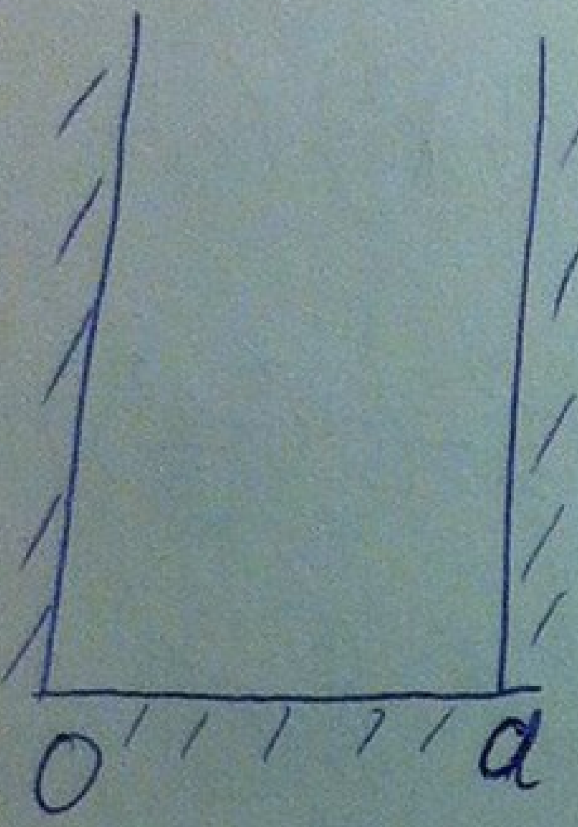
(8)



$$|\psi_n\rangle = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi n x}{a}\right)$$

$\hat{x}(E)$

$\hat{p}(E)$

$H|\psi_n\rangle = E_n|\psi_n\rangle$ – энергетич. представление

$$x_{mn} = \int_0^a \psi_m^*(x)\, x\, \psi_n(x)\,dx$$

Правило отбора.

$$p_{mn} = \int_0^a \psi_m^*(x)\frac{\hbar}{i}\frac{d}{dx}\psi_n(x)\,dx$$

$$x_{mn} = \frac{2}{a}\int_0^a \sin\left(\frac{\pi m x}{a}\right)\cdot\sin\left(\frac{\pi n x}{a}\right)dx = \frac{1}{a}\left[\int_0^a \cos\left(\frac{\pi x}{a}(m-n)\right)dx - \int_0^a \cos\left(\frac{\pi x}{a}(m+n)\right)dx\right] =$$

$$= \frac{1}{\pi}\left[\int_0^a \frac{1}{m-n}\cos\left(\frac{\pi x}{a}(m-n)\right)d\left(\frac{\pi x}{a}(m-n)\right) - \int_0^a \cos\left(\frac{\pi x}{a}(m+n)\right)d\left(\frac{\pi x}{a}(m+n)\right)\frac{1}{m+n}\right] =$$

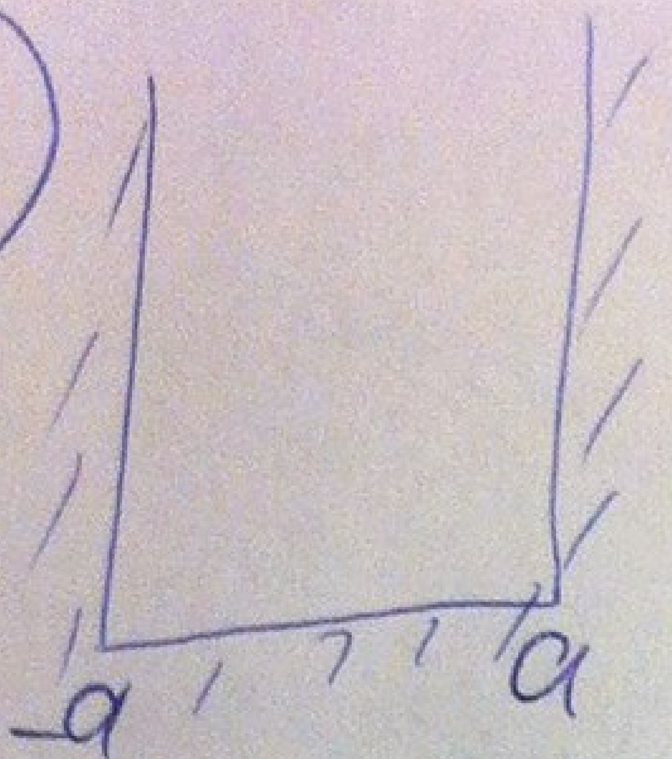
$$= \frac{1}{\pi}\left[\frac{1}{m-n}\sin\left(\frac{\pi x}{a}(m-n)\right)\Big|_0^a - \frac{1}{m+n}\sin\left(\frac{\pi x}{a}(m-n)\right)\Big|_0^a\right] = \frac{1}{\pi}\left[\frac{1}{m-n}\sin\pi(m-n) - \frac{1}{m+n}\sin\pi(m+n)\right]$$

$$p_{mn} = \frac{2}{a}\int_0^a \sin\left(\frac{\pi m x}{a}\right)\cdot\frac{\hbar}{i}\frac{\pi n}{a}\cos\left(\frac{\pi n x}{a}\right)dx = \frac{\hbar}{i}\frac{\pi n}{a^2}\left[\int_0^a \sin\left[\frac{\pi x}{a}(m+n)\right]dx + \int_0^a \sin\left[\frac{\pi x}{a}(m-n)\right]dx\right] =$$

$$= \frac{\hbar}{i}\frac{n}{a}\left[\frac{-1}{m+n}\cos\left[\frac{\pi x}{a}(m+n)\right]\Big|_0^a + \frac{-1}{m-n}\cos\left[\frac{\pi x}{a}(m-n)\right]\Big|_0^a\right] = \frac{\hbar}{i}\frac{n}{a}\left[2 - \frac{\cos\pi(m+n)}{m+n} - \frac{\cos\pi(m-n)}{m-n}\right]$$

$$\langle E|\hat{x}|\tilde{E}\rangle \int dx\,d\tilde{x}\,\psi_E^*(x)\langle x|\hat{x}|\tilde{x}\rangle\psi_{\tilde{E}}(\tilde{x}) = \quad ?$$

(9)



$\psi(x) = N(a^2-x^2)$ $N=?$ $\psi' = N(-2x)$

$\psi'' = -2N$

а) fidelity $\psi_1(x)$ и ψ_0

б) $\langle E\rangle = ?$ $D_E = ?$

Нормировка

$$1 = \int_0^a N^2(a^2-x^2)^2dx = N^2\int_{-a}^{a}(a^4-2a^2x^2+x^4)dx = N^2\left(a^4x - 2a^2\frac{x^3}{3}+\frac{x^5}{5}\right)\Bigg|_{-a}^{a} =$$

$$= 2N^2\left(a^5-\frac{2a^5}{3}+\frac{a^5}{5}\right) = 2N^2a^5\left(1-\frac{2}{3}+\frac{1}{5}\right) = 2N^2a^5\left(\frac{1}{3}+\frac{1}{5}\right) = N^2a^5\frac{16}{15} \Rightarrow N = \sqrt{\frac{15}{16a^5}}$$

$$\psi(x) = \sqrt{\frac{15}{16a^5}}(a^2-x^2)$$

$$F = \frac{|\langle\varphi|\psi\rangle|^2}{\langle\varphi|\varphi\rangle\langle\psi|\psi\rangle} = |\langle\varphi|\psi\rangle|^2$$

$$-\frac{\hbar^2}{2m}\psi'' = E\psi$$

$$\psi'' + \frac{2m}{\hbar^2}E\psi = 0$$

$$-2N + \frac{2m}{\hbar^2}EN(a^2-x^2) = 0$$

$$E = \frac{\hbar^2}{m}\frac{1}{a^2-x^2}$$

$$\langle E\rangle = \frac{\hbar^2}{m}\cdot N^2\int_{-a}^{a}\frac{(a^2-x^2)^2}{a^2-x^2}dx = \frac{\hbar^2}{m}\cdot N^2\int_{-a}^{a}(a^2-x^2)dx =$$

$$= \frac{\hbar^2}{m}N^2\left(a^2x-\frac{x^3}{3}\right)\Bigg|_{-a}^{a} = \frac{\hbar^2}{m}N^2\left(a^3-\frac{a^3}{3}-\left(-a^3+\frac{a^3}{3}\right)\right) =$$

$$= \frac{\hbar^2}{m}N^2\, 2\cdot\frac{2}{3}a^3 = \frac{4\hbar^2N^2}{3m}a^3 = \frac{4}{3}\frac{\hbar^2}{m}a^3\cdot\frac{15}{16a^5} =$$

$$= \frac{\hbar^2}{ma^2}\frac{5}{4}$$

$$\langle E\rangle^2 = \frac{\hbar^4\, 25}{m^2a^4\cdot 16}$$

$$\langle E^2\rangle = \frac{\hbar^4}{m^2}\cdot N^2\int_{-a}^{a}\frac{(a^2-x^2)^2}{(a^2-x^2)^2}dx = \frac{\hbar^4}{m}N^2\cdot x\Big|_{-a}^{a} = \frac{2\hbar^4}{m}N^2a = \frac{2\hbar^4a}{m}\frac{15}{16a^5} = \frac{15}{8}\frac{\hbar^4}{ma^4}$$

$$D_E = \langle E^2\rangle - \langle E\rangle^2 = \frac{\hbar^4}{m^2a^4}\left[\frac{15}{8}-\frac{25}{16}\right] = \frac{5}{16}\frac{\hbar^4}{m^2a^4}$$

$$\psi_2 = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)$$

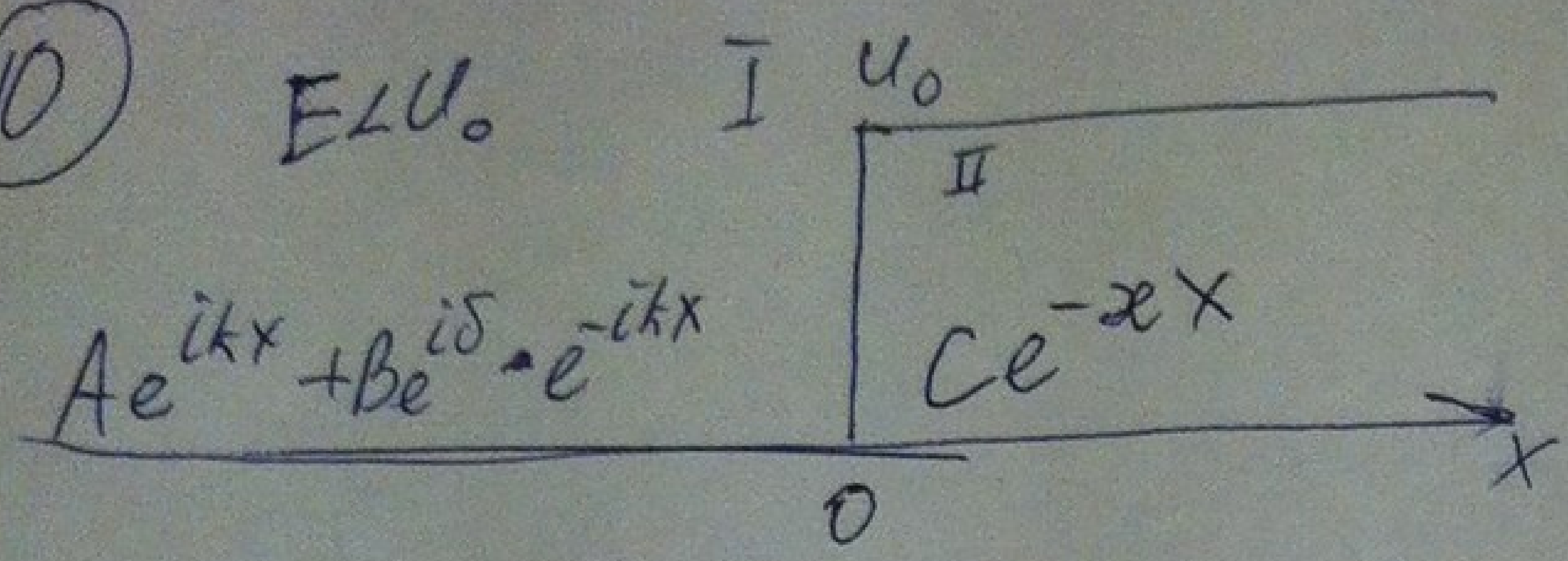
$$\psi_1 = \sqrt{\frac{2}{a}}\cos\frac{\pi x}{a}$$

$$\cancel{F = \sqrt{\frac{15}{16a^5}}\cdot\sqrt{\frac{2}{a}}\int_{-a}^{a}\sin\left(\frac{\pi x}{a}\right)\cdot(a^2-x^2)dx = \sqrt{\frac{15}{16a^2}}\sqrt{\frac{2}{a}}\cdot\frac{a^3}{(\pi)^3}}$$

$$F = \sqrt{\frac{15}{16a^5}}\sqrt{\frac{2}{a}}\int_{-a}^{a}(a^2-x^2)\cos\left(\frac{\pi x}{a}\right)dx = \sqrt{\frac{15}{16a^5}}\sqrt{\frac{2}{a}}\,\frac{a\left(((2+\pi^2)a^2-\pi^2x^2)\sin\left(\frac{\pi x}{a}\right)-2\pi ax\cos\left(\frac{\pi x}{a}\right)\right)}{\pi^3}\Bigg|_{-a}^{a} =$$

$$= \sqrt{\frac{15}{16a^5}}\sqrt{\frac{2}{a}}\left[\frac{-2\pi a^2\cdot(-1)}{\pi^3} - \frac{+2\pi a^2\cdot(-1)}{\pi^3}\right] = \sqrt{\frac{15}{16a^5}}\sqrt{\frac{2}{a}}\cdot\frac{4a^2}{\pi^2} = \sqrt{\frac{15\cdot16}{8}}\frac{1}{a^3}\frac{a^2}{\pi^2} = \frac{\sqrt{30}}{a\pi^2}$$

(10) $E<U_0$



I: $-\frac{\hbar^2}{2m}\psi''-E\psi=0$

$\psi''+\frac{2mE}{\hbar^2}\psi=0$

$k=\sqrt{\frac{2mE}{\hbar^2}}$

II $-\frac{\hbar^2}{2m}\psi''+(U-E)\psi=0$

$\psi''+\frac{2m}{\hbar^2}(U+E)\psi=0$

$\frac{2m}{\hbar^2}(U-E)=\varkappa^2$

$\left.\begin{array}{l}\psi_I(0)=\psi_{II}(0)\\ \psi_I'(0)=\psi_{II}'(0)\end{array}\right\}$

$\left.\begin{array}{l}A+Be^{i\delta}=C\\ ikA-ikBe^{i\delta}=-\varkappa C\end{array}\right\}$

$ikA-ikBe^{i\delta}=-\varkappa(A+Be^{i\delta})$

примем $A=B$

$Bik-ikBe^{i\delta}=-\varkappa(B+Be^{i\delta})$

$ik-ike^{i\delta}=-\varkappa(1+e^{i\delta})$

$ik-ike^{i\delta}+\varkappa+\varkappa e^{i\delta}=0$

$e^{i\delta}(\varkappa-ik)=-(\varkappa+ik)$

$e^{i\delta}=\frac{-(\varkappa+ik)}{\varkappa-ik}=$

$=-\frac{(\varkappa+ik)^2}{(\varkappa-ik)(\varkappa+ik)}=-\frac{(\varkappa+ik)^2}{\varkappa^2+k^2}=\frac{\varkappa^2+2ik\varkappa-k^2}{-(\varkappa^2+k^2)}$

---

(11) $\psi_k(x)=\begin{cases}e^{ikx}+Ae^{-ikx}, & x<0 \quad k=\sqrt{\frac{2mE}{\hbar^2}}\\ Be^{i\varkappa x}, & x>0 \quad \varkappa=\sqrt{\frac{2m(E-U_0)}{\hbar^2}}\end{cases}$

$\left.\begin{array}{l}\psi_I(0)=\psi_{II}(0)\\ \psi_I'(0)=\psi_{II}'(0)\end{array}\right\} \Rightarrow \begin{array}{l}1+A=B\\ k(1-A)=\varkappa B\end{array}$

$A=\frac{k-\varkappa}{k+\varkappa}$

$B=\frac{2k}{k+\varkappa}$

$|R|=|A|^2=\left(\frac{k-\varkappa}{k+\varkappa}\right)^2$

$|T|=|B|^2\frac{\varkappa}{k}=\left|\frac{2k}{k+\varkappa}\right|^2\frac{\varkappa}{k}$

Д/з на 28.04.14

№1

$[\psi_n(x)] = \frac{1}{\sqrt{M}}$ $\quad \int_{-\infty}^{+\infty} dx\, \psi_n^*(x)\psi_m(x) = \delta_{n,m}$ $\quad [\delta] = \frac{1}{E}$

$[\psi_E(x)] = \frac{1}{\sqrt{M \cdot Дж}}$ $\quad \int_{-\infty}^{+\infty} dx\, \psi_{E'}^*(x)\psi_E(x) = \delta(E'-E)$

$[\psi_n(p)] = \frac{1}{\sqrt{kг \cdot \frac{м}{с}}}$

$[\psi_E(p)] = \frac{1}{\sqrt{kг \frac{м}{с} \cdot Дж}}$

№2 $H = \frac{p^2}{2m}$ $\quad [H, p] = 0$

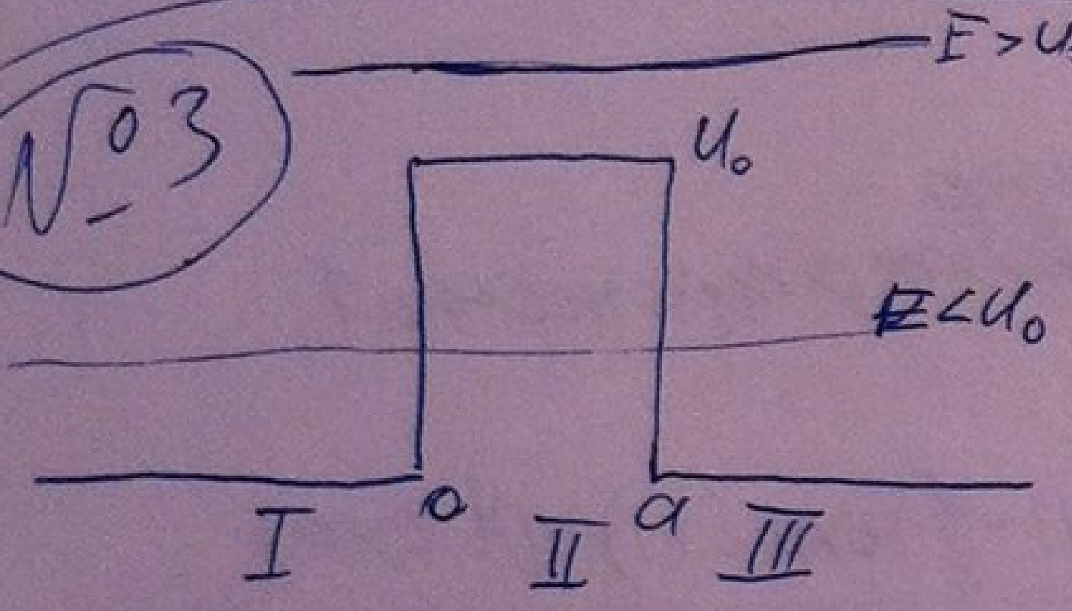
Собственные функции, обладающие чётностью?

sin и cos

$N\cos\frac{p}{\hbar}x$ $\quad N = ?$

$N'\sin\frac{p}{\hbar}x$ $\quad N' = ?$

№3



$R(E) = ?$

$T(E) = ?$

Графики для

$\beta \gg 1$

$\beta \ll 1$

а) $E > U_0$

I, III $\quad -\frac{\hbar^2}{2m}\psi''(x) - E\psi(x) = 0$

$\psi_I(x) = B_1\exp(ikx) + A\exp(-ikx)$

$\psi_{III}(x) = B_3\exp(ik(x-a))$

II $\quad -\frac{\hbar^2}{2m}\psi''(x) + V(x)\psi(x) = E\psi(x)$

$\psi_{II} = B_2\exp(i\varkappa x) + A_2\exp(-i\varkappa x)$

$k = \sqrt{\frac{2mE}{\hbar^2}}, \quad \varkappa = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}, \quad B_1 = 1$

$$\left.\begin{array}{l} \psi_I(0) = \psi_{II}(0) \\ \psi_I'(0) = \psi_{II}'(0) \\ \psi_{II}(a) = \psi_{III}(a) \\ \psi_{II}'(a) = \psi_{III}'(a) \end{array}\right\}$$

$$\left.\begin{array}{l} 1 + A = B_2 + A_2 \\ k(1-A) = \varkappa(B_2 - A_2) \\ B_3 = B_2\exp(i\varkappa a) + A_2\exp(-i\varkappa a) \\ kB_3 = \varkappa(B_2\exp(i\varkappa a) - A_2\exp(i\varkappa a)) \end{array}\right\}$$

$$\left.\begin{array}{l} 1 + A = B_2 + A_2 \\ 1 - A = \frac{\varkappa}{k}(B_2 - A_2) \end{array}\right\} \Rightarrow 2 = B_2\left(\frac{\varkappa}{k}+1\right) + A_2\left(1 - \frac{\varkappa}{k}\right)$$

$$\left.\begin{array}{l} B_3 = B_2\exp(i\varkappa a) + A_2\exp(-i\varkappa a) \\ B_3 = \frac{\varkappa}{k}(B_2\exp(i\varkappa a) - A_2\exp(-i\varkappa a)) \end{array}\right\} \Rightarrow 0 = B_2\exp(i\varkappa a)\left(1 - \frac{\varkappa}{k}\right) + A_2\exp(-i\varkappa a)\left(1 + \frac{\varkappa}{k}\right) \Rightarrow$$

$$\Rightarrow \frac{2 - A_2\left(1 - \frac{\varkappa}{k}\right)}{\left(1 + \frac{\varkappa}{k}\right)} = B_2 \Rightarrow \frac{2 - A_2\left(1 - \frac{\varkappa}{k}\right)}{\left(1 + \frac{\varkappa}{k}\right)}\exp(i\varkappa a)\left(1 - \frac{\varkappa}{k}\right) + A_2\exp(-i\varkappa a)\left(1 + \frac{\varkappa}{k}\right) \Rightarrow$$

$$\Rightarrow \frac{2\left(1 - \frac{\varkappa}{k}\right)}{\left(1 + \frac{\varkappa}{k}\right)}\exp(i\varkappa a) = A_2\left(\frac{\left(1 - \frac{\varkappa}{k}\right)^2}{\left(1 + \frac{\varkappa}{k}\right)}\exp(i\varkappa a) - \left(1 + \frac{\varkappa}{k}\right)\exp(-i\varkappa a)\right)$$

$$2\left(1 - \frac{\varkappa}{k}\right)\exp(i\varkappa a) = A_2\left(\left(1 - \frac{\varkappa}{k}\right)^2\exp(i\varkappa a) - \left(1 + \frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)\right)$$

$$A_2=\frac{2\left(1-\frac{\varkappa}{k}\right)\exp(i\varkappa a)}{\left(1-\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)}$$

$$\frac{2-A_2\left(1-\frac{\varkappa}{k}\right)}{\left(1+\frac{\varkappa}{k}\right)}=B_2=\frac{2}{\left(1+\frac{\varkappa}{k}\right)}-\frac{\left(1-\frac{\varkappa}{k}\right)}{\left(1+\frac{\varkappa}{k}\right)}\cdot\frac{2\left(1-\frac{\varkappa}{k}\right)\exp(i\varkappa a)}{\left(1-\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)}=$$

$$=\frac{2}{1+\frac{\varkappa}{k}}\left[\frac{\left(1-\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)-\left(1-\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)}{\text{— '' —}}\right]=-\frac{\frac{2}{\left(1+\frac{\varkappa}{k}\right)}\left(1+\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)}{\text{— '' —}}$$

$$A=B_2+A_2-1=\frac{-2\left(1+\frac{\varkappa}{k}\right)\exp(-i\varkappa a)+2\left(1-\frac{\varkappa}{k}\right)\exp(i\varkappa a)-\left(1-\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)+\left(1+\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)}{\text{— '' —}}$$

$$=\frac{\exp(i\varkappa a)\left(2\left(1-\frac{\varkappa}{k}\right)-\left(1-\frac{\varkappa}{k}\right)^2\right)+\exp(-i\varkappa a)\left(\left(1+\frac{\varkappa}{k}\right)^2-2\left(1+\frac{\varkappa}{k}\right)\right)}{\text{— '' —}}=$$

$$=\frac{\exp(i\varkappa a)\left(2-\frac{2\varkappa}{k}-1+\frac{2\varkappa}{k}-\frac{\varkappa^2}{k^2}\right)+\exp(-i\varkappa a)\left(1+\frac{2\varkappa}{k}+\frac{\varkappa^2}{k^2}-2-\frac{2\varkappa}{k}\right)}{\text{— '' —}}=$$

$$=\frac{\exp(i\varkappa a)\left(1-\frac{\varkappa^2}{k^2}\right)+\exp(-i\varkappa a)\left(\frac{\varkappa^2}{k^2}-1\right)}{\text{— '' —}}$$ — ~~коэффициент~~ амплитуда отражения

$$B_3=B_2\exp(i\varkappa a)+A_2\exp(-i\varkappa a)=\frac{-2\left(1+\frac{\varkappa}{k}\right)\exp(-i\varkappa a)\cdot\exp(i\varkappa a)+2\left(1-\frac{\varkappa}{k}\right)\exp(i\varkappa a)\exp(-i\varkappa a)}{\text{— '' —}}=$$

$$=\frac{-2-\frac{2\varkappa}{k}+2-\frac{2\varkappa}{k}}{\text{— '' —}}=\frac{-\frac{4\varkappa}{k}}{\text{— '' —}}$$ — ~~коэффициент~~ амплитуда прохождения.

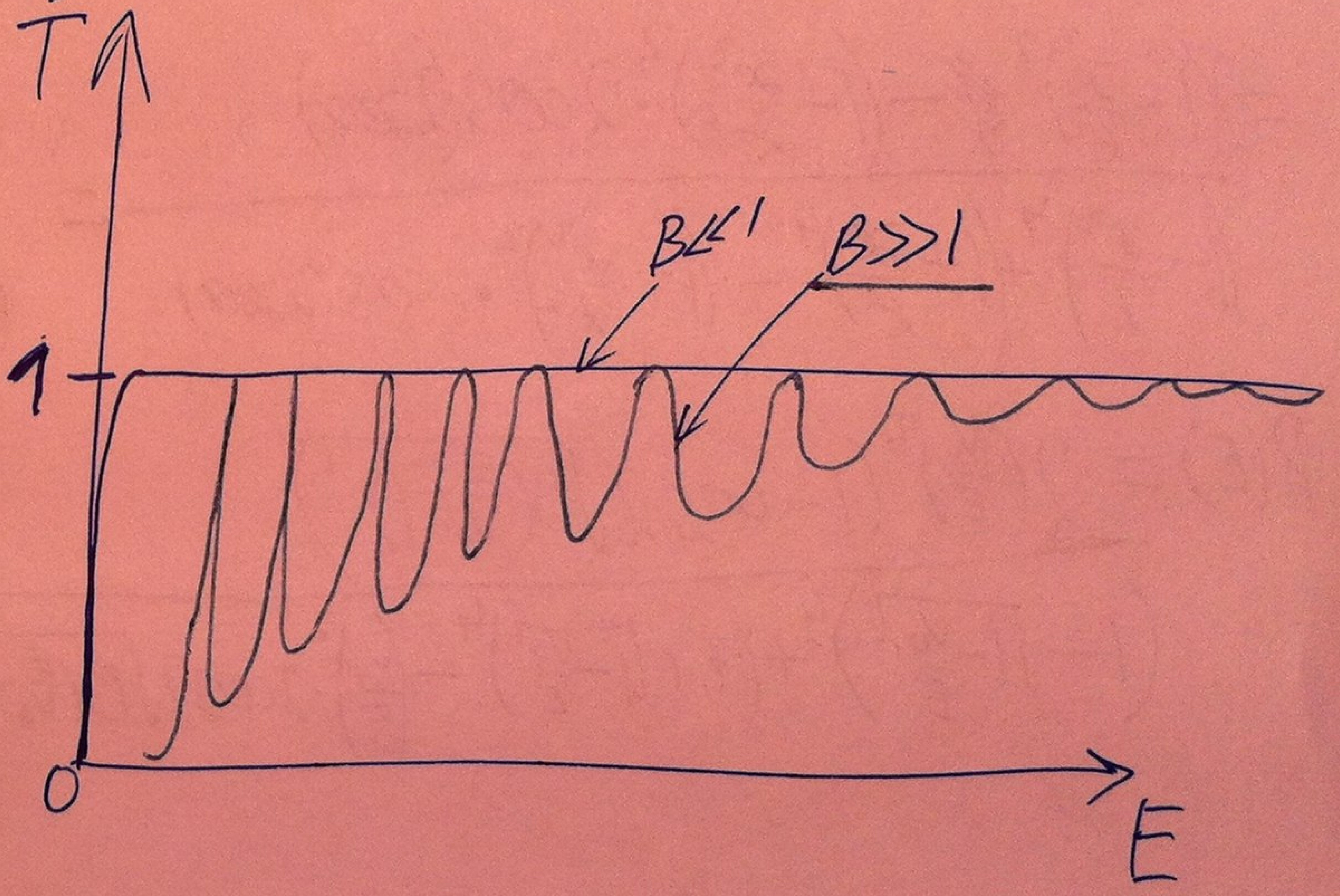
$$~~R=|A|^2=\frac{2\left(1-\frac{\varkappa}{k}\right)\exp(i\varkappa a)\cdot 2\left(1-\frac{\varkappa}{k}\right)\exp(-i\varkappa a)}{\left[\left(1-\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)\right]\left[\left(1-\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)\right]}=$$

$$=\frac{4\left(1-\frac{\varkappa}{k}\right)^2}{\ldots}=$$

$$T = |B_3|^2 = \frac{\left(-\frac{4\varkappa}{k}\right)\cdot\left(-\frac{4\varkappa}{k}\right)}{\left[\left(1-\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)\right]\left[\left(1-\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)\right]} =$$

$$= \frac{16\frac{\varkappa^2}{k^2}}{\left(1-\frac{\varkappa}{k}\right)^4+\left(1+\frac{\varkappa}{k}\right)^4-\left(1-\frac{\varkappa^2}{k^2}\right)^2\cdot 2\cos(2\varkappa a)}$$

$$T(E) = \frac{16\cdot\left(1-\frac{V_0}{E}\right)}{\left(1-\sqrt{1-\frac{V_0}{E}}\right)^4+\left(1+\sqrt{1-\frac{V_0}{E}}\right)^4-\left(\frac{V_0}{E}\right)^2\cdot 2\cos\left(2\sqrt{B\left(\frac{E}{V_0}-1\right)}\right)}$$



$$\cancel{\left(1+\frac{\varkappa^2}{k^2}\right)^2\exp(-2\varkappa a)+\left(1+\frac{\varkappa^2}{k^2}\right)\exp(2\varkappa a)+\left(1-\frac{i\varkappa}{k}\right)^4+\left(1+\frac{i\varkappa}{k}\right)^4}$$

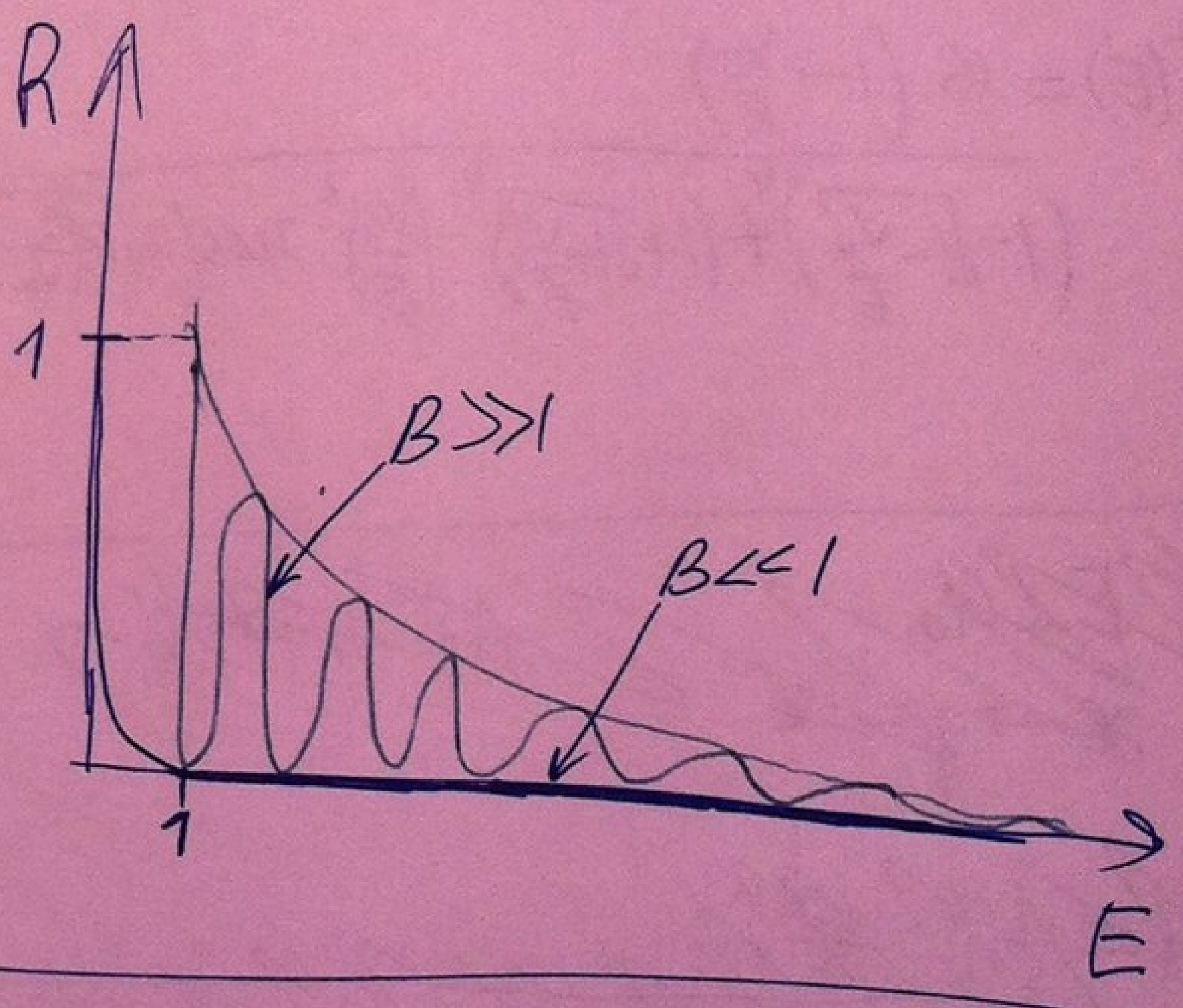
$$\cancel{=4\left(1+\frac{\varkappa^2}{k^2}\right)}$$

$$R=|A|^2=\frac{\left[\exp(i\varkappa a)\left(1-\frac{\varkappa^2}{k^2}\right)+\exp(-i\varkappa a)\left(\frac{\varkappa^2}{k^2}-1\right)\right]\left[\exp(-i\varkappa a)\left(1-\frac{\varkappa^2}{k^2}\right)+\exp(i\varkappa a)\left(\frac{\varkappa^2}{k^2}-1\right)\right]}{\left[\left(1-\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)\right]\left[\left(1-\frac{\varkappa}{k}\right)^2\exp(-i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\exp(i\varkappa a)\right]}=$$

$$=\frac{\left(1-\frac{\varkappa^2}{k^2}\right)^2+\left(\frac{\varkappa^2}{k^2}-1\right)^2+\exp(2i\varkappa a)\left(1-\frac{\varkappa^2}{k^2}\right)\left(\frac{\varkappa^2}{k^2}-1\right)+\exp(-2i\varkappa a)\left(\frac{\varkappa^2}{k^2}-1\right)\left(1-\frac{\varkappa^2}{k^2}\right)}{\left(1-\frac{\varkappa}{k}\right)^4+\left(1+\frac{\varkappa}{k}\right)^4-\left(1-\frac{\varkappa}{k}\right)^2\left(1+\frac{\varkappa}{k}\right)^2\exp(2i\varkappa a)-\left(1+\frac{\varkappa}{k}\right)^2\left(1-\frac{\varkappa}{k}\right)^2\exp(-2i\varkappa a)}=$$

$$=\frac{2\left(1-\frac{\varkappa^2}{k^2}\right)^2-\left(1-\frac{\varkappa^2}{k^2}\right)^2\cdot 2\cos(2\varkappa a)}{\left(1-\frac{\varkappa}{k}\right)^4+\left(1+\frac{\varkappa}{k}\right)^4-\left(1-\frac{\varkappa^2}{k^2}\right)^2\cdot 2\cos(2\varkappa a)}=\frac{2\left(1-\frac{\varkappa^2}{k^2}\right)^2\left(1-\cos(2\varkappa a)\right)}{\left(1-\frac{\varkappa}{k}\right)^4+\left(1+\frac{\varkappa}{k}\right)^4-\left(1-\frac{\varkappa^2}{k^2}\right)\cdot 2\cos(2\varkappa a)}$$

$$R(E)=\frac{2\left(\frac{V_0}{E}\right)^2\left(1-\cos 2\sqrt{B\left(\frac{E}{V_0}-1\right)}\right)}{\left(1-\sqrt{1-\frac{V_0}{E}}\right)^4+\left(1+\sqrt{1-\frac{V_0}{E}}\right)^4-\left(\frac{V_0}{E}\right)^2\cdot 2\cos 2\sqrt{B\left(\frac{E}{V_0}-1\right)}}$$



б) $E<U_0$ $\quad \varkappa\to q=i\varkappa=\sqrt{\frac{2m(V_0-E)}{\hbar^2}}$

$$R=|A|^2=\frac{2\left(1+\frac{\varkappa^2}{k^2}\right)^2\left(1-\cos(2i\varkappa a)\right)}{\left(1-\frac{i\varkappa}{k}\right)^4+\left(1+\frac{i\varkappa}{k}\right)^4-\left(1+\frac{\varkappa^2}{k^2}\right)^2\cdot 2\cos(2i\varkappa a)}=\frac{2\left(1+\frac{\varkappa^2}{k^2}\right)^2\left(1-\mathrm{ch}(2\varkappa a)\right)}{\left(1-\frac{i\varkappa}{k}\right)^4+\left(1+\frac{i\varkappa}{k}\right)^4-\left(1+\frac{\varkappa^2}{k^2}\right)^2\cdot 2\,\mathrm{ch}(2\varkappa a)}=$$

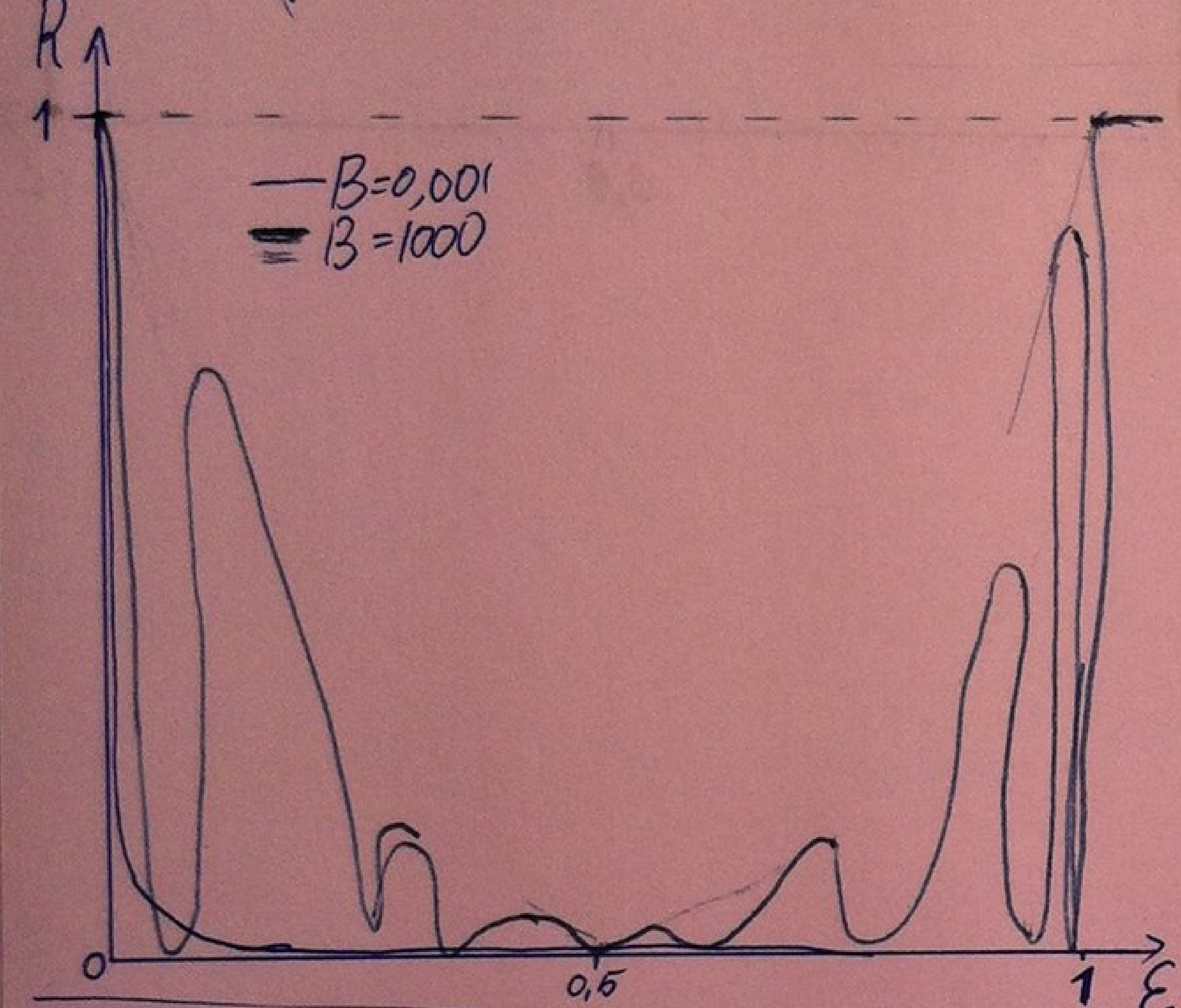
$$=\frac{2\left(1+1-\frac{V_0}{E}\right)^2\left(1-\mathrm{ch}\,2\sqrt{B\left(\frac{E}{V_0}-1\right)}\right)}{\left(1-\sqrt{\frac{V_0}{E}-1}\right)^4+\left(1+\sqrt{\frac{V_0}{E}-1}\right)^4-\left(1+1-\frac{V_0}{E}\right)^2\cdot 2\,\mathrm{ch}\left(2\sqrt{B\left(\frac{E}{V_0}-1\right)}\right)}$$

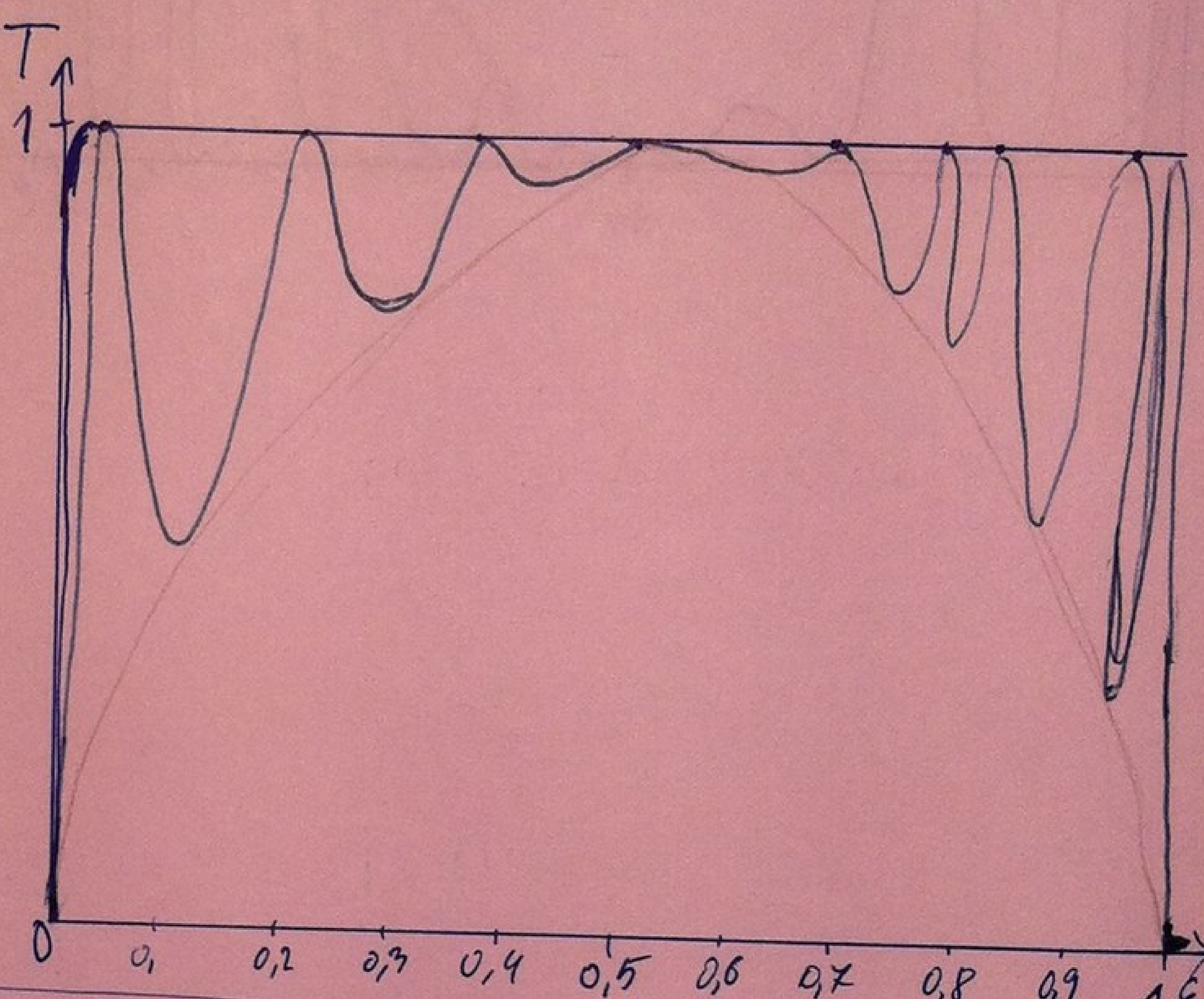
$$\varepsilon=\frac{E}{V_0}$$

$$T=\frac{16\frac{(i\varkappa)^2}{k^2}}{\left(1-\frac{i\varkappa}{k}\right)^4+\left(1+\frac{i\varkappa}{k}\right)^4-\left(1+\frac{\varkappa^2}{k^2}\right)^2\cdot 2\cos(2i\varkappa a)}=$$

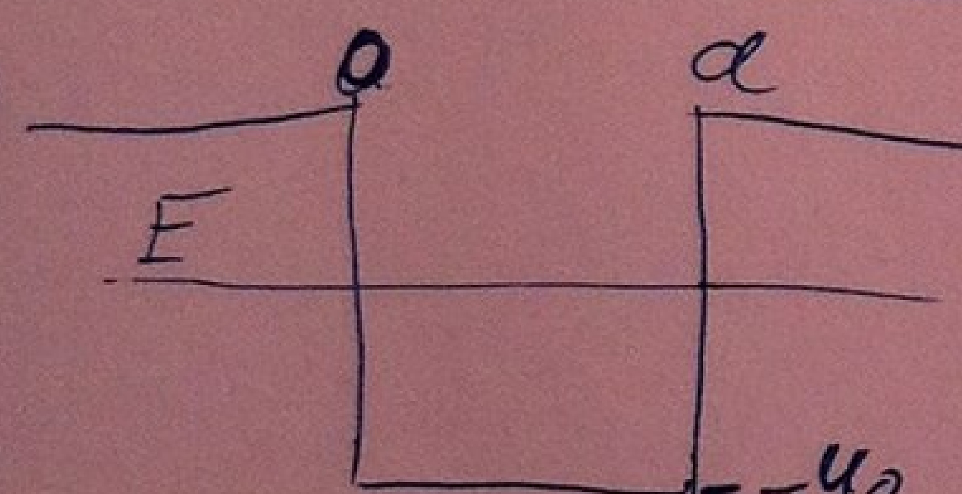
$$=-\frac{16\frac{\varkappa^2}{k^2}}{\left(1-\frac{i\varkappa}{k}\right)^4+\left(1+\frac{i\varkappa}{k}\right)^4-\left(1+\frac{\varkappa^2}{k^2}\right)^2\cdot 2ch(2\varkappa a)}$$

$$T(E)=-\frac{16\left(1-\frac{V_0}{E}\right)}{\left(1-\sqrt{\frac{V_0}{E}-1}\right)^4+\left(1+\sqrt{\frac{V_0}{E}-1}\right)^4-\left(2-\frac{V_0}{E}\right)^2\cdot 2ch\left(2\sqrt{B\left(\frac{E}{V_0}-1\right)}\right)}$$





b)



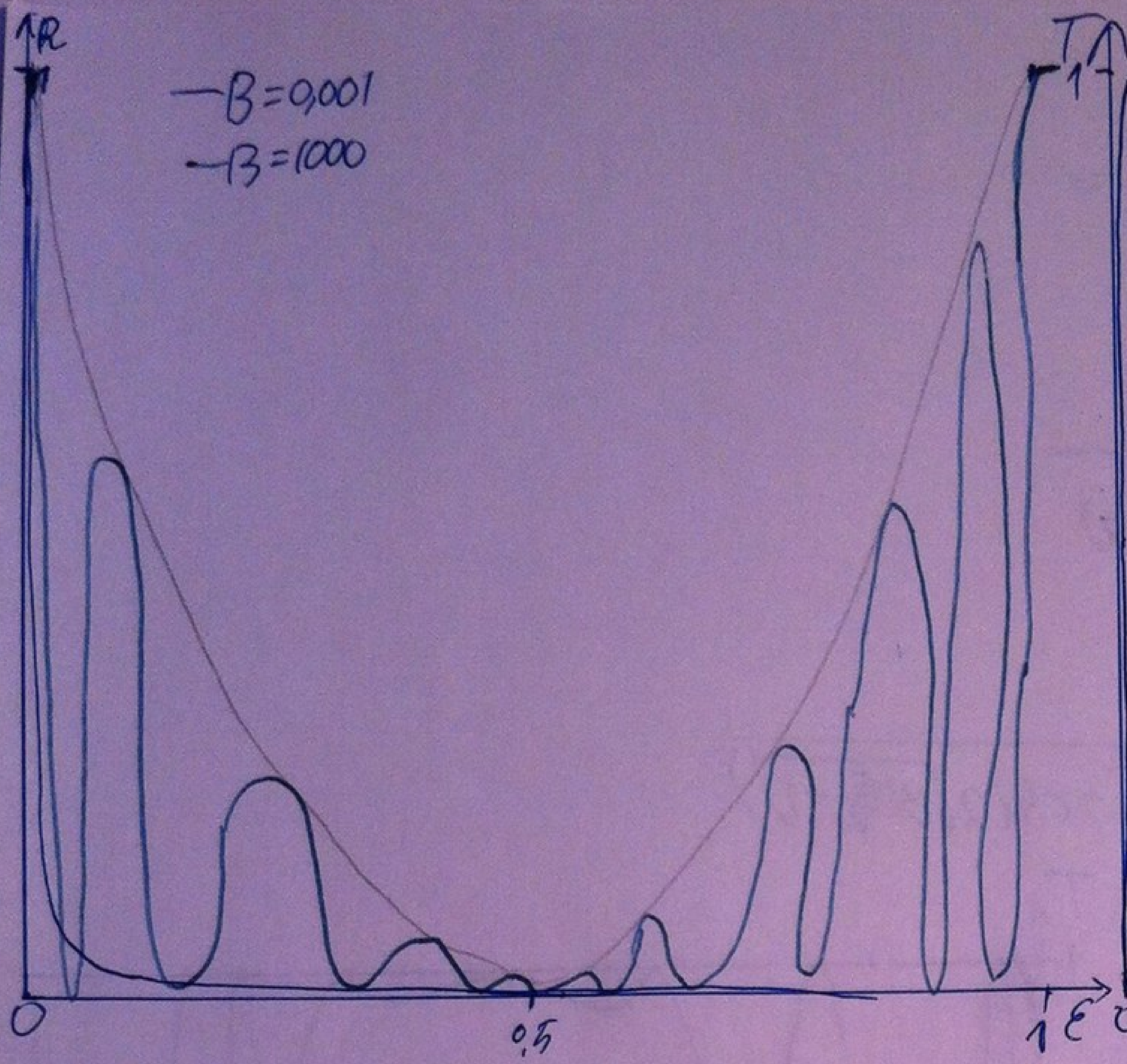
$u_0 \to -u_0$

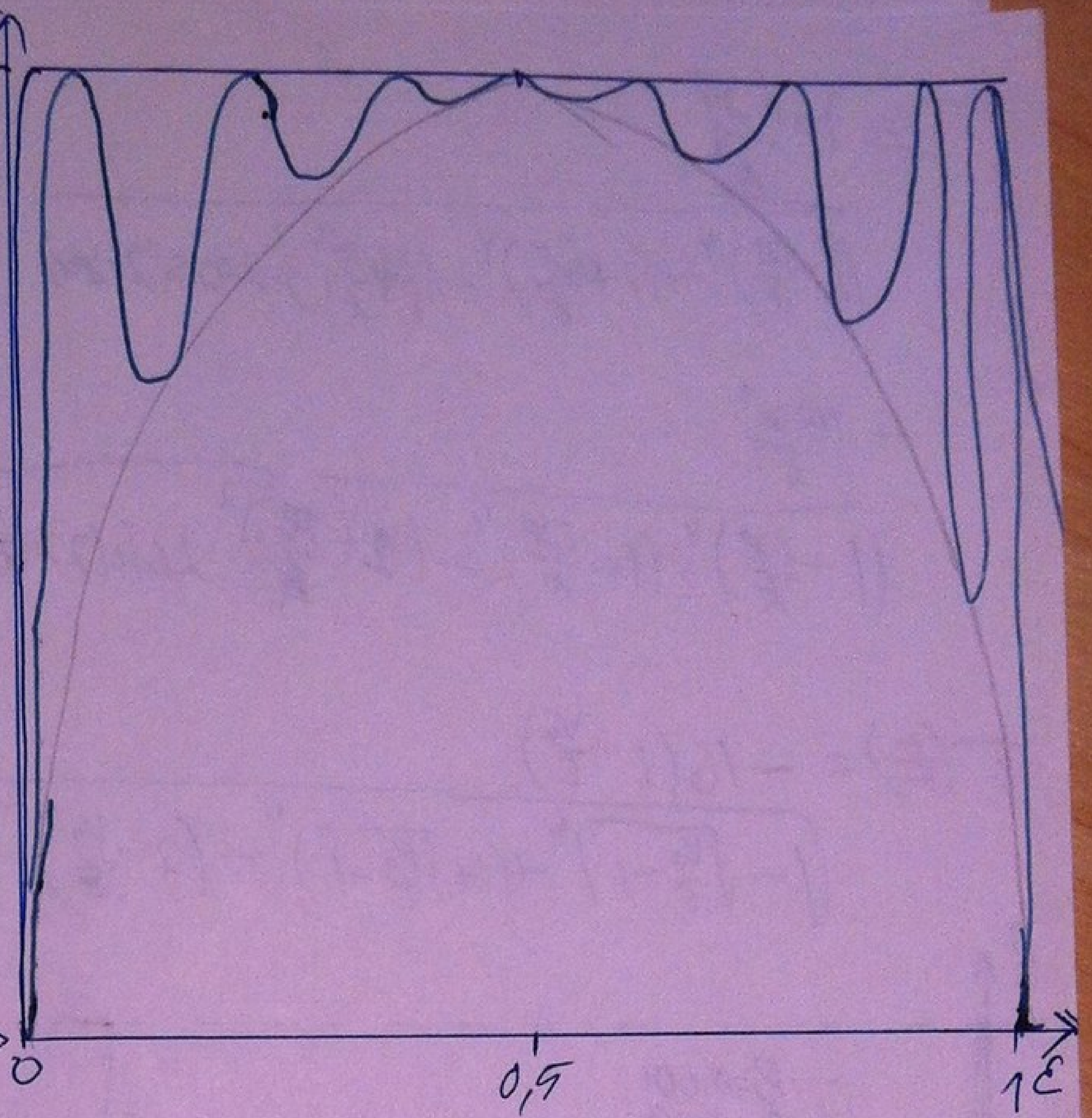
$$T(E)=-\frac{16\left(1-\frac{-V_0}{E}\right)}{\left(1-\sqrt{\frac{-V_0}{E}-1}\right)^4+\left(1+\sqrt{\frac{-V_0}{E}-1}\right)^4-\left(2-\frac{-V_0}{E}\right)^2\cdot 2ch\left(2\sqrt{B\left(\frac{E}{-V_0}-1\right)}\right)}=$$

$$=-\frac{16\left(1+\frac{V_0}{E}\right)}{\left(1-\sqrt{-\left(1+\frac{V_0}{E}\right)}\right)^4+\left(1+\sqrt{-\left(1+\frac{V_0}{E}\right)}\right)^4-\left(2+\frac{V_0}{E}\right)^2\cdot 2ch\left(2\sqrt{-B\left(1+\frac{E}{V_0}\right)}\right)}$$

$$R(E)=\frac{2\left(2+\frac{V_0}{E}\right)^2\left(1-ch2\sqrt{-B\left(1+\frac{E}{V_0}\right)}\right)}{\left(1-\sqrt{-\left(1+\frac{V_0}{E}\right)}\right)^4+\left(1+\sqrt{-\left(\frac{V_0}{E}+1\right)}\right)^4-\left(2+\frac{V_0}{E}\right)^2\cdot 2ch\left(2\sqrt{-B\left(1+\frac{E}{V_0}\right)}\right)}$$

$\varepsilon=\frac{E}{V}$, $E>0$, $V<0$





(2) $$\lim_{L\to\infty} N_1^2 \int_{-L}^{+L} \cos\frac{p}{\hbar}x\cdot\cos\frac{p'}{\hbar}x\,dx = \lim_{L\to\infty}\frac{N_1^2}{2}\left[\int_{-L}^{L}\cos\left(\frac{p-p'}{\hbar}x\right)dx + \int_{-L}^{L}\cos\left(\frac{p+p'}{\hbar}x\right)dx\right] =$$

$$= \lim_{L\to\infty}\frac{N_1^2}{2}\left[\frac{\sin\left(\frac{p-p'}{\hbar}\right)x}{(p-p')/\hbar}\Bigg|_{-L}^{L} + \frac{\sin\left(\frac{p+p'}{\hbar}\right)}{(p+p')/\hbar}\Bigg|_{-L}^{L}\right] = \lim_{L\to\infty}\frac{N_1^2}{2}\left[\frac{2\sin\frac{p-p'}{\hbar}L}{(p-p')/\hbar} + \frac{2\sin\frac{p+p'}{\hbar}L}{(p+p')/\hbar}\right] =$$

$$= \lim_{L\to\infty} N_1^2\left[\frac{\sin k'L}{k'} + \frac{\sin k''L}{k''}\right] = \lim_{L\to\infty}\frac{\sin k'L}{k'} + \lim_{L\to\infty}\frac{\sin k''L}{k''} = 2\pi\hbar\delta(p-p') + 2\pi\hbar\delta(p+p') -$$

$$N_1 = \frac{1}{\sqrt{2\pi\hbar}}$$

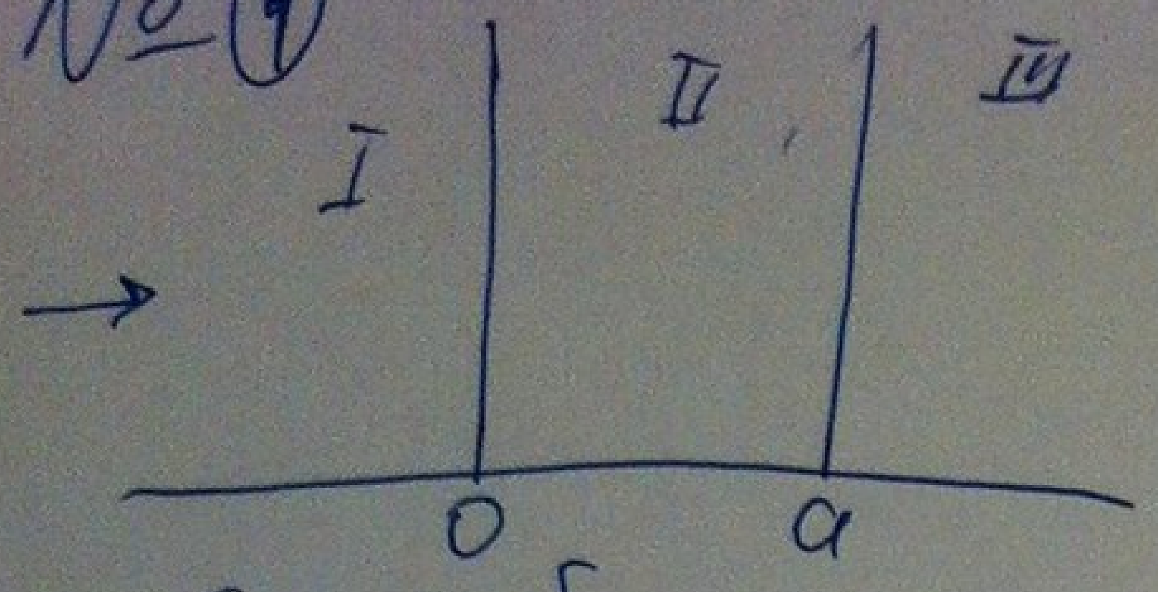
$$\lim_{L\to\infty} N_2^2\int_{-L}^{L}\sin\frac{p}{\hbar}x\cdot\sin\frac{p'}{\hbar}x\,dx = \lim_{L\to\infty}\frac{N_2^2}{2}\left[\int_{-L}^{L}\cos\frac{(p-p')}{\hbar}x\,dx - \int_{-L}^{L}\cos\frac{(p+p')}{\hbar}x\,dx\right] =$$

$$= \lim_{L\to\infty}\frac{N_2^2}{2}\left[\frac{\sin(p-p')x/\hbar}{(p-p')/\hbar}\Bigg|_{-L}^{L} - \frac{\sin(p+p')x/\hbar}{(p+p')/\hbar}\Bigg|_{-L}^{L}\right] = \lim_{L\to\infty} N_2^2\left[\frac{\sin k'L}{k'} - \frac{\sin k''L}{k''}\right] = \lim_{L\to\infty}\frac{\sin k'L}{k'} - \lim_{L\to\infty}\frac{\sin k''L}{k''} =$$

$$= 2\pi\hbar\delta(p-p') - 2\pi\hbar\delta(p+p')$$

$$N_2 = \frac{1}{\sqrt{2\pi\hbar}}$$

№ (4)



$q\delta(x) + q\delta(x-a)$

$R(E) = ?$

$T(E) = ?$

$k = \sqrt{\frac{2mE}{\hbar^2}} > 0$

$\mathrm{I}\quad -\frac{\hbar^2}{2m}\psi'' - E\psi = 0 \qquad x < 0$

$\psi_{\mathrm{I}} = 1\cdot \exp(ikx) + D\exp(-ikx)$

$\mathrm{II}\quad -\frac{\hbar^2}{2m}\psi'' - E\psi = 0 \qquad 0 < x < a$

$\psi_{\mathrm{II}} = A\sin(kx) + B\cos(kx)$

$\mathrm{III}\quad -\frac{\hbar^2}{2m}\psi'' - E\psi = 0 \qquad x > a$

$\psi_{\mathrm{III}} = C\exp(ik(x-a))$

Г.У.:
$$\begin{cases} \psi_{\mathrm{I}}(0) = \psi_{\mathrm{II}}(0) \\ \psi_{\mathrm{II}}(a) = \psi_{\mathrm{III}}(a) \\ \psi'_{\mathrm{II}}(0) - \psi'_{\mathrm{I}}(0) = +\frac{2mq}{\hbar^2}\psi(0) \\ \psi'_{\mathrm{III}}(a) - \psi'_{\mathrm{II}}(a) = \frac{2mq}{\hbar^2}\psi(0) \end{cases}$$

$$\begin{cases} 1 + D = B \\ A\sin(ka) + B\cos(ka) = C \\ kA - kB\cdot 0 - ik\cdot 1 + ikD = \frac{2mq}{\hbar^2}(1+D) \\ ikC - kA\cos(ka) + kB\sin(ka) = \frac{2mq}{\hbar^2}C \end{cases}$$

Вольфрам решил.

$$D = -\frac{2\cdot\frac{2mq}{\hbar^2}\cdot k\cos(ka) + \left(\frac{2mq}{\hbar^2}\right)^2\sin(ka)}{2\cdot\frac{2mq}{\hbar^2}\cdot k\cos(ka) - 2ik^2\cos(ka) + \left(\frac{2mq}{\hbar^2}\right)^2\sin(ka) - 2i\frac{2mq}{\hbar^2}\cdot k\sin(ka) - 2\cdot k^2\sin(ka)}$$ амплитуда отражения

$$B = -\frac{2k\left(ik\cos(ka) + i\frac{2mq}{\hbar^2}\sin(ka) + k\sin(ka)\right)}{\text{——— // ———}}$$

~~амплитуда прохождения~~

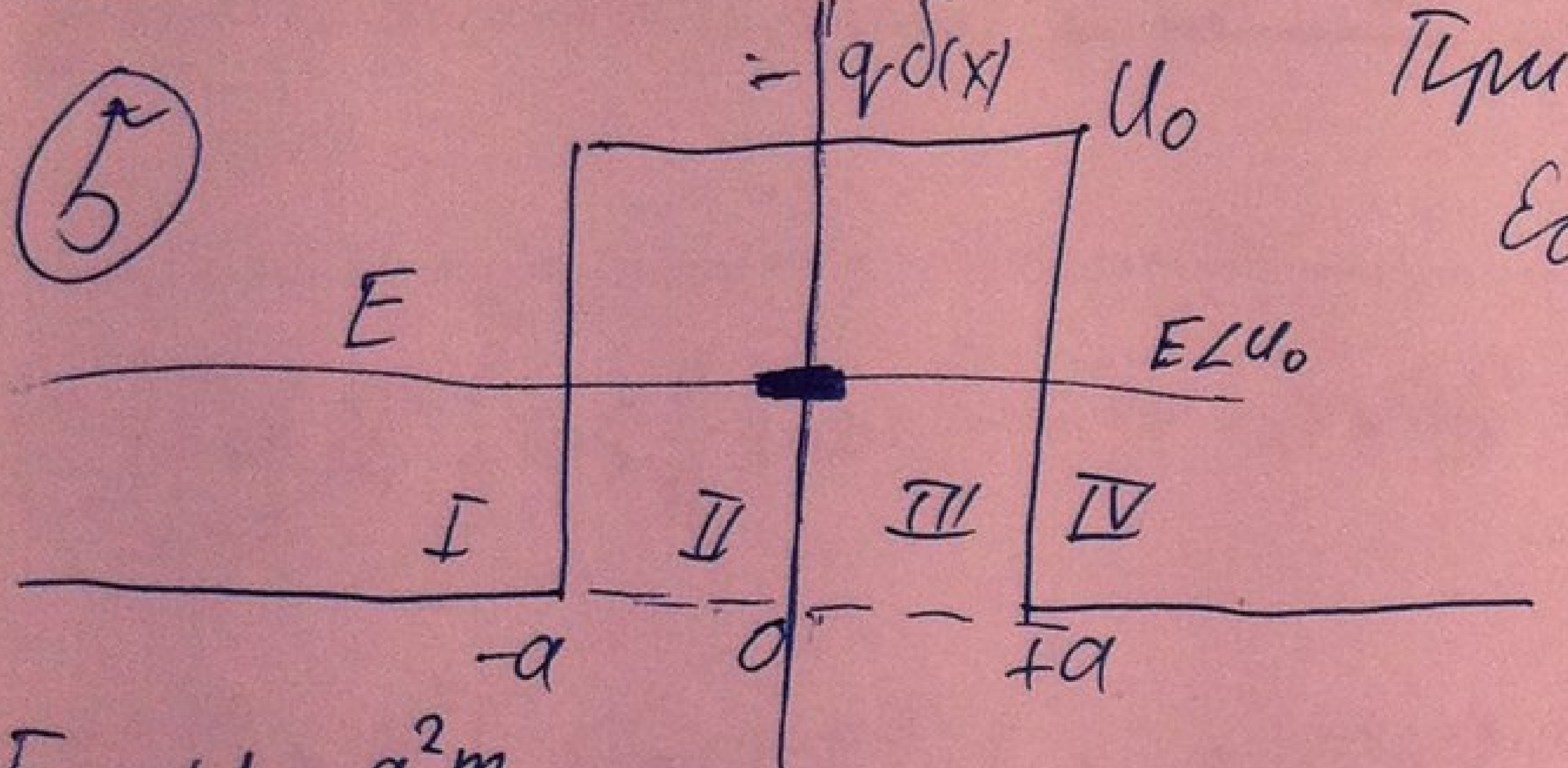
$$A = \frac{2i\left(\frac{2mq}{\hbar^2}\cdot k\cos(ka) - ik^2\cos(ka) - k^2\sin(ka)\right)}{\text{——— // ———}}$$

$$C = \frac{-2i(k^2\cos(ka) + k^2\sin(ka))}{\text{—— // ——}} = \frac{-2ik^2}{\text{—— // ——}}$$ — амплитуда прохождения

$$R = |D|^2 = \frac{2\cdot\left(\frac{2mq}{\hbar^2}\right)^2\left(2\cdot k\cos(ak) + \frac{2mq}{\hbar^2}\cdot\sin(ak)\right)^2}{\left(\frac{2mq}{\hbar^2}\right)^4 + 4\left(\frac{2mq}{\hbar^2}\right)^2\cdot k^2 + 8k^4 - \left(\frac{2mq}{\hbar^2}\right)^2\left(\left(\frac{2mq}{\hbar^2}\right)^2 - 4k^2\right)\cdot\cos(2ak) + 4\cdot\left(\frac{2mq}{\hbar^2}\right)^3\cdot k\cdot\sin(2ak)}, \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$T = |C|^2 = \frac{-2ik^2}{\text{—//—}}\cdot\frac{2ik^2}{\text{—//—}} = \frac{8k^4}{\left(\frac{2mq}{\hbar^2}\right)^4 + 4\left(\frac{2mq}{\hbar^2}\right)^2\cdot k^2 + 8k^4 - \left(\frac{2mq}{\hbar^2}\right)^2\left(\left(\frac{2mq}{\hbar^2}\right)^2 - 4k^2\right)\cos(2ak) + 4\left(\frac{2mq}{\hbar^2}\right)^3 k\sin(2ak)}$$

⑤



При каких q, U_0, a $T(E)=1$?

Если $T=1 \Rightarrow R=0$

$$k=\sqrt{\frac{2mE}{\hbar^2}} \qquad \varkappa=\sqrt{\frac{2m(E-U_0)}{\hbar^2}}$$

$$\psi_I = \exp(ik(x+a))$$

$$\psi_{IV} = \exp(ik(x-a))$$

$$\psi_{II} = B\exp(-\varkappa x) + C\exp(\varkappa x)$$

$$\psi_{III} = D\exp(-\varkappa x) + F\exp(+\varkappa x)$$

$$E_\delta = U_0 - \frac{q^2 m}{2\hbar^2}$$

$$\begin{cases}\psi_I(-a)=\psi_{II}(-a)\\ \psi_I'(-a)=\psi_{II}'(-a)\\ \psi_{III}(a)=\psi_{IV}(a)\\ \psi_{III}'(a)=\psi_{IV}'(a)\\ \psi_{II}(0)=\psi_{III}(0)\\ \psi_{III}'-\psi_{II}'=-\frac{2mq}{\hbar^2}\psi(0)\end{cases}$$

$$\begin{cases}1=B\exp(\varkappa a)+C\exp(-\varkappa a)\\ ik=-\varkappa B\exp(\varkappa a)+\varkappa C\exp(-\varkappa a)\\ D\exp(-\varkappa a)+F\exp(\varkappa a)=1\\ -\varkappa D\exp(-\varkappa a)+\varkappa F\exp(\varkappa a)=ik\\ B+C=D+F\\ \cancel{(\varkappa B+\varkappa C)+(-\varkappa D+\varkappa F)=-\frac{2mq}{\hbar^2}(B+C)}\\ (-\varkappa D+\varkappa F)-(-\varkappa B+\varkappa C)=-\frac{2mq}{\hbar^2}(B+C)\end{cases}$$

$$\left.\begin{aligned}1&=B\exp(\varkappa a)+C\exp(-\varkappa a)\\ \frac{ik}{\varkappa}&=-B\exp(\varkappa a)+C\exp(-\varkappa a)\end{aligned}\right\} \Rightarrow 1+\frac{ik}{\varkappa}=2C\exp(-\varkappa a) \Rightarrow C=\frac{1+\frac{ik}{\varkappa}}{2\exp(-\varkappa a)}$$

$$1=B\cdot\exp(\varkappa a)+\frac{1+\frac{ik}{\varkappa}}{2\exp(\varkappa a)}\cdot\exp(-\varkappa a)=B\exp(\varkappa a)+\left(1+\frac{ik}{\varkappa}\right)\frac{1}{2} \Rightarrow B=\frac{\left(1-\frac{ik}{\varkappa}\right)\frac{1}{2}}{\exp(\varkappa a)}$$

$$\left.\begin{aligned}D\exp(-\varkappa a)+F\exp(\varkappa a)&=1\\ -D\exp(-\varkappa a)+F\exp(\varkappa a)&=\frac{ik}{\varkappa}\end{aligned}\right\} \Rightarrow 1+\frac{ik}{\varkappa}=2F\exp(+\varkappa a) \Rightarrow F=\frac{1+\frac{ik}{\varkappa}}{2\exp(+\varkappa a)}$$

$$1=D\exp(-\varkappa a)+\frac{\frac{ik}{\varkappa}+1}{2\exp(+\varkappa a)}\exp(\varkappa a) \Rightarrow D\exp(-\varkappa a)+\left(1+\frac{ik}{\varkappa}\right)\frac{1}{2} \Rightarrow D=\frac{\left(1-\frac{ik}{\varkappa}\right)\frac{1}{2}}{\exp(-\varkappa a)}$$

$$\left[\left(-\varkappa B+\varkappa C\right)+\varkappa D-\varkappa F\right]=-\frac{2mq}{\hbar^2}(B+C) \Rightarrow -\frac{2mq}{\hbar^2}=\frac{\varkappa(B-C+F-D)}{(B+C)}$$

$$q=\frac{\hbar^2\varkappa\left(\left(1-\frac{ik}{\varkappa}\right)/2\exp(\varkappa a)-\left(1+\frac{ik}{\varkappa}\right)/2\exp(-\varkappa a)+\left(1+\frac{ik}{\varkappa}\right)/2\exp(\varkappa a)-\left(1-\frac{ik}{\varkappa}\right)/2\exp(-\varkappa a)\right)}{2m\left(\frac{1-\frac{ik}{\varkappa}}{2\exp(\varkappa a)}+\frac{1+\frac{ik}{\varkappa}}{2\exp(-\varkappa a)}\right)}=$$

$$=\frac{\hbar^2\varkappa}{2m}\left[\frac{2/\exp(\varkappa a)-2/\exp(-\varkappa a)}{\left(1-\frac{ik}{\varkappa}\right)\exp(-\varkappa a)+\left(1+\frac{ik}{\varkappa}\right)\exp(\varkappa a)}\right]=\frac{\hbar^2\varkappa}{m}\left[\frac{\exp(-\varkappa a)-\exp(\varkappa a)}{(\exp(-\varkappa a)+\exp(\varkappa a))+\frac{ik}{\varkappa}(\exp(\varkappa a)-\exp(-\varkappa a))}\right]$$

$$=\frac{\hbar^2\varkappa}{m}\left[\frac{-2Sh(\varkappa a)}{2Ch(\varkappa a)+\frac{ik}{\varkappa}\cdot 2Sh(\varkappa a)}\right] \Rightarrow q=\frac{\hbar^2\varkappa}{m}\left[\frac{Sh(\varkappa a)}{Ch(\varkappa a)+\frac{ik}{\varkappa}\cdot Sh(\varkappa a)}\right]$$

Если мы знаем, что $E=E_\delta=U_0-\frac{q^2m}{2\hbar^2}$

$$U_0-E=\frac{q^2m}{2\hbar^2}\Rightarrow q=\sqrt{\frac{2\hbar^2(U_0-E)}{m}}$$

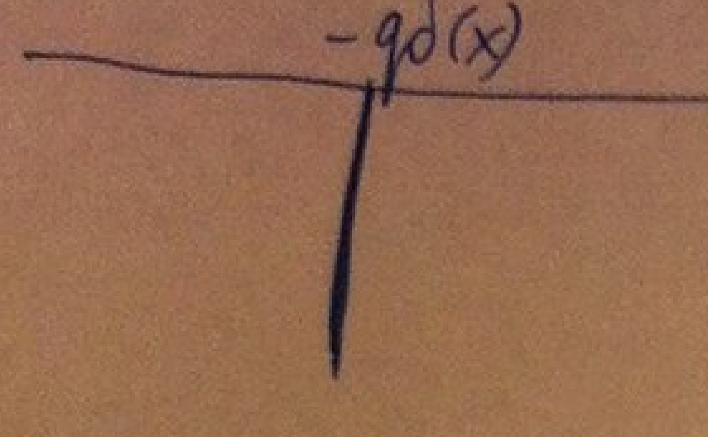
$$\sqrt{\frac{2\hbar^2(U_0-E)}{m}}=\frac{\hbar^2}{m}\sqrt{\frac{2m(E-V_0)}{\hbar^2}}\left[\frac{Sh(\varkappa a)}{Ch(\varkappa a)+\frac{ik}{\varkappa}Sh(\varkappa a)}\right]$$

$$\sqrt{\frac{2\hbar^2(U_0-E)}{m}\cdot\frac{\hbar^2}{2m(E-V_0)}}\cdot\frac{m}{\hbar^2}=\left[\frac{Sh(\varkappa a)}{Ch(\varkappa a)+\frac{ik}{\varkappa}Sh(\varkappa a)}\right]=i \quad \Rightarrow Sh(\varkappa a)=iCh(\varkappa a)-\frac{k}{\varkappa}Sh(\varkappa a)$$

$$Sh(\varkappa a)\left(1+\frac{k}{\varkappa}\right)=iCh(\varkappa a)$$

$$-iSh(\varkappa a)\left(1+\frac{k}{\varkappa}\right)=Ch(\varkappa a)$$

$$-q\delta(x) \qquad V(p)=\int dx\,\frac{e^{-\frac{ipx}{\hbar}}}{\sqrt{2\pi\hbar}}\,q\delta(x)=\frac{q}{\sqrt{2\pi\hbar}}$$



$$\tilde{U}(p)=\frac{1}{2\pi\hbar}\int U(x)\exp(-ipx/\hbar)\,dx=-\frac{q}{2\pi\hbar}$$

$$\frac{p^2}{2m}\varphi(p)-\frac{q}{2\pi\hbar}\underbrace{\int_{-\infty}^{+\infty}\varphi(p)\,dp}_{C}=E\varphi(p)\quad,\ E=-|E|<0$$

$$\varphi(p)=\frac{qm}{\pi\hbar}\frac{C}{p^2+2m|E|}$$

$$C=\int_{-\infty}^{+\infty}\varphi(p)\,dp=\frac{qm}{\pi\hbar}\cdot C\int_{-\infty}^{+\infty}\frac{dp}{(p^2)^2+2m|E|}=\frac{q\cdot C}{\hbar}\sqrt{\frac{m}{2|E|}}\Rightarrow$$

$$\Rightarrow\sqrt{|E|}=\frac{q}{\hbar}\frac{\sqrt{m}}{\sqrt{2}}\Rightarrow|E|=\frac{q^2m}{2\hbar^2}\Rightarrow E=-\frac{q^2m}{2\hbar^2}$$

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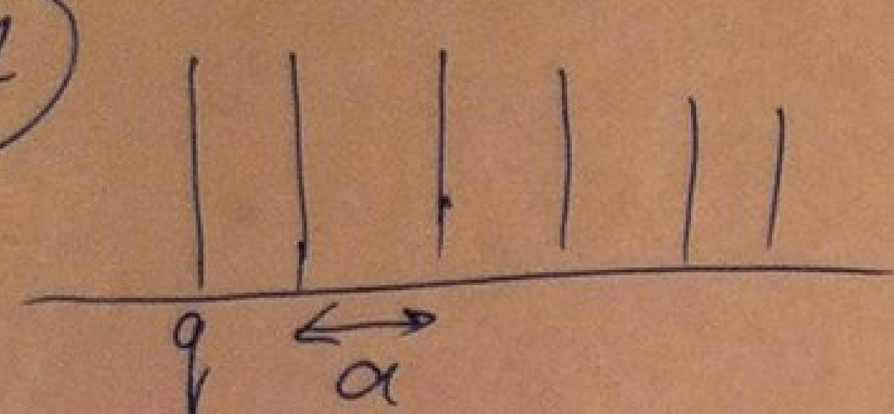


$V(x+a)=V(x)$

$\psi_1(x+a)=\psi(x)e^{\frac{ipa}{\hbar}}$

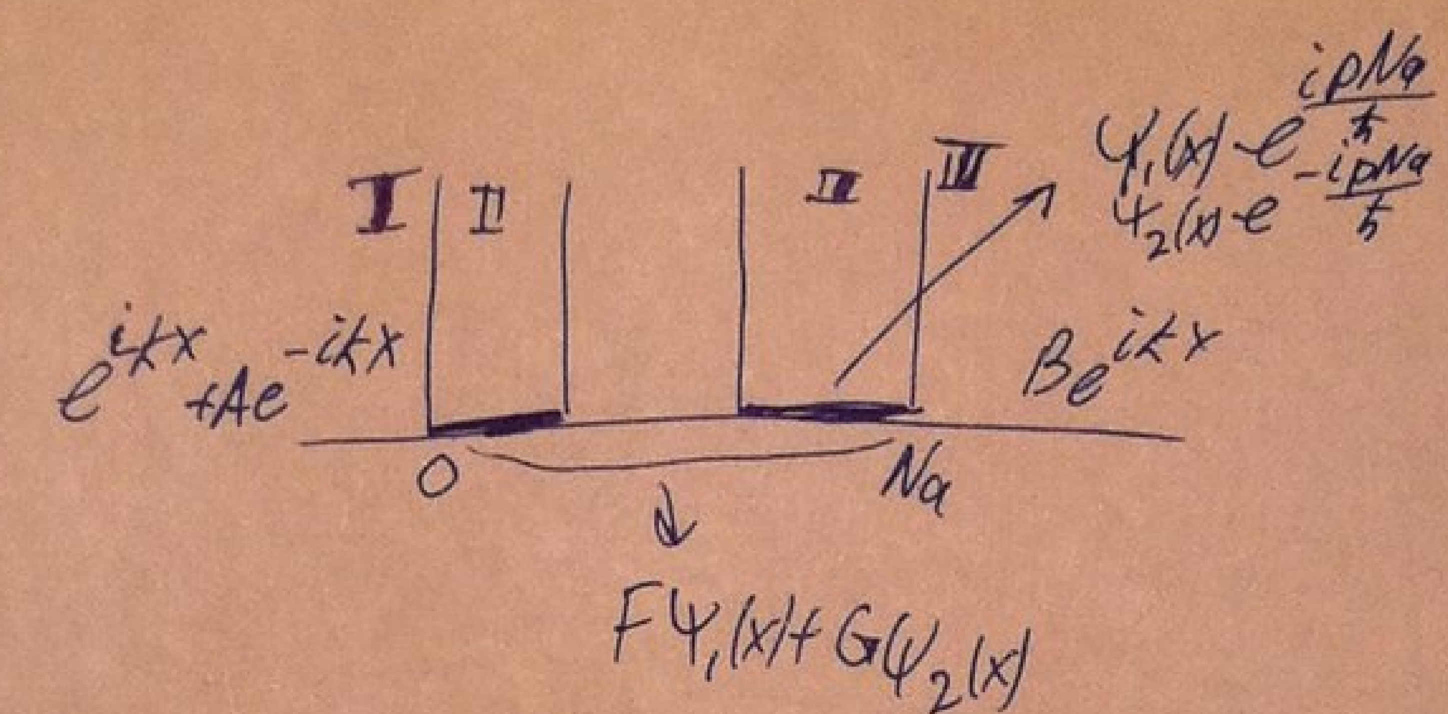
$\psi_2(x+a)=\psi_2(x)e^{-\frac{ipa}{\hbar}}$ – разр. зона

$T(E)=?$

$R(E)=?$

$\psi_1(x+a)=\psi_1 e^{\frac{\varkappa a}{\hbar}}$

$\psi_2(x+a)=\psi_2 e^{-\frac{\varkappa a}{\hbar}}$ запр. зона



$\psi_1=e^{ikx}+Ae^{-ikx}$

$\psi_2=Fe^{ikx}+Ge^{-ikx}$

$\psi_3=Be^{ik(x-Na)}$

$$\begin{cases}\psi_1(0)=\psi_2(0)\\ \psi_2'-\psi_1'(0)=\frac{2mq}{\hbar^2}\psi(0)\\ \psi_2(Na)=\psi_3(Na)\\ \psi_3'-\psi_2'=\frac{2mq}{\hbar^2}\psi(Na)\end{cases}\qquad\begin{cases}1+A=F+G\\ ikF-ikG-ik+ikA=\frac{2mq}{\hbar^2}(1+A)\\ B=Fe^{ikNa}+Ge^{-ikNa}\\ ikB-ikFe^{ikNa}+ikGe^{-ikNa}=\frac{2mq}{\hbar^2}(B)\end{cases}$$

$$A=\frac{-\left[\left(\frac{2mq}{\hbar^2}\right)^2(-1+e^{2ikNa})+i\frac{2mq}{\hbar^2}k(2+e^{2ikNa})+2k^2e^{2ikNa}\right]}{\left(\frac{2mq}{\hbar^2}\right)^2(-1+e^{2ikNa})-i\frac{2mq}{\hbar^2}(-4+e^{2ikNa})-2k^2(-2+e^{2ikNa})}$$

$$F=\frac{-k\left(2k(-2+e^{2ikNa})-i\left(\frac{2mq}{\hbar^2}\right)(2+e^{2ikNa})\right)}{\text{———}\ //\ \text{———}}$$

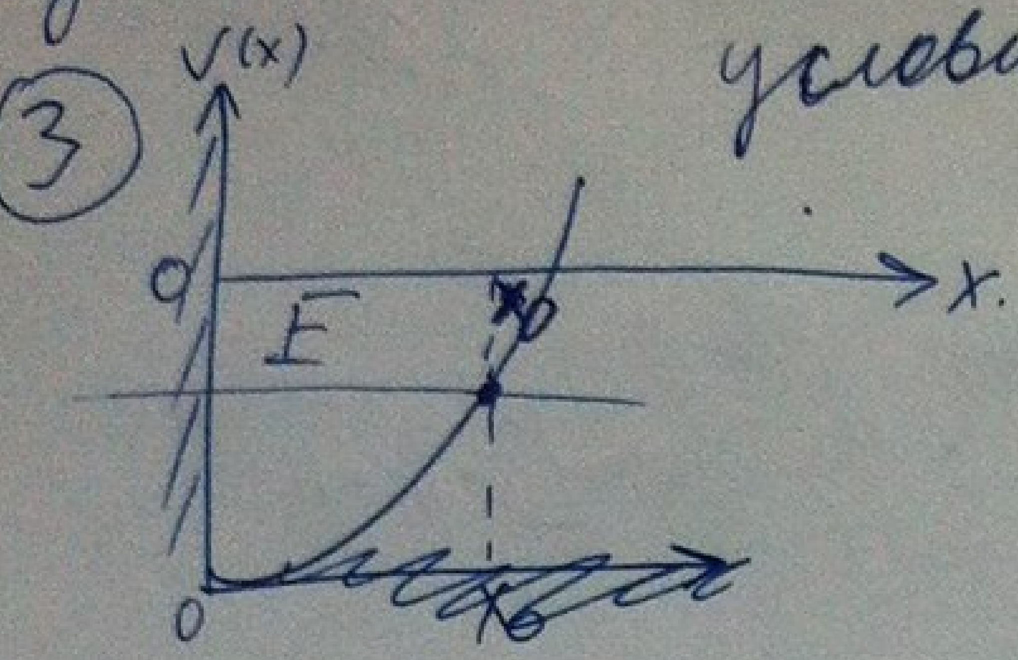
$$G=\frac{-k\left(2k+3i\left(\frac{2mq}{\hbar^2}\right)\right)e^{2ikNa}}{\text{———}\ //\ \text{———}}$$

$$B=-\frac{k\left(2k-i\frac{2mq}{\hbar^2}\right)e^{ikNa}(-1+e^{2ikNa})}{\text{———}\ //\ \text{———}}$$

$R=|A|^2=$

Дз на 5.05.2014.

③ условие квантования Бора-Зоммерфельда?



$$\begin{cases} x<x_0: \ \psi_1(x) = \dfrac{C}{\sqrt{k(x)}} \sin\left(\int_x^{x_0} k(x)\,dx + \dfrac{\pi}{4}\right) \\ x>x_0: \ \psi_2(x) = \dfrac{C}{2\sqrt{\varkappa(x)}} \exp\left(-\int_{x_0}^{x} \varkappa(x)\,dx\right) \\ x \le 0: \ \psi(0)=0 \end{cases}$$

$$k = \sqrt{\frac{2m}{\hbar^2}(E-V(x))}$$

$$\varkappa = \sqrt{\frac{2m}{\hbar^2}(V(x)-E)}$$

$$\psi_1(0)=0 \Rightarrow \frac{C}{\sqrt{k(x)}} \sin\left(\int_0^{x_0} k(x)\,dx + \frac{\pi}{4}\right) = 0 \Rightarrow \int_0^{x_0} k(x)\,dx + \frac{\pi}{4} = \pi(n+1) \Rightarrow$$

$$\Rightarrow \int_0^{x_0} \sqrt{\frac{2m}{\hbar^2}(E-V(x))}\,dx = \pi(n+3/4)$$

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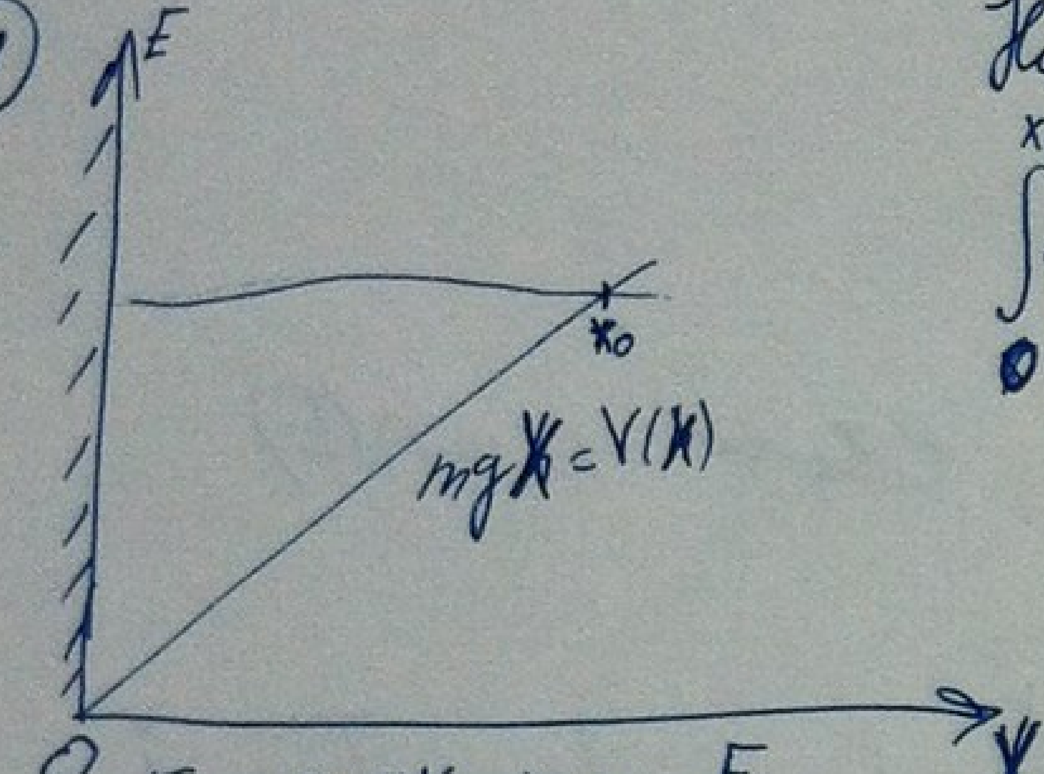
④ Найти квазиклассические уровни энергии. $E_n = ?$



$E = mgx_0 \Rightarrow x_0 = \dfrac{E}{mg}$

$$\int_0^{x_0} \sqrt{\frac{2m}{\hbar^2}(E-V(x))}\,dx = \pi(n+3/4) \quad , V(x) = mgx$$

$$\int_0^{x_0} \sqrt{\frac{2m}{\hbar^2}(E-mgx)}\,dx = \int_0^{x_0} \sqrt{\frac{2m}{\hbar^2}} \sqrt{mg} \sqrt{\frac{E}{mg}-x}\,dx =$$

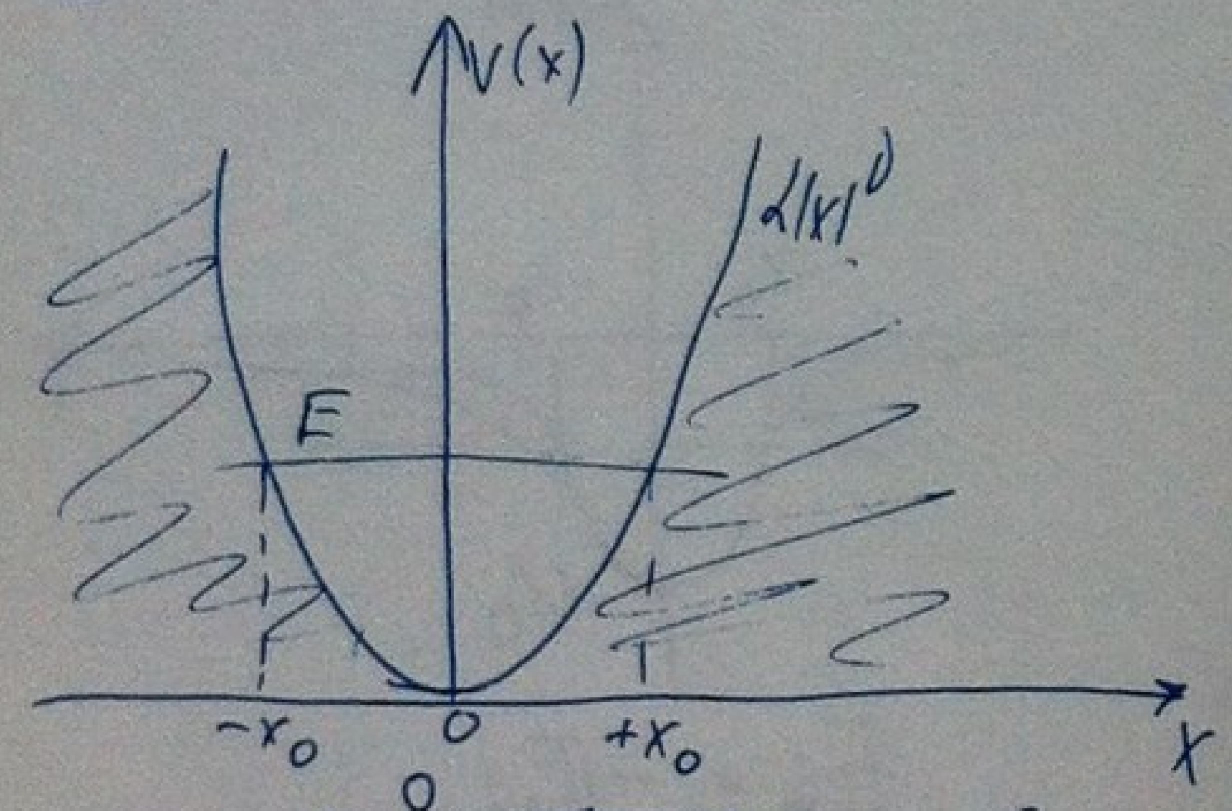
$$= \sqrt{\frac{2m^2g}{\hbar^2}} \cdot \int_{x_0}^{0} \sqrt{\frac{E}{mg}-x}\, d\left(\frac{E}{mg}-x\right) = \sqrt{\frac{2m^2g}{\hbar^2}} \cdot \left(\frac{E}{mg}-x\right)^{3/2} \cdot \frac{2}{3} \Bigg|_{\frac{E}{mg}}^{0} =$$

$$= \sqrt{\frac{2m^2g}{\hbar^2}} \cdot \sqrt{\frac{E^3}{(mg)^3}} \cdot \frac{2}{3} = \sqrt{\frac{8E^3}{9\hbar^2 g^2 m}} = \pi(n+3/4)$$

$$\Rightarrow E = \frac{\pi^{2/3}(n+3/4)^{2/3}\left(9\hbar^2 g^2 m\right)^{1/3}}{8^{1/3}} = \left(\frac{\pi^2 (n+3/4)^2\, 9\hbar^2 g^2 m}{8}\right)^{1/3}$$

---

⑤ $V(x) = \alpha |x|^\nu$; $E_n = C \cdot n^\mu$ $\quad \mu = ?$ $\quad$ Если $\nu = 2$, то $\mu = 1$ (Проверка)



Используя правило квантования Бора-Зоммерфельда

$$\int_{x_1}^{x_2} k(x)\,dx = \pi(n+1/2)$$

$E = \alpha x_0^\nu \Rightarrow x_0 = \sqrt[\nu]{\dfrac{E}{\alpha}} = \left(\dfrac{E}{\alpha}\right)^{1/\nu}$

$-x_0 = -\left(\dfrac{E}{\alpha}\right)^{1/\nu}$

$x_0 \cdot \left(\dfrac{\alpha}{E}\right)^{1/\nu} = 1$

$$\int_{-x_0}^{x_0} \sqrt{\frac{2m}{\hbar^2}(E-\alpha|x|^\nu)}\,dx = \pi(n+1/2)$$

$$\int_{-x_0}^{0} \sqrt{\frac{2m}{\hbar^2}(E-\alpha(-x)^\nu)}\,dx + \int_0^{x_0} \sqrt{\frac{2m}{\hbar^2}(E-\alpha x^\nu)}\,dx = \pi(n+1/2)$$

$$I = \sqrt{\frac{2mE}{\hbar^2}} \left(\int_{-x_0}^{0} \sqrt{1-\frac{\alpha(-x)^\nu}{E}}\,dx + \int_0^{x_0} \sqrt{1-\frac{\alpha x^\nu}{E}}\,dx\right) = 2\sqrt{\frac{2mE}{\hbar^2}} \left(\frac{E}{\alpha}\right)^{1/\nu} \int_0^{x_0} \sqrt{1-\left(x\cdot\left(\frac{\alpha}{E}\right)^{1/\nu}\right)^\nu}\, d\left(x \cdot \left(\frac{\alpha}{E}\right)^{1/\nu}\right) =$$

$$= 2\sqrt{\frac{2mE}{\hbar^2}} \cdot \left(\frac{E}{\alpha}\right)^{1/\nu} \cdot \int_0^1 \sqrt{1-t^\nu}\,dt = 2\sqrt{\frac{2mE}{\hbar^2}} \cdot \left(\frac{E}{\alpha}\right)^{1/\nu} \cdot C \quad , C = \int_0^1 \sqrt{1-t^\nu}\,dt = const$$

$$I = \pi(n+1/2)$$

$$C\cdot 2\sqrt{\frac{2mE}{\hbar^2}}\left(\frac{E}{\alpha}\right)^{1/\nu}=\pi(n+1/2)\Rightarrow \cancel{2C\sqrt{\frac{2m}{\hbar^2}}\cdot\left(\frac{1}{\alpha}\right)^{1/\nu}(E)^{1/\nu+2}=\pi(n+1/2)}\Rightarrow\cancel{(E)^{1/\nu+2}=\frac{\pi(n+1/2)\cdot\hbar(\alpha)^{1/\nu}}{2\cdot C\cdot\sqrt{2m}}}$$

$$\cancel{E=\left(\frac{\pi(n+1/2)\hbar}{2\cdot C\cdot\sqrt{2m}}\right)^{\nu+2}\cdot\alpha^{2}}\Rightarrow 2\cdot C\sqrt{\frac{2m}{\hbar^2}}\cdot\left(\frac{1}{\alpha}\right)^{1/\nu}\cdot(E)^{1/2}\cdot E^{1/\nu}=\pi(n+1/2)$$

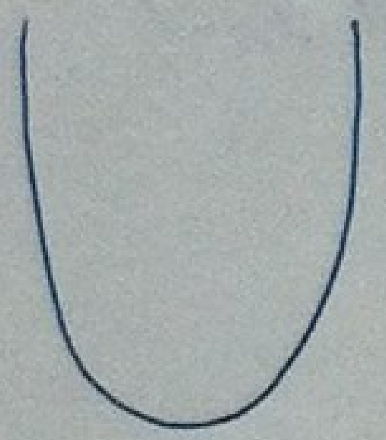
$$E^{(1/2+1/\nu)}=E^{\left(\frac{\nu+2}{2\nu}\right)}=\frac{\pi(n+1/2)(\alpha)^{1/\nu}\cdot\hbar}{2\cdot C\cdot\sqrt{2m}}\Rightarrow E_n=\left(\frac{\pi(n+1/2)\cdot\hbar}{C\cdot\sqrt{8m}}\right)^{2\nu/(\nu+2)}\cdot\alpha^{2/(\nu+2)}\sim C\cdot(n+1)^{2\nu/\nu+2}$$

Если $\nu=2$, то $\frac{2\nu}{\nu+2}=\frac{4}{4}=1$, верно

~~6~~ $\cancel{\psi(x)=\frac{1}{\sqrt{k}}\sin\left(\int_a^x k(x)dx+\frac{\pi}{4}\right)}$ $\psi(a)=?$

a

7



$\rho(E)=?$ плотность состояний

$$\int_{x_1}^{x_2}k(x)dx=\sqrt{\frac{2m}{\hbar^2}}\int_{x_1}^{x_2}\sqrt{E-U(x)}\,dx=\pi(n+1/2)\quad,\ \text{если } n=n(E)$$

$$\rho(E)=\frac{d(n+1/2)}{dE}=\frac{1}{\pi}\sqrt{\frac{2m}{\hbar^2}}\cdot\frac{1}{2}\int_{x_1}^{x_2}\frac{1}{\sqrt{E-U(x)}}dx=\frac{1}{2\pi\hbar}\underbrace{2\int_{x_1}^{x_2}\frac{1}{\sqrt{\frac{2}{m}(E-U(x))}}dx}_{T\text{-период финитного движения}}=\frac{T}{2\pi\hbar}=\frac{1}{\hbar\omega(E)}$$

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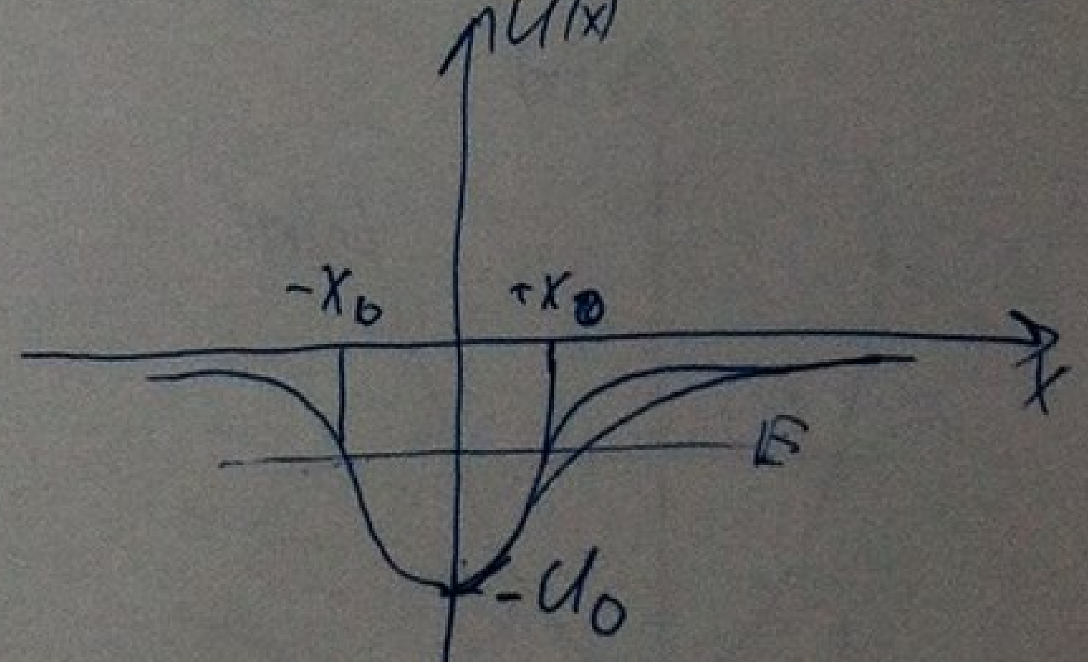
9 $U(x)=-\dfrac{U_0a^4}{(x^2+a^2)^2}$ При каких параметрах потенциала появляется новый уровень?



$$\int_{x_1}^{x_2}k(x)dx=\sqrt{\frac{2m}{\hbar^2}}\int_{x_1}^{x_2}\sqrt{E_n-U(x)}\,dx=\pi(n+1/2)$$

$n=0,1,2\ldots$
$n=N-1$
$E_{N-1}\to 0$

$$\sqrt{\frac{2m}{\hbar^2}}\int_{-x_0}^{+x_0}\sqrt{E_n+\frac{U_0a^4}{(x^2+a^2)^2}}\,dx=\pi(N-1/2)$$

$$\sqrt{\frac{2m}{\hbar^2}}\cdot\sqrt{U_0}\cdot a\int_{-x_0}^{x_0}\sqrt{\frac{E_n}{U_0}+\frac{1}{(1+y^2)^2}}\,dy=\pi(N-1/2)$$

$$\sqrt{\frac{2mU_0a^2}{\hbar^2}}=(N-1/2)$$

$$\frac{2mU_0a^2}{\hbar^2}=(N-1/2)^2$$

$$E_n=-|E_n|$$

$$-|E_n|=-\frac{U_0a^4}{(a^2+x^2)^2}=-\frac{U_0}{\left(1+\frac{x^2}{a^2}\right)^2}$$

$$\frac{|E_n|}{U_0}=\left(1+\frac{x^2}{a^2}\right)^{-2}$$

$$\sqrt{\frac{U_0}{|E_n|}}=1+\frac{x^2}{a^2}$$

$$x^2=\left(\sqrt{\frac{U_0}{|E_n|}}-1\right)a^2$$

$$x_0=\pm a\left(\sqrt{\frac{U_0}{|E|}}-1\right)^{1/2}$$

Уточним ответ 1) при $|x| >> a$

$$\psi'' + \frac{\alpha}{x^4}\psi = 0 \quad , \alpha = \frac{2mU_0a^4}{\hbar^2} = \xi^2 a^2 \qquad a << x < a\xi$$

$$\begin{cases} \psi = A_\pm \sqrt{|x|}\cdot J_{1/2}(\sqrt{\alpha}/|x|) \Longleftrightarrow \psi = A_\pm x\sqrt{\frac{2}{\pi\sqrt{\alpha}}}\sin\frac{\sqrt{\alpha}}{x} = A_\pm x\sqrt{\frac{2}{\pi\sqrt{\alpha}}}\sin\frac{\xi a}{x} \\ \psi_{kB} = \frac{C}{\sqrt{k(x)}}\sin\left(\int_x^\infty k(x)dx + \gamma_1\right) = \frac{C}{\sqrt{k(x)}}\sin\left(\frac{\pi\xi}{2} - \xi\,\mathrm{arctg}\frac{x}{a} + \gamma_1\right) \approx C\cdot x\,(2mU_0a^4)^{-1/4}\sin\left(\frac{\xi a}{x} + \gamma_1\right) \end{cases}$$

$\Rightarrow \gamma_1 = 0$

$$\psi_{kB} = \frac{C'}{\sqrt{k(x)}}\sin\left(\int_{-\infty}^{x} k(x)dx + \gamma_2\right) \qquad a << -x << a\xi \quad \Rightarrow \gamma_2 = 0$$

$$\int_{-\infty}^{+\infty} k(x)dx = \pi N \Rightarrow \xi = N \Rightarrow \frac{2mU_0a^2}{\hbar^2} = N^2$$

Галицкий 9.12

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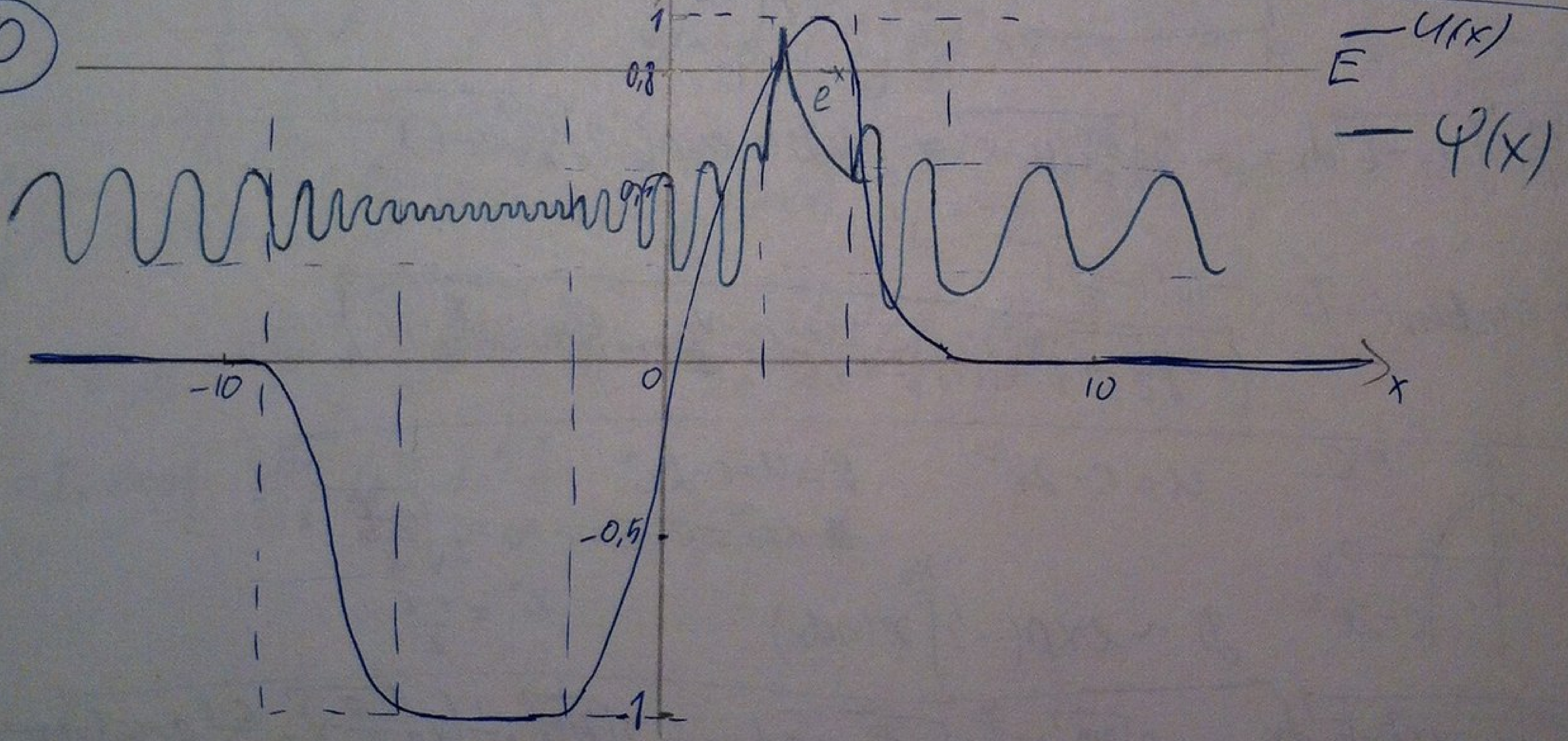
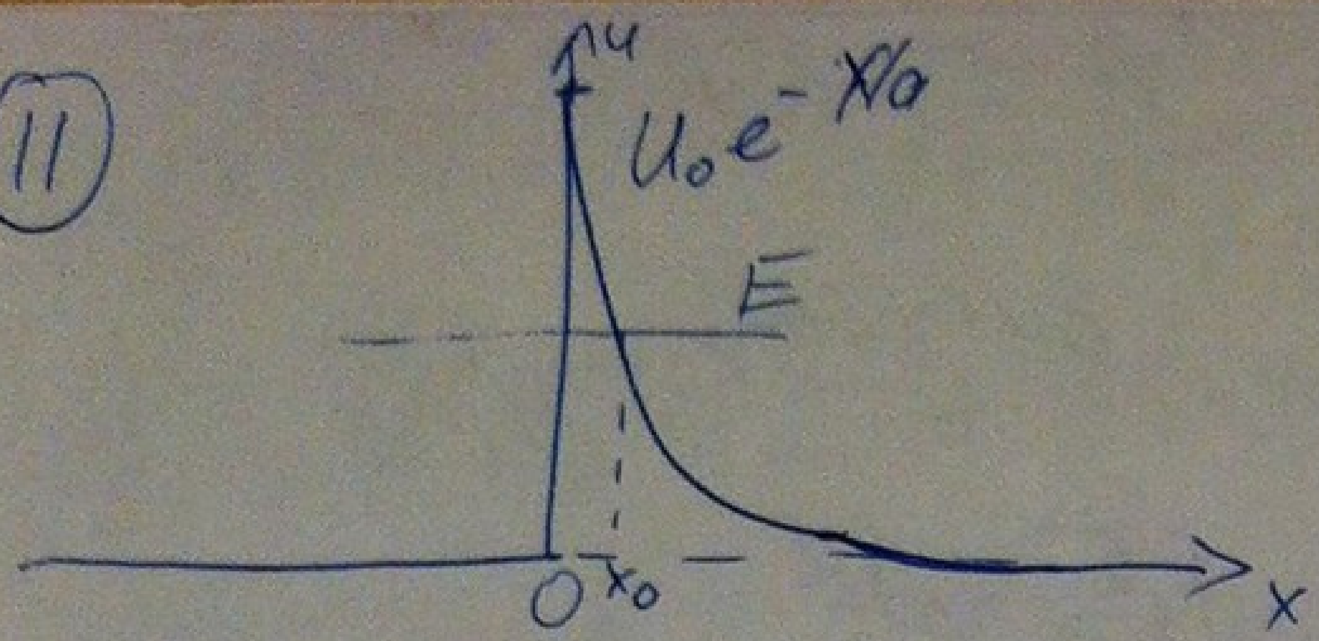


$$\psi(x) = \frac{1}{\sqrt{\frac{2m(E-U(x))}{\hbar^2}}}\sin\left(\int_a^x \sqrt{\frac{2m(E-U(x))}{\hbar^2}}dx + \frac{\pi}{4}\right) \text{ — в разреш. зоне}$$

$$\psi(x) = \frac{1}{\sqrt{\frac{2m}{\hbar^2}(U(x)-E)}}\exp\left(-\int_x^a \sqrt{\frac{2m}{\hbar^2}(U(x)-E)}\right) \text{ — в запр. зоне}$$

(11)



$$U(x)=\begin{cases}0, & x<0\\ U_0 e^{-x/a}, & x>0\end{cases}$$

$$D \sim \exp\left(-2\int_{x_1}^{x_2} \varkappa(x)\,dx\right)$$

$E=U_0e^{-x_0/a}$

$\ln\frac{E}{U_0}=-x_0/a$

$x_0=-a\ln\frac{E}{U_0}=a\ln\left(\frac{U_0}{E}\right)$

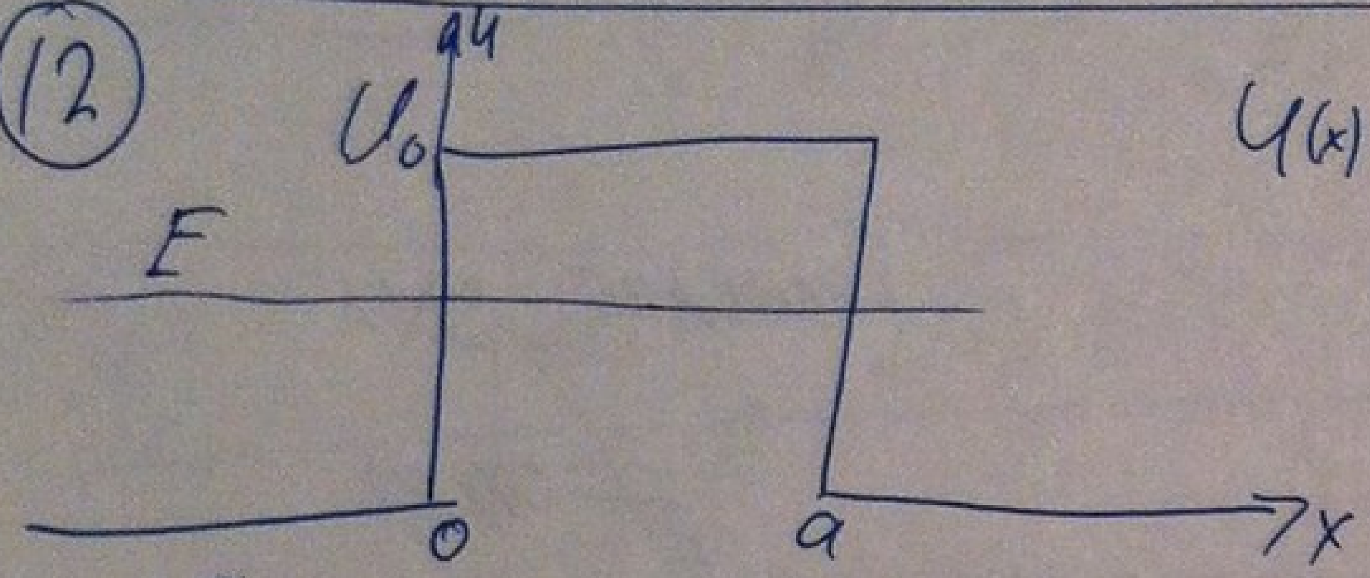
$y_0=\ln\left(\frac{E}{U_0}\right)$

$$\cancel{D\approx\exp}\ -2\int_0^{x_0}\sqrt{\frac{2m}{\hbar^2}}\sqrt{U_0e^{-x/a}-E}\,dx=+2\sqrt{\frac{2mE}{\hbar^2}}a\int_0^{\ln\left(\frac{E}{U_0}\right)}\sqrt{\frac{U_0}{E}\exp(+y)-1}\,dy=$$

$$=2\sqrt{\frac{2mE}{\hbar^2}}a\cdot 2\left(\sqrt{-1+\frac{U_0}{E}\exp(y)}-\mathrm{arctg}\left(\sqrt{-1+\frac{U_0}{E}\exp(y)}\right)\right)\Bigg|_0^{\ln\left(\frac{E}{U_0}\right)}=$$

$$=4\sqrt{\frac{2mE}{\hbar^2}}a\cdot\left(-\sqrt{\frac{U_0}{E}-1}+\mathrm{arctg}\sqrt{-1+\frac{U_0}{E}}\right)\Rightarrow D\sim\exp\left(-4\sqrt{\frac{2ma^2E}{\hbar^2}}\left[\sqrt{\frac{U_0}{E}-1}-\mathrm{arctg}\sqrt{\frac{U_0}{E}-1}\right]\right)$$

(12)



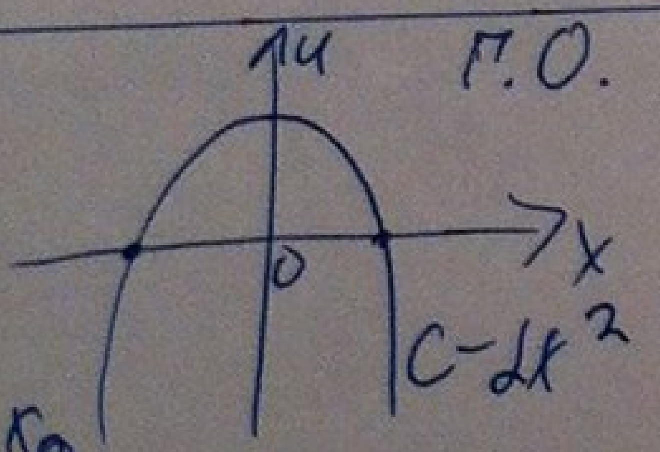
$$U(x)=\begin{cases}0, & x<0\\ U_0, & 0<x<a\\ 0, & x>a\end{cases}$$

$$D\sim\exp\left(-2\int_{x_1}^{x_2}\varkappa(x)\,dx\right)$$

$$-2\int_0^a\sqrt{\frac{2m}{\hbar^2}}\sqrt{U_0-E}\,dx=-2\sqrt{\frac{2m}{\hbar^2}(U_0-E)}\,a\Rightarrow D\sim\exp\left(-2\sqrt{\frac{2ma^2}{\hbar^2}(U_0-E)}\right)$$

Точн. ответ: $T=\dfrac{-16\left(1-\frac{V_0}{E}\right)}{\left(1-\sqrt{\frac{V_0}{E}-1}\right)^4+\left(1+\sqrt{\frac{U_0}{E}-1}\right)^4-\left(2-\frac{V_0}{E}\right)\cdot 2\,\mathrm{ch}\left(2\sqrt{B\left(\frac{E}{V_0}-1\right)}\right)}$

(13)



П.О. $U=C-\alpha x^2$

$E=U=C-\alpha x_0^2$

$\alpha x_0^2=C-E\Rightarrow x_0=\pm\sqrt{\frac{C-E}{\alpha}}$

$x_0^2=\frac{C-E}{\alpha}$

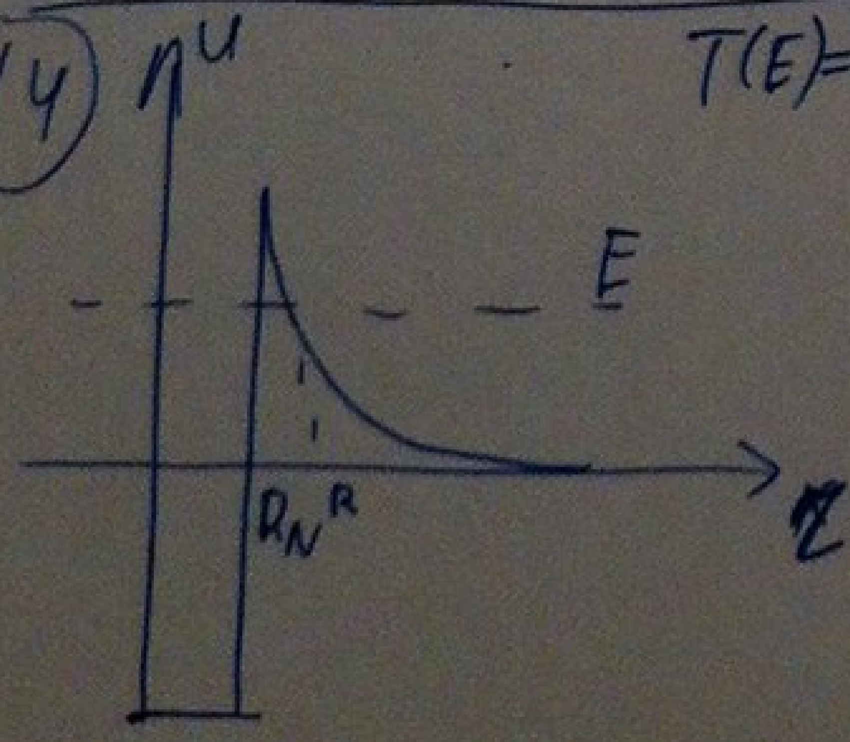
$$D\sim\exp\left(-2\int_{x_1}^{x_2}\varkappa(x)\,dx\right)$$

$$-2\int_{-x_0}^{x_0}\sqrt{\frac{2m}{\hbar^2}}\sqrt{C-\alpha x^2-E}\,dx=-2\sqrt{\frac{2m}{\hbar^2}}\sqrt{\alpha}\int_{-x_0}^{x_0}\sqrt{\frac{C-E}{\alpha}-x^2}\,dx=-2\sqrt{\frac{2m\alpha}{\hbar^2}}\cdot\frac{1}{2}\left(x\sqrt{\frac{C-E}{\alpha}-x^2}+\frac{C-E}{\alpha}\mathrm{arctg}\left(\frac{x}{\sqrt{\frac{C-E}{\alpha}-x^2}}\right)\right)\Bigg|_{-x_0}^{x_0}=$$

$$=-2\sqrt{\frac{2m\alpha}{\hbar^2}}\left(x\sqrt{\frac{C-E}{\alpha}-x^2}+\frac{C-E}{\alpha}\mathrm{arctg}\left(\frac{x}{\sqrt{\frac{C-E}{\alpha}-x^2}}\right)\right)\Bigg|_0^{x_0}=-2\sqrt{\frac{2m\alpha}{\hbar^2}}\left(\frac{C-E}{\alpha}\cdot\frac{\pi}{2}\right)=-\sqrt{\frac{2m\alpha}{\hbar^2}}\left(\frac{C-E}{\alpha}\right)\cdot\pi$$

$$D\sim\exp\left(\sqrt{\frac{2m\alpha}{\hbar^2}}\left(\frac{E-C}{\alpha}\right)\right)=\exp\left(-\sqrt{\frac{2m\alpha}{\hbar^2}}\left(\frac{C-E}{\alpha}\right)\right)$$

(14)



$T(E)=?$ $T\sim\frac{1}{D}$

$$V(r)=\begin{cases}-V_0, & r<R_N\\ \frac{2(Z-2)e^2}{r}, & r\geq R_N\end{cases}$$

$E_\alpha=\frac{2(Z-2)e^2}{R}\Rightarrow$

$\Rightarrow R=\frac{2(Z-2)e^2}{E_\alpha}$

$$D\sim\exp\left(-2\int_{R_N}^{R}\sqrt{\frac{2M_\alpha}{\hbar^2}}\sqrt{V(r)-E_\alpha}\,dr\right)$$

$$-2\int_{R_N}^{R}\sqrt{\frac{2M_\alpha}{\hbar^2}}\sqrt{V(r)-E_\alpha}\,dr=-2\int_{R_N}^{R}\sqrt{\frac{2M_\alpha}{\hbar^2}}\sqrt{\frac{2(Z-2)e^2}{r}-\frac{2(Z-2)e^2}{R}}\,dr=$$

$(xp_y - yp_x)| =$

$$= -2\int_{R_N}^{R}\sqrt{\frac{2M_\alpha}{\hbar^2}E_\alpha}\sqrt{\frac{R}{r}-1}\,dr = \left[\begin{matrix} r = Rx^2 \\ x=\sin\varphi\end{matrix}\right] = -2\sqrt{\frac{2M_\alpha}{\hbar^2}}\cdot 2\sqrt{E_\alpha}\cdot R\int_{\sqrt{R_N/R}}^{1}\sqrt{1-x^2}\,dx = -2\sqrt{\frac{2M_\alpha}{\hbar^2}}\cdot 2\sqrt{E_\alpha}\,R\int_{\varphi_0}^{\pi/2}\cos^2\varphi\,d\varphi =$$

$$= -2\sqrt{\frac{2M_\alpha}{\hbar^2}}\sqrt{E_\alpha}\,R\left(\frac{\pi}{2}-\varphi_0-\frac12\sin(2\varphi_0)\right) \quad ; \sin\varphi_0=\sqrt{\frac{R_N}{R}}, \quad \varphi_0\ll 1$$

$$-2\sqrt{\frac{2M_\alpha}{\hbar^2}}\sqrt{E_\alpha}\,R\left(\frac{\pi}{2}-\varphi_0-\frac12\sin(2\varphi_0)\right) \approx -2\sqrt{\frac{2M_\alpha}{\hbar^2}}\sqrt{E_\alpha}\,R\left(\frac{\pi}{2}-2\varphi_0\right) \approx -2\sqrt{\frac{2M_\alpha}{\hbar^2}}\sqrt{E_\alpha}\,R\left(\frac{\pi}{2}-2\sqrt{\frac{R_N}{R}}\right) =$$

$$= -2\sqrt{\frac{2M_\alpha}{\hbar^2}}\sqrt{E_\alpha}\cdot\frac{2(Z-2)e^2}{E_\alpha}\left(\frac{\pi}{2}-2\sqrt{\frac{R_N E_\alpha}{2(Z-2)e^2}}\right) = -2\sqrt{\frac{2M_\alpha}{\hbar^2}}\cdot\frac{2(Z-2)e^2}{\sqrt{E_\alpha}}\left(\frac{\pi}{2}-2\sqrt{\frac{R_N E_\alpha}{2(Z-2)e^2}}\right)$$

$T=\frac{1}{\nu\cdot D}$, где $\nu\simeq\frac{\hbar}{M_\alpha R_N^2}$, $\frac{\hbar}{M_\alpha R_N}\sim v_\alpha$

$$T\approx\frac{M_\alpha R_N^2}{\hbar}\exp\left(-\frac{2}{\hbar}\sqrt{2M_\alpha}\,\frac{2(Z-2)e^2}{\sqrt{E_\alpha}}\left(\frac{\pi}{2}-2\sqrt{\frac{R_N E_\alpha}{2(Z-2)e^2}}\right)\right)$$

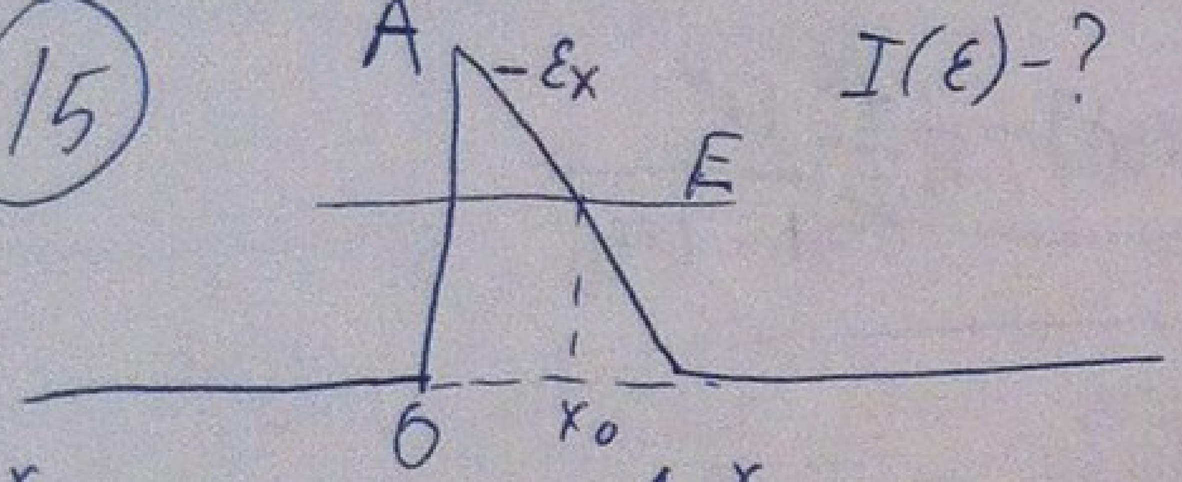
Закон Гейгера-Неттола

$A\approx 148 \quad B\approx 53{,}5$

$$\lg\tau=\frac{A}{\sqrt{E_\alpha}}-B$$

$[\tau]=c$
$[E_\alpha]=$ МэВ

15



$I(\varepsilon)$ - ?

$U(x)=A-\varepsilon x$

$$D\sim\exp\left(-\frac{2}{\hbar}\int_0^{x_0}\sqrt{2m}\sqrt{A-\varepsilon x}\,dx\right)$$

$E=A-\varepsilon x_0$
$A-E=\varepsilon x_0$
$x_0=\frac{A-E}{\varepsilon}$

$$\int_0^x\sqrt{A-\varepsilon x}\,dx=-\frac{1}{\varepsilon}\int_0^x\sqrt{A-\varepsilon x}\,d(A-\varepsilon x)=-\frac{1}{\varepsilon}\cdot(A-\varepsilon x)^{3/2}\cdot\frac{2}{3}\Big|_0^{x_0}=-\frac{1}{\varepsilon}\cdot\frac{2}{3}\left(E^{3/2}-A^{3/2}\right)$$

$$D\sim\exp\left(+\frac{2\sqrt{2m}}{\hbar}\frac{1}{\varepsilon}\frac{2}{3}\left(E^{3/2}-A^{3/2}\right)\right)=\exp\left(-\frac{4\sqrt{2m}}{3\hbar\varepsilon}\left(A^{3/2}-E^{3/2}\right)\right)$$

$$I=I_0\exp\left[-\frac{4\sqrt{2m}}{3\hbar\varepsilon}\left(A^{3/2}-E^{3/2}\right)\right]$$

№ 6 $\quad -\frac{\hbar^2}{2m}\psi''(x)+(U(x)-E)\psi=0$

$$\psi=\frac{1}{\left(\frac{2m}{\hbar^2}(E-U(x))\right)^{1/4}}\sin\left(\int_a^x\sqrt{\frac{2m}{\hbar^2}(E-U(x))}\,dx+\frac{\pi}{4}\right)$$

$$\cancel{\psi'=\frac{1}{\sqrt{\frac{2m}{\hbar^2}}}\cdot\left(+\frac12\right)\frac{1\cdot U'(x)}{(E-U(x))^{3/2}}\sin()+\frac{1}{\sqrt{\frac{2m}{\hbar^2}}\sqrt{E-U(x)}}\cdot\cos()\cdot\sqrt{\frac{2m}{\hbar^2}}\sqrt{E-U(x)}\,\frac12(E-U(x))^{-1/2}\cdot(-U'(x))} =$$

$$\cancel{=\frac{1}{2\sqrt{\frac{2m}{\hbar^2}}}\frac{U'(x)}{(E-U(x))^{3/2}}\cdot\sin()-\cos()\cdot\frac{1}{2}\frac{U'(x)}{\sqrt{E-U(x)}}=\frac{U'(x)}{2}\frac{1}{\sqrt{E-U(x)}}\left(\frac{1}{\sqrt{\frac{2m}{\hbar^2}}}\frac{1}{E-U(x)}\cdot\sin-\cos\right)}$$

$$\psi'=\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\left(+\frac14\right)\frac{U'(x)}{(E-U(x))^{5/4}}\cdot\sin+\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}(E-U(x))^{1/4}}\cos()\cdot\left(\frac{2m}{\hbar^2}\right)^{1/2}\cdot(E-U(x))^{1/2}\cdot\cancel{\frac12(E-U(x))^{-1/2}\cdot(-U'(x))}=$$

$$=\frac14\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\frac{U'(x)}{(E-U(x))^{5/4}}\sin()+\frac{\left(\frac{2m}{\hbar^2}\right)^{1/2}(E-U(x))^{1/2}}{\left(\frac{2m}{\hbar^2}\right)^{1/4}(E-U(x))^{1/4}}\cos()=\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}(E-U(x))^{1/4}}\left(\frac14\frac{U'(x)}{E-U(x)}\sin()-\left(\frac{2m}{\hbar^2}\right)^{1/2}\cos()(E-U(x))^{1/2}\right)$$

$$\Psi'' = \left(\frac{1}{4}\frac{U'(x)}{E-U(x)}\sin() - \left(\frac{2m}{\hbar^2}\right)^{1/2}\cos()\cdot(E-U(x))^{1/2}\right)\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\cdot\left(+\frac{1}{4}\right)\frac{U'(x)}{(E-U(x))^{5/4}} +$$

$$+\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\frac{1}{(E-U(x))^{1/4}}\left(+\left(\frac{2m}{\hbar^2}\right)^{1/2}\sin()\cdot\left(\frac{2m}{\hbar^2}\right)^{1/2}(E-U(x)) + \left(\frac{2m}{\hbar^2}\right)^{1/2}\cos()\cdot\frac{1}{2}(E-U(x))^{-1/2}U'(x) + \frac{1}{4}\left(\frac{U''(x)}{E-U(x)}\sin() + U'(x)\left(\frac{(-1)}{(E-U(x))^2}\sin() + \frac{1\cdot\cos()}{E-U(x)}\cdot\left(\frac{2m}{\hbar^2}\right)^{1/2}(E-U)^{1/2}\right)\right)\right)$$

$$= \frac{1}{4}\frac{U'\sin()}{E-U(x)}\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\cdot\frac{1}{4}\frac{U'(x)}{(E-U(x))^{5/4}} - \left(\frac{2m}{\hbar^2}\right)^{1/2}\cdot\cos()\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\frac{1}{4}\frac{U'(x)(E-U(x))^{1/2}}{(E-U(x))^{5/4}} +$$

$$+\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\frac{1}{(E-U(x))^{1/4}}\frac{2m}{\hbar^2}(E-U(x))\sin() + \frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\frac{1}{(E-U(x))^{1/4}}\frac{1}{4}\frac{U''(x)\sin()}{(E-U(x))} + \frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\frac{1}{(E-U(x))^{1/4}}\frac{1}{4}\frac{U'(x)\cos()}{(E-U(x))}\left(\frac{2m}{\hbar^2}\right)^{1/2}(E-U)^{1/2} -$$

exp

$$-\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\frac{1}{(E-U(x))^{1/4}}\frac{1}{4}\frac{U'\sin()}{(E-U(x))^2} = \underbrace{\frac{1}{\left(\frac{2m}{\hbar^2}\right)^{1/4}}\frac{1}{(E-U(x))^{1/4}}\sin()}_{\Psi}\left[\frac{2m}{\hbar^2}(E-U(x)) + \frac{1}{4}\frac{U''}{(E-U(x))}\right]$$

$$-\frac{\hbar^2}{2m}\Psi\left[\frac{2m}{\hbar^2}(E-U(x)) + \frac{1}{4}\frac{U''}{(E-U(x))}\right] + (V(x)-E)\Psi = 0$$

$$(E-U(x)) - \frac{1}{4}\frac{\hbar^2}{2m}\frac{U''}{(E-U(x))} + (E-V(x)) = 0 \Rightarrow (V(x)-E) = (E-U(x)) - \frac{1}{4}\frac{\hbar^2}{2m}\frac{U''}{E-U(x)}$$

$$\cancel{\frac{1}{4}\frac{\hbar^2}{2m}\frac{U''(x)}{(E-U(x))} = 2(E-U(x))}$$

$$\cancel{U''(x) = 8(E-U(x))^2\frac{16m}{\hbar^2}}$$

Дз на 12.05.2014.

① Th_{232} $E_\alpha = 4{,}01$ МэВ $\tau = ?$

Ra_{226} $E_\alpha = 4{,}78$ МэВ $\tau = 1602$ г.

Po_{212} $E_\alpha = 8{,}78$ МэВ $\tau = ?$

$R_N = 1{,}25 \cdot 10^{-15}$ м $\cdot$ (а.е.м)$^{1/3}$

$$T = \frac{M_\alpha R_N^2}{\hbar} exp\left(+\frac{2}{\hbar}\sqrt{2M_\alpha}\,\frac{2(Z-2)e^2}{\sqrt{E_\alpha}}\left(\frac{\pi}{2} - 2\sqrt{\frac{R_N E_\alpha}{2(Z-2)e^2}}\right)\right)$$

$$\ln \tau = \frac{A}{\sqrt{E_\alpha}} - B$$

$$T = \frac{M_\alpha c^2 \cdot R_N^2}{\hbar c \cdot c} exp\left(+\frac{2\sqrt{2M_\alpha c^2}}{\hbar c}\,\frac{2(Z-2)e^2}{\sqrt{E_\alpha}}\left(\frac{\pi}{2} - 2\sqrt{\frac{R_N E_\alpha}{2(Z-2)e^2}}\right)\right)$$

$$T = A\, exp\left(\frac{2\sqrt{2M_\alpha c^2}}{\cancel{\ }}\,\frac{2(Z-2)}{\sqrt{E_\alpha}}\cdot\frac{e^2}{\hbar c}\left(\frac{\pi}{2} - 2\sqrt{\frac{R_N \cdot E_\alpha}{2(Z-2)e^2}}\right)\right)$$

$$A = 1602 \cdot 3{,}155 \cdot 10^7 \cdot exp\left(-\frac{2\sqrt{2 \cdot 3{,}73 \cdot 10^9}}{\sqrt{4{,}78 \cdot 10^6}} \cdot 2 \cdot 86 \cdot \frac{1}{137}\left(\frac{\pi}{2} - 2\sqrt{\frac{1{,}25 \cdot (222)^{1/3} \cdot 4{,}78 \cdot 10^6}{2 \cdot 86 \cdot 1{,}44 \cdot 10^6}}\right)\right)$$

$$= 5{,}38695 \cdot 10^{-25}$$

1) Th_{232}

$$T = \frac{3{,}73 \cdot 10^9 эВ \cdot 1{,}25 фм \cdot 10^{-15}_м \cdot (228)^{1/3}}{200 \cdot 10^6 эВ \cdot фм \cdot 3 \cdot 10^8 \frac{м}{с}} exp\left(+\frac{2\sqrt{2 \cdot 3{,}73 \cdot 10^9 эВ}}{200 \cdot 10^6 эВ \cdot фм}\,\frac{2 \cdot 88 \cdot 1{,}44 \cdot 10^6 эВ \cdot фм}{\sqrt{4{,}01 \cdot 10^6 эВ}}\left(\frac{\pi}{2} - 2\sqrt{\frac{1{,}25 фм \cdot (228)^{1/3} \cdot 4{,}01 \cdot 10^6}{2 \cdot 88 \cdot 1{,}44 \cdot 10^6 эВ \cdot фм}}\right)\right) =$$

$= \cancel{\ldots}$ $1{,}256 \cdot 10^{21}$ с $\approx 3{,}98 \cdot 10^{13}$ лет.

2) Po_{212}

$$T = \frac{3{,}73 \cdot 10^9 эВ \cdot 1{,}25 фм \cdot 10^{-15} м \cdot (208)^{1/3}}{200 \cdot 10^6 эВ \cdot фм \cdot 3 \cdot 10^8 \frac{м}{с}} exp\left(\frac{2\sqrt{2 \cdot 3{,}73 \cdot 10^9 эВ} \cdot 2 \cdot 82}{137\sqrt{8{,}78 \cdot 10^6}}\left(\frac{\pi}{2} - 2\sqrt{\frac{1{,}25 фм (208)^{1/3} \cdot 4{,}01 \cdot 10^6}{2 \cdot 82 \cdot 1{,}44 \cdot 10^6 эВ \cdot фм}}\right)\right) =$$

$= 3{,}7 \cdot 10^{-6}$ с

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② U_{238} $E = 4{,}2$ МэВ $t = 4{,}51 \cdot 10^9$ г. $R_N = ?$

U_{232} $E = 5{,}32$ МэВ $t = 72$ г $R_N = ?$

U_{227} $E = 6{,}8$ МэВ $t = 1{,}3$ мин. $R_N = ?$

$$\left(\frac{\pi}{2} - \left(\ln\frac{t}{A}\right)\cdot\frac{\sqrt{E_\alpha}}{2\sqrt{2M_\alpha c^2}\cdot 2(Z-2)}\,\frac{\hbar c}{e^2}\right)^2 \frac{1}{4}\,\frac{2(Z-2)e^2}{E_\alpha} = R_N$$

1) U_{238}

$$R_N = \frac{90 \cdot 1{,}44 фм}{2 \cdot 4{,}2}\left(\frac{\pi}{2} - \left(\ln\frac{4{,}51 \cdot 10^9}{5{,}39 \cdot 10^{-25}}\right)\cdot\frac{\sqrt{4{,}2} \cdot 137}{2\sqrt{2 \cdot 3{,}73 \cdot 10^9} \cdot 2 \cdot 90}\right) = 24{,}22 фм$$

2) U_{232}

$$R_N = \frac{90 \cdot 1{,}44 фм}{2 \cdot 5{,}32}\left(\frac{\pi}{2} - \left(\ln\frac{4{,}51 \cdot 10^9}{5{,}39 \cdot 10^{-25}}\right)\cdot\frac{\sqrt{5{,}32} \cdot 137}{2\sqrt{2 \cdot 3{,}73 \cdot 10^9} \cdot 2 \cdot 90}\right) = 19{,}12 фм$$

3) U_{227}

$$R_N = \frac{9 \cdot 1{,}44 фм}{2 \cdot 6{,}8}\left(\frac{\pi}{2} - \left(\ln\frac{4{,}51 \cdot 10^9}{5{,}39 \cdot 10^{-25}}\right)\cdot\frac{\sqrt{6{,}8} \cdot 137}{2\sqrt{2 \cdot 3{,}73 \cdot 10^9} \cdot 2 \cdot 90}\right) = 14{,}96 фм$$

3) 2 независимых Г.О.

$[a a^+] = 1$

$[b b^+] = 1$

$[a, b] = [a, b^+] = 0$

$[l_i, l_j] = i\varepsilon_{ijk} l_k$

$[l_x, l_z] = i l_z$

$$[l_x, l_z] = i\varepsilon_{ikj} l_y = -i\varepsilon_{ijk} l_y = -i l_y$$

$$[l_y, l_z] = i\varepsilon_{jki} l_x = i\varepsilon_{ijk} l_x = i l_x$$

$$l_x = \frac{1}{\hbar}[\bar{x} \times \bar{p}_x] \qquad a a^+ = a^+ a + 1$$

$$\bar{l} = \frac{1}{\hbar}[\bar{r} \times \bar{p}]$$

$$\left.\begin{aligned} l_x &= \frac{a^+ b + a b^+}{2} \\ l_y &= \frac{a^+ b - b a^+}{2i} \\ l_z &= ? \\ l^2 &= ? \end{aligned}\right\} \Rightarrow [l_x, l_y] = i\varepsilon_{xyz} l_z = i l_z$$

$$i l_z = \frac{1}{4i}[(a^+ b + a b^+), (a^+ b - b^+ a)] = \frac{1}{4i}\left([a^+ b, a^+ b] + [a^+ b, -b^+ a] + [a b^+, a^+ b] + [a b^+, -b^+ a]\right)$$

$$= \frac{1}{4i}\left(a^+[b, a^+ b] + [a^+, a^+ b]b + a^+[b, -b^+ a] + [a^+, -b^+ a]b + a[b^+, a^+ b] + [a, a^+ b]b^+ + a[b^+, -b^+ a] + [a, -b^+ a]b^+\right) =$$

$$= \frac{1}{4i}\Big(a^+\underset{0}{[b, a^+]}b + a^+ a^+\underset{0}{[b, b]} + a^+\underset{0}{[a^+, b]}b + \underset{0}{[a^+, a^+]}bb + a^+(-b)\underset{0}{[b, +a]} + a^+\underset{+1}{[b, +b^+]}(-a) + (-b^+)\underset{-1}{[a^+, a]}b + \underset{0}{[a^+, b^+]}(-a)b +$$

$$+ a a^+\underset{-1}{[b^+, b]} + a\underset{0}{[b^+, a^+]}b + a^+\underset{0}{[a, b]}b^+ + \underset{+1}{[a, a^+]}b b^+ + a(-b^+)\underset{0}{[b^+, a]} + a\underset{0}{[b^+, b^+]}(-a) + (-b^+)\underset{0}{[a, a]}b^+ + \underset{0}{[a, +b^+]}(-a)b^+\Big) =$$

$$= \frac{1}{4i}\left(-a^+ a + b^+ b - a a^+ + b b^+\right) = \cancel{\frac{1}{4i}(-(aa^+ + a^+a) + (bb^+ + b^+b)) = \frac{1}{4i}(+1 - 1) = 0}$$

$$\cancel{l^2 = l_x^2 + l_y^2 + l_z^2} \quad = \frac{1}{4i}\left(-(a a^+ + a^+ a) + (b b^+ + b^+ b)\right) = \frac{1}{4i}\left(-(a^+ a + a^+ a + 1) + (b^+ b + b^+ b + 1)\right) = \frac{1}{2i}(b^+ b - a^+ a)$$

$$l_z^2 = \frac{1}{4}(b^+ b - a^+ a)(b b^+ - a a^+) = \frac{1}{4}\left(b^+ b \cdot b b^+ - b^+ b \cdot a a^+ - a^+ a \cdot b b^+ + a^+ a \cdot a a^+\right)$$

$$l_x^2 = \frac{1}{4}(a^+ b + a b^+)(a b^+ + a^+ b) = \frac{1}{4}\left(a^+ b \cdot a b^+ + a^+ b \cdot a^+ b + a b^+ \cdot a b^+ + a b^+ a^+ b\right)$$

$$l_y = \frac{1}{4}(a^+ b - b^+ a)(a b^+ - b a^+) = \frac{1}{4}\left(a^+ b \cdot a b^+ - a^+ b \cdot b a^+ - b^+ a \cdot a b^+ + b^+ a \cdot b a^+\right)$$

$$l^2 = l_z^2 + l_x^2 + l_y^2 = \frac{1}{4}\left(+b^+ b \cdot b b^+ + a^+ a \cdot a a^+ + 2(a^+ b \cdot a b^+) - b^+ b a a^+ - a^+ a b b^+ + (a^+ b)^2 + (a b^+)^2 + a b^+ a^+ b - a^+ b b a^+ - b^+ a \cdot a b^+ + b^+ a b a^+\right) =$$

$$= \frac{1}{4}\left(b^+ b \cdot b b^+ + a^+ a \cdot a a^+ + 2(b \cdot b^+ a^+ a) - b^+ b a a^+ - b b^+ a^+ a + (b a^+)^2 + (b^+ a)^2 + b^+ b a a^+ - b^2 (a^+)^2 - (b^+)^2 a^2 + b^+ b a a^+\right) =$$

$$= \frac{1}{4}\left(b^+ b \cdot b b^+ + a^+ a a a^+ + b b^+ a^+ a + (b a^+)^2 + (b^+ a)^2 + b^+ b a a^+ - b^2 (a^+)^2 - (b^+)^2 a^2\right)$$

④ 1Г.О. $[aa^+]=1$ $l_\pm = l_1 \pm i l_2$

$l_+ = a^+(2l - a^+a)^{1/2}$

$l_- = (2l - a^+a)^{1/2} \cdot a$

1) $[l_3, l_\pm] = l_3 l_\pm - l_\pm l_3 = l_3(l_1 \pm i l_2) - (l_1 \pm i l_2) l_3 =$

$= (l_3 l_1 - l_1 l_3) \pm i(l_3 l_2 - l_2 l_3) = \underbrace{[l_3, l_1]}_{\substack{i\varepsilon_{312} l_2 \\ -i\varepsilon_{132} l_2 \\ +i\varepsilon_{123} l_2}} \pm i \underbrace{[l_3, l_2]}_{\substack{i\varepsilon_{321} l_1 \\ i\varepsilon_{132} l_1 \\ -i\varepsilon_{123} l_1}} =$

$= i l_2 \pm l_1$

2) $[l_+, l_-] = [l_1 + i l_2, l_1 - i l_2] = [l_1, l_1 - i l_2] + i[l_2, l_1 - i l_2] =$

$= \overset{0}{[l_1, l_1]} - i\underbrace{[l_1, l_2]}_{i\varepsilon_{123} l_3} + i\underbrace{[l_2, l_1]}_{\substack{i\varepsilon_{213} l_3 \\ -i\varepsilon_{123} l_3}} + \overset{0}{[l_2, l_2]} = 2 l_3$

---

⑤ $l_z|\psi\rangle = m|\psi\rangle$ $[\hat A, \hat B] = i\hat C \Rightarrow D_A \cdot D_B = \frac{1}{4}\langle C\rangle^2$

$D_{l_x} \cdot D_{l_y} \geq ?$ $[l_x, l_y] = i l_z \Rightarrow D_{l_x} \cdot D_{l_y} = \frac{1}{4}|\langle l_z\rangle|^2 = \ldots^2$

$L = 1$

$\langle C^+ C\rangle \geq 0$ $\frac{1}{4} l_z^2 |l_m\rangle = \frac{1}{4} m^2 |l_m\rangle$

$\langle l_x^2 + l_y^2\rangle = \langle 1^2 - l_z^2\rangle = i(l+1) - m^2$, $\langle l_x^2\rangle = \langle l_y^2\rangle$ $\Rightarrow \langle l_x^2\rangle = \langle l_y^2\rangle = \frac{[l(l+1) - m^2]}{2}$

~~6~~ ⑦ $Y_{1,1}(\theta,\varphi)$, $Y_{1,0}(\theta,\varphi)$, $Y_{1,-1}(\theta,\varphi)$ — совершить преобразования. (найти закон)

ψ, ξ, ζ — углы Эйлера

$Y_{11} = \sqrt{\frac{3}{8\pi}} \frac{x+iy}{z}$

$Y_{1,0} = \sqrt{\frac{3}{4\pi}} \frac{z}{r}$

$Y_{1,-1} = \sqrt{\frac{3}{8\pi}} \frac{x-iy}{r}$

При вращении системы координат, определяемом углами Эйлера α, β, γ, с.ф. преобразуются след. образом:

$Y_{l,m}(\theta,\varphi) = \sum_{m'=-l}^{l} D^l_{mm'}(\alpha,\beta,\gamma) \cdot Y_{lm'}(\theta', \varphi')$

$D^l_{mm'}(\alpha,\beta,\gamma)$ — обобщ. сферич. ф-ции (функции — ~~...~~ Вигнера)

$D^l_{m0}(\alpha,\beta,\gamma) = \sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\beta,\alpha)$

$D^l_{0m}(\alpha,\beta,\gamma) = (-1)^m \sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\beta,\gamma)$

$D^l_{00}(\alpha,\beta,\gamma) = P_l(\cos\beta)$

e^i

Кванты на 12.09.

$H = \frac{p^2}{2m} + \frac{kx^2}{2}$

$E = \hbar\omega\left(n + \frac{1}{2}\right)$

$\omega = \sqrt{\frac{k}{m}}$

$k = 1,\ m = 1,\ \hbar = 1$

№1) $E = \dfrac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$

$$\langle\psi|\psi\rangle = \int_{-a}^{a}\left(1 - \frac{x^2}{a^2}\right)^2 dx = \int_{-a}^{a}\left(1 + \frac{x^4}{a^4} - \frac{2x^2}{a^2}\right)dx =$$

$$= \left(x + \frac{x^5}{5a^4} - \frac{2x^3}{3a^2}\right)\Big|_{-a}^{a} = a + \frac{a^5}{5a^4} - \frac{2a^3}{3a^2} - \left(-a - \frac{a^5}{5a^4} + \frac{2a^3}{3a^2}\right)$$

$$= 2a + \frac{2a}{5} - \frac{4a}{3} = a\,\frac{30 + 6 - 20}{15} = \frac{16}{15}a$$

$$\left\langle\psi\left|\frac{p^2}{2} + \frac{x^2}{2}\right|\psi\right\rangle \qquad \to \left(\frac{\hbar^2}{2m}\right) \qquad p = -i\hbar\frac{\partial}{\partial x}$$

$$p^2 = -\hbar^2\frac{\partial^2}{\partial x^2}$$

$$= \int_{-a}^{a}\left(1 - \frac{x^2}{a^2}\right)\left(-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{x^2}{2}\right)\left(1 - \frac{x^2}{a^2}\right)dx =$$

$$= \int_{-a}^{a}\left(1 - \frac{x^2}{a^2}\right)\left(\frac{\cancel{2}}{\cancel{2}a^2} + \frac{x^2}{2} - \frac{x^4}{2a^2}\right)dx =$$

$$= \int_{-a}^{a}\left(\frac{1}{a^2} + \frac{x^2}{2} - \frac{x^4}{\cancel{2}a^2} - \frac{x^2}{a^4} \cancel{- \frac{x^4}{2a^2}} + \frac{x^6}{2a^4}\right)dx =$$

$$= \left(\frac{x}{a^2} + \frac{x^3}{6} - \frac{x^5}{5a^2} - \frac{x^3}{3a^4} + \frac{x^7}{14a^4}\right)\Big|_{-a}^{a} =$$

$$= 2\frac{a}{a^2} + 2\frac{a^3}{6} - 2\frac{a^5}{5a^2} - 2\frac{a^3}{3a^4} + 2\frac{a^7}{14a^4} =$$

$$= \frac{2}{a} + \frac{a^3}{3} - \frac{2a^3}{5} - \frac{2}{3a} + \frac{a^3}{7} = \frac{4}{3a} + \frac{8}{105}a^3$$

$$\frac{1}{3} - \frac{2}{5} + \frac{1}{7} = \frac{35 - 42 + 15}{105} = \frac{8}{105}$$

$$E(a) = \frac{\frac{4}{3a} + \frac{8}{105}a^3}{\frac{16a}{15}} = \frac{4}{3a}\cdot\frac{15}{16a} + \frac{8a^3}{105}\cdot\frac{15}{16a} = \frac{5}{4a^2} + \frac{a^2}{14}$$

$$\frac{\partial E}{\partial a} = -2\frac{5}{4}\frac{1}{a^3} + \frac{2a}{14} = -\frac{5}{2a^3} + \frac{a}{7} = 0$$

$$\frac{5}{2a^3} = \frac{a}{7} \quad ; \quad 35 = 2a^4$$

$$a^4 = \frac{35}{2} \quad ; \quad a_0 = \left(\frac{35}{2}\right)^{1/4}$$

$$E(a_0) = \frac{5}{4}\left(\frac{2}{35}\right)^{1/2} + \frac{1}{14}\left(\frac{35}{2}\right)^{1/2} = 0{,}598 \approx 0{,}6$$

0,299   0,299

0,239   4,183   0,5 – точн

№2) $H = \frac{p^2}{2} + \frac{x^4}{2}$

б)
$$\psi = \begin{cases} 1 - \frac{x^2}{a^2}, & |x| < a \\ 0, & |x| \geq a \end{cases}$$

$$\langle \psi | \psi \rangle = \frac{16a}{15}$$

$$\langle \psi | \frac{p^2}{2} + \frac{x^4}{2} | \psi \rangle ,$$

$$\frac{1}{2}\langle \psi | p^2 | \psi \rangle = -\frac{1}{2}\langle \psi | \frac{\partial^2}{\partial x^2} | \psi \rangle =$$

$$= -\frac{1}{2}\int_{-a}^{a}\left(1-\frac{x^2}{a^2}\right)\frac{\partial^2}{\partial x^2}\left(1-\frac{x^2}{a^2}\right)dx = +\frac{1}{2}\int_{-a}^{a}\left(1-\frac{x^2}{a^2}\right)\cdot\frac{2}{a^2}\,dx =$$

$$= \frac{1}{a^2}\int_{-a}^{a}\left(1-\frac{x^2}{a^2}\right)dx = \frac{1}{a^2}\left(x - \frac{x^3}{3a^2}\right)\Big|_{-a}^{a} = \frac{1}{a^2}\left(2a - 2\frac{a^3}{3a^2}\right) =$$

$$= \frac{1}{a^2}\left(2a - \frac{2}{3}a\right) = \frac{4}{3a}$$

$$\frac{1}{2}\langle \psi | x^4 | \psi \rangle = \frac{1}{2}\int_{-a}^{a}\left(1-\frac{x^2}{a^2}\right)x^4\left(1-\frac{x^2}{a^2}\right)dx =$$

$$= \frac{1}{2}\int_{-a}^{a}x^4\left(1+\frac{x^4}{a^4}-\frac{2x^2}{a^2}\right)dx = \frac{1}{2}\int_{-a}^{a}\left(x^4 + \frac{x^8}{a^4} - \frac{2x^6}{a^2}\right)dx =$$

$$= \frac{1}{2}\left(\frac{x^5}{5} + \frac{x^9}{9a^4} - \frac{2x^7}{7a^2}\right)\Big|_{-a}^{a} = \qquad \frac{9}{45}+\frac{5}{45}$$

$$= \cancel{\tfrac{1}{2}}\left(\cancel{2}\frac{a^5}{5} + \cancel{2}\frac{a^9}{9a^4} - \cancel{2}\frac{2a^7}{7a^2}\right) = a^5\left(\frac{1}{5}+\frac{1}{9}-\frac{2}{7}\right) =$$

$$= a^5\left(\frac{14}{45} - \frac{2}{7}\right) = a^5\,\frac{98-90}{315} = a^5\cdot\frac{8}{315}$$

$$\langle \psi | \frac{p^2}{2} + \frac{x^4}{2} | \psi \rangle = \frac{4}{3a} + \frac{8a^5}{315}$$

$$E = \frac{\frac{4}{3a} + \frac{8a^5}{315}}{\frac{16a}{15}} = \frac{\cancel{4}}{\cancel{3}a}\cdot\frac{\cancel{15}\,5}{\cancel{16}\,4a} + \frac{\cancel{8}a^{\cancel{5}\,4}}{\cancel{315}\,21}\cdot\frac{\cancel{15}}{\cancel{16}\,2\cancel{a}} = \frac{5}{4a^2} + \frac{a^4}{42}$$

$$\frac{\partial E}{\partial a} = -\frac{5}{4}\,\frac{2}{a^3} + \frac{4a^3}{42} = -\frac{5}{2a^3} + \frac{a^3}{10{,}5} = 0$$

$$\frac{5}{2a^3} = \frac{a^3}{10{,}5}\ ;\quad 2a^6 = 52{,}5\ ;\quad a^6 = 26{,}25\ ;$$

$$a_0 = (26{,}25)^{1/6}\ ;\quad E(a_0) = \overset{0{,}42}{\frac{5}{4\cdot(26{,}25)^{1/3}}} + \overset{0{,}21}{\frac{(26{,}25)^{2/3}}{42}} =$$

$= 0{,}63$ точн $0{,}53$

a) $\psi = e^{-\alpha x^2}$, $\quad H = \frac{p^2}{2} + \frac{x^4}{2}$

$$\langle \psi | \psi \rangle = \int_{-\infty}^{+\infty} e^{-\alpha x^2} e^{-\alpha x^2}\,dx = \int_{-\infty}^{+\infty} e^{-2\alpha x^2}\,dx =$$

$$= \left[\begin{matrix} 2\alpha x^2 = y^2 \\ \sqrt{2\alpha}\,x = y\ ;\ dx = \frac{1}{\sqrt{2\alpha}}\,dy \end{matrix}\right] = \int_{-\infty}^{+\infty} e^{-y^2}\,dy\,\frac{1}{\sqrt{2\alpha}} =$$

$$= \sqrt{\frac{\pi}{2\alpha}}$$

$$\langle \psi | \frac{p^2}{2} | \psi \rangle = -\frac{1}{2} \langle \psi | \frac{\partial^2}{\partial x^2} | \psi \rangle =$$

$$= -\frac{1}{2} \int_{-\infty}^{+\infty} e^{-\alpha x^2} \frac{\partial^2}{\partial x^2} \left(e^{-\alpha x^2} \right) dx =$$

Задача 15.1. Вычислить с использованием вариационного метода энергию основного состояния одномерного гармонического осциллятора.

Решение. Гамильтониан одномерного гармонического осциллятора имеет вид:

$$\widehat{H} = -\frac{\hbar^2}{2M}\frac{d^2}{dx^2} + \frac{M\omega^2 x^2}{2}$$

В качестве пробной функции возьмём функцию $\psi(x,\alpha) = Ae^{-\frac{1}{2}\alpha x^2}$. Эта функция не имеет узлов и стремится к нулю при $x \to \pm\infty$. Условие нормировки для функции ψ даёт нам:

$$1 = \int_{-\infty}^{\infty} \psi^* \psi \, dx = |A|^2 \int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \frac{|A|^2}{\sqrt{\alpha}}\sqrt{\pi},$$

или

$$A = \left(\frac{\alpha}{\pi}\right)^{1/4}$$

Теперь вычисляем интеграл

$$J(\alpha)=\int_{-\infty}^{\infty}dx\sqrt{\frac{\alpha}{\pi}}e^{-\frac{1}{2}\alpha x^2}\left(-\frac{\hbar^2}{2M}\frac{d^2}{dx^2}+\frac{M\omega^2x^2}{2}\right)e^{-\frac{1}{2}\alpha x^2}=$$

$$=\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty}dx\,e^{-\frac{1}{2}\alpha x^2}\left(-\frac{\hbar^2}{2M}\left(\alpha^2x^2-\alpha\right)+\frac{M\omega^2x^2}{2}\right)e^{-\frac{1}{2}\alpha x^2}=$$

$$=\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty}dx\left[\left(-\alpha^2\frac{\hbar^2}{2M}+\frac{M\omega^2}{2}\right)x^2+\alpha\frac{\hbar^2}{2M}\right]e^{-\alpha x^2}=$$

$$=\sqrt{\frac{\alpha}{\pi}}\left[\left(-\alpha^2\frac{\hbar^2}{2M}+\frac{M\omega^2}{2}\right)\frac{\sqrt{\pi}}{2\alpha^{3/2}}+\alpha\frac{\hbar^2}{2M}\frac{\sqrt{\pi}}{\sqrt{\alpha}}\right]=$$

$$=\frac{\hbar^2\alpha}{4M}+\frac{M\omega^2}{4\alpha}$$

При вычислении данного интеграла мы использовали, что

$$\int_{-\infty}^{\infty}dx\,e^{-\alpha x^2}=\frac{\sqrt{\pi}}{\sqrt{\alpha}},\quad \int_{-\infty}^{\infty}dx\,x^2\,e^{-\alpha x^2}=\frac{\sqrt{\pi}}{2\alpha^{3/2}}$$

Минимум $J(\alpha)$ достигается при $\alpha_0=\dfrac{M\omega}{\hbar}$, при этом значение $J(\alpha_0)=E_0=\dfrac{\hbar\omega}{2}$, а волновая функция имеет вид:

$$\psi_0=\left(\frac{M\omega}{\pi\hbar}\right)^{1/4}e^{-\frac{M\omega}{2\hbar}x^2},$$

что в точности совпадает с видом волновой функции, полученной решением уравнения Шрёдингера. ♦

$$\langle U\rangle = A^2\int_0^\infty (x^2+2\alpha x^2)^2 e^{-2\alpha x} x^2\,dx =$$

$$= A^2\int_0^\infty (x^6 + 2\alpha x^6 + \alpha^2 x^6) e^{-2\alpha x}\,dx =$$

$$= -\frac{A^2}{2\alpha}\int_0^\infty \left(\frac{t^6}{64\alpha^4} + \frac{t^5}{16\alpha^4} + \frac{t^4}{16\alpha^4}\right) e^{-t}\,dt =$$

$$= \frac{A^2}{2\alpha}\cdot\frac{1}{\alpha^4}\left\{\frac{3}{2} + \frac{1}{16}\cdot 120 + \frac{720}{64}\right\} =$$

$$= \frac{A^2}{2\alpha^5}\cdot\left\{\frac{3}{2} + \frac{15}{2} + \frac{45}{4}\right\} =$$

$$= \frac{A^2}{2\alpha^5}\cdot\frac{81}{4} = \frac{81A^2}{8\alpha^3}$$

$$\langle E\rangle = \frac{3}{4\alpha}A^2 + \frac{81}{8\alpha^3}A^2 =$$

$$= \frac{3}{4\alpha}\cdot\frac{2\alpha^3}{7} + \frac{81}{8\alpha^4}\cdot\frac{2\alpha^3}{7} =$$

$$= \frac{1}{28}\left[6\alpha^2 + \frac{81}{\alpha^2}\right]$$

$$\frac{\partial\langle E\rangle}{\partial\alpha} = 12\alpha - \frac{243}{\alpha^3} = 0;$$

$$12\alpha^4 = \cancel{243}\ 162$$

$$\cancel{\alpha = \sqrt{\frac{9}{2}}}$$

$$\alpha = \sqrt[4]{\frac{162}{12}}$$

$$E_1 = \frac{9}{7}\sqrt{\frac{3}{2}} \simeq 1,5747$$

Д/З: 8.1.5, 8.14, 8.1.6, 8.1.10

Семинар № 2.

$$\hat{H} = \hat{H}_0 + \varepsilon\hat{V}$$

$$\varepsilon \ll 1$$

$$E_m = \sum_{n=0}^{\infty}\varepsilon^n E_m^{(n)}$$

$$\psi_m = \sum_{n=0}^{\infty}\varepsilon^n\psi_m^n$$

$$E_m^{(1)} = \langle\psi_m^{(0)}|\hat{V}|\psi_m^{(0)}\rangle$$

$$\psi_n^{(1)} = \sum_{m\neq n}\frac{\langle\psi_m^{(0)}|V|\psi_n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}}\psi_n^{(1)}$$

$$E_m^{(2)} = \sum_{m\neq n}\frac{|\langle\psi_n^{(0)}|\hat{V}|\psi_n^{(0)}\rangle|^2}{E_n^{(0)} - E_m^{(0)}}$$

(1)

$$\hat{H}_0 = \begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}$$

$$\hat{V} = \begin{pmatrix}A & B\\ B & A\end{pmatrix}$$

$$|\psi_1\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|\psi_2\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$E_2^{(1)} = \langle\psi_2|V|\psi_2\rangle = \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}A & B\\ B & A\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} =$$

$$= \begin{pmatrix}B & A\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = A\cdot\varepsilon$$

$$E_1^{(1)} = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}A & B\\ B & A\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}A & B\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = A$$

$$E_1^2 = \frac{\langle\psi_2|V|\psi_1\rangle^2}{E_1 - E_2}$$

$(1\,0)\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $(1\,0)\begin{pmatrix} B \\ A \end{pmatrix} = B$

$E_1^{(2)} = \frac{B^2}{E_1 - E_2}$

$E_2^{(2)} = \frac{B^2}{E_2 - E_1}$

(2)

$\hat{H} = \frac{\hat{p}^2 + \hat{x}^2}{2}$ (у доски)

$\hat{V} = \varepsilon \hat{x}^2$

(3)

$\hat{V} = \varepsilon\, \hat{x}^3$

$\hat{H} = \frac{\hat{p}^2}{2m} + c_1 \frac{u_0}{a^2}\frac{\hat{x}^2}{2} + c_2 \frac{u_0}{a^3}\hat{x}^3$

$\varepsilon \sim ?$

логично, что $\varepsilon \sim \frac{x_0}{a}$

x_0 – амплитуда

a – ширина ямы

$\omega = \sqrt{\frac{u_0}{Ma^2}}$

$\Delta p\, \Delta x \sim \hbar$

$x_0 \sim \sqrt{\frac{\hbar}{M\omega}}$

$\varepsilon \sim \sqrt{\frac{\hbar}{M\omega a^2}} = \sqrt[4]{\frac{\hbar^2\, Ma^2}{M^2 a^4 u_0}} =$

$= \sqrt[4]{\frac{\hbar^2}{Ma^2 u_0}} = \frac{1}{B^{\frac{1}{4}}}$

$\omega \sim \sqrt{\frac{m}{M}} \sim 10^{-2}$

$B \sim 10^4 \Rightarrow \varepsilon \sim 0.1$

(4)

$E_n^{(2)} = \varepsilon^2 \sum_{m \neq n} \frac{\left(\langle n | \left(\frac{a + a^+}{\sqrt{2}}\right)^3 | m \rangle\right)^2}{E_n^{(0)} - E_m^{(0)}} =$

$V_{n\,n-3} = \langle n | \hat{a}^{+3} | n-3 \rangle = \sqrt{(n-2)(n-1)n}$

$V_{n\,n+3} = \langle n | \hat{a}^3 | n+3 \rangle = \sqrt{(n+3)(n+2)(n+1)}$

$V_{n\,n-1} = \langle n | \hat{a}\hat{a}^{+2} + \hat{a}^+\hat{a}\hat{a}^+ + \hat{a}^{+2}\hat{a} | n-1 \rangle =$

$= (n-1)\sqrt{n} + n\sqrt{n} + (n+1)\sqrt{n} = 3n\sqrt{n}$

$V_{n\,n+1} = 3(n+1)\sqrt{n+1}$

$= \frac{\varepsilon^2}{8}\left(\frac{V_{n\,n-3}^2 - V_{n\,n+3}^2}{3} + V_{n\,n-1}^2 - V_{n\,n+1}^2\right) =$

$= \frac{\varepsilon^2}{8} \cdot (30n^2 + 30n + 11)$

$E_{n+1} - E_n = \hbar\omega(n + \frac{1}{2})$

$(n+1)^2 - n^2 \approx 2n$

$E_{n+1} - E_n = 1 - \frac{15}{2} n \varepsilon^2 > 0$

$n\varepsilon^2 < \frac{2}{15}$, т. е. при $n \sim 13$ Т.В.

не применима

для В.Ф. $\varepsilon \cdot n^{\frac{3}{2}} \ll 1$

(5)

δ – яма

$\hat{H} = \left(\frac{\hat{p}^2}{2m} - q\delta(x)\right) - (e\mathcal{E}x)$

Первая поправка нулевая.

$|0\rangle = \sqrt{\varkappa}\, e^{-\varkappa|x|}, \quad \varkappa = \frac{m q}{\hbar^2}$

$\langle \beta | \varepsilon x | 0 \rangle$

$\langle k | \varepsilon x | 0 \rangle$

$|k\rangle = \frac{1}{\sqrt{\pi}} \sin kx$

$d = \chi \mathcal{E}$

$\chi = -\frac{\partial^2 E_n}{\partial \mathcal{E}^2} = -2\frac{E_n^{(2)}}{\mathcal{E}^2}$

$$\int_{-\infty}^{+\infty} \sqrt{\varkappa}\, e^{-\varkappa|x|} \frac{1}{\sqrt{\pi}} \cdot \sin kx \cdot x\,dx =$$

$$= 2\int_0^{+\infty} \sqrt{\varkappa}\, e^{-\varkappa x} \cdot \frac{1}{\sqrt{\pi}} \cdot \sin kx \cdot x\,dx =$$

$$= 2\sqrt{\frac{\varkappa}{\pi}} \int_0^{+\infty} e^{-\varkappa x} \cdot \left(\frac{e^{ikx} - e^{-ikx}}{2i}\right) \cdot x\,dx =$$

$$= \frac{1}{i}\sqrt{\frac{\varkappa}{\pi}} \int_0^{+\infty} x \cdot e^{-(\varkappa \pm ik)x}\,dx = \frac{1}{i}\sqrt{\frac{\varkappa}{\pi}} \cdot$$

$$\cdot \frac{1}{(\varkappa \pm ik)^2} \int_0^{+\infty} y e^{-y}\,dy = \frac{1}{i}\sqrt{\frac{\varkappa}{\pi}} \cdot \left(\frac{1}{(\varkappa - ik)^2} - \frac{1}{(\varkappa + ik)^2}\right) =$$

$$= \frac{2 \cdot 2ik \cdot \varkappa}{(\varkappa^2 + k^2)^2} \cdot \frac{1}{i} \sqrt{\frac{\varkappa}{\pi}}$$

$E_0 = -\frac{\hbar^2 \varkappa^2}{2m}; \quad E_k = \frac{\hbar^2 k^2}{2m}$

$$\int_0^{+\infty} \frac{4k^2\varkappa^2}{(\varkappa^2+k^2)^4} \cdot \frac{\varkappa}{\pi} \cdot \frac{dk}{\frac{\hbar^2}{2m}(k^2+\varkappa^2)} = \frac{8m\varkappa^3}{\pi\hbar^2} \int_0^{+\infty} \frac{k^2\,dk}{(\varkappa^2+k^2)^5} =$$

$$= \int_0^{+\infty} \frac{y^2\,dy}{(1+y^2)^5} = \frac{5}{256}\pi$$

$$\chi = \frac{5}{4} \frac{e^2}{q \varkappa^3}$$

2/3. : 8.2.5, 8.2.9

Семинар.

$\hat{H} = \hat{H}_0 + \varepsilon \hat{V}$

$$\hat{H}_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_1 \end{pmatrix}$$

$$V = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$$

$|\psi\rangle = a_1 f_1 + a_2 f_2 + \Sigma a_n f_n$

$\hat{H}|\psi\rangle = a_1 E_1 f_1 + a_2 E_1 f_2 + \varepsilon \hat{V}(a_1 f_1 + a_2 f_2) + \Sigma a_n (E_n + \varepsilon \hat{V}) f_n = E\psi$

$$\begin{cases} a_1 E_1 + \varepsilon(a_1 V_{11} + a_2 V_{12}) + \Sigma a_n \overset{0}{\varepsilon V_{1n}} = a_1 E \\ a_2 E_1 + \varepsilon(a_1 V_{21} + a_2 V_{22}) + \Sigma a_n \overset{0}{\varepsilon V_{2n}} = a_2 E \end{cases}$$

Пренебрегаем

$$\begin{vmatrix} E_1 + \varepsilon V_{11} - E & \varepsilon V_{12} \\ \varepsilon V_{21} & E_1 + \varepsilon V_{22} - E \end{vmatrix} = 0$$

$E^{(1)}$ и $E^{(2)}$ получим из определителя.

Теперь сама задачка.

$$\begin{vmatrix} E_1 + \varepsilon A - E & \varepsilon B \\ \varepsilon B & E_1 + \varepsilon A - E \end{vmatrix} = 0$$

$$(E_1 + \varepsilon A - E)^2 - (\varepsilon B)^2 = 0$$

$$E = E_1 + \varepsilon A \pm \varepsilon B$$

2

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{x^2 + y^2}{2} + \varepsilon \hat{x}\hat{y}$$

$E_1 - ?$

$\frac{\varepsilon}{2}\langle 0,1|\,\hat{a}^{+}\hat{b}^{+} + \hat{a}\hat{b} + \hat{a}\hat{b}^{+} + \hat{a}^{+}\hat{b}\,|1,0\rangle = \frac{\varepsilon}{2}$

$V_{01,10}$

$$V = \begin{vmatrix} 0 & \frac{\varepsilon}{2} \\ \frac{\varepsilon}{2} & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} E_1 - E & \frac{\varepsilon}{2} \\ \frac{\varepsilon}{2} & E_1 - E \end{vmatrix}$$

$$E_1^{(1)} = \pm\frac{\varepsilon}{2}$$

$$E = E_1^{0} \pm \frac{\varepsilon}{2}$$

③ (у доски)

$$U = -q\,\delta(x-a) - q\,\delta(x+a)$$

$$E = -\frac{mq^2}{2\hbar^2}$$

$$\psi_1 = \sqrt{\varkappa}\, e^{-\varkappa|x \pm a|}$$

④

$$H = H_0 + \varepsilon V$$

$$H_0 = \begin{vmatrix} E_1 & 0 & 0 \\ 0 & E_1 & 0 \\ 0 & 0 & E_2 \end{vmatrix}$$

$$\varepsilon V = \begin{vmatrix} 0 & 0 & \varepsilon V \\ 0 & 0 & \varepsilon V \\ \varepsilon V & \varepsilon V & 0 \end{vmatrix}$$

1 способ:

$V_{11} = 0 \Rightarrow$ 1-я поправка нулевая.

$$E_1^{(2)} = \frac{|V_{13}|^2}{E_1 - E_2} = \frac{\varepsilon^2 V^2}{E_1 - E_2}$$

$$E_2^{(2)} = \frac{2\varepsilon^2 V^2}{E_2 - E_1}$$

$$\hat{H} = \begin{pmatrix} E_1 - E & 0 & \varepsilon V \\ 0 & E_1 - E & \varepsilon V \\ \varepsilon V & \varepsilon V & E_2 - E \end{pmatrix}$$

$$(E_1 - E)\cdot\left\{(E_1 - E)(E_2 - E) - \varepsilon^2 V^2\right\} + \varepsilon V\left\{-(E_1 - E)\,\varepsilon V\right\} = 0$$

$$(E_1 - E)\left\{E_1E_2 - E(E_2 + E_1) + E^2 - \varepsilon^2\right\}$$

$$E = \frac{(E_2 + E_1) \pm \sqrt{(E_2 - E_1)^2 + 8\varepsilon^2 V^2}}{2} \approx$$

$$\approx \frac{E_2 + E_1 \pm (E_2 - E_1)\cdot\left(1 + \frac{4\varepsilon^2 V^2}{(E_2 - E_1)^2}\right)}{2}$$

2) $E = E_2 + \frac{2\varepsilon^2 V^2}{(E_2 - E_1)}$

3) $E = E_1 - \frac{2\varepsilon^2 V^2}{(E_2 - E_1)}$ — ответ

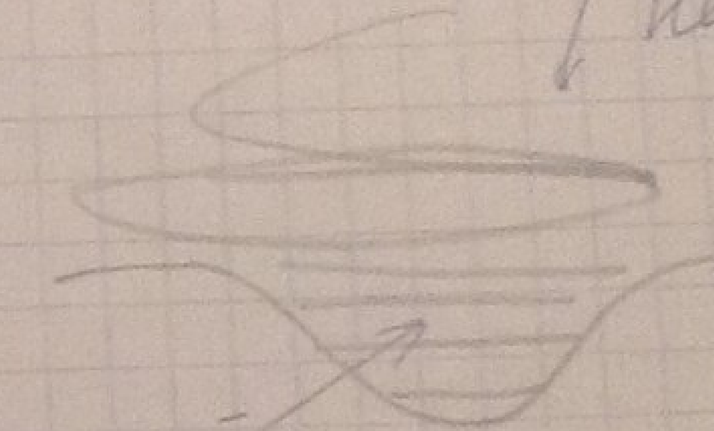
Функция Грина: $G(x, x', t - t')$

$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - U(x)\right) G(x, x', t - t') = i\hbar\,\delta(x - x')\,\delta(t - t')$$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x) - E\right] G(x, x', E) = \delta(x - x')$$

$\rho(E)$ — плотность сос-ий.

непрерывный спектр



дискретный

$$\rho(E) = \frac{1}{\pi L}\,\mathrm{Im}\,\mathrm{Tr}\,G(x, x', E + i\gamma) = \frac{1}{\pi L}\int_{-\frac{L}{2}}^{\frac{L}{2}} G(x, x', E + i\gamma)\,dx$$

$$\hat{G} = \frac{1}{\hat{H} - E}$$

Дворянов Денис 422 гр.

$$\hat{H}_0 = \frac{\hat{p}^2 + \hat{x}^2}{2}$$

$$\hat{V} = \varepsilon \hat{x}$$

$$E^{(1)} = V_{nn} = \langle n | \varepsilon x | n \rangle =$$

$$= \cancel{\frac{\varepsilon}{\sqrt{2}} \cdot \langle n | x | n \rangle} = \frac{\varepsilon}{\sqrt{2}} \langle n | \hat{a}^+ + \hat{a} | n \rangle =$$

$$= \frac{\varepsilon}{\sqrt{2}} \cdot 0$$

+

$$E_m^{2} = \sum_{m \neq n} \frac{|V_{mn}|^2}{E_m^{(0)} - E_n^{(0)}}$$

$$|V_{m,m+1}|^2 = \left| \frac{\varepsilon}{\sqrt{2}} \langle m | \cancel{\hat{a}^+}^{0} + \hat{a} | m+1 \rangle \right|^2 =$$

$$= \left| \frac{\varepsilon}{\sqrt{2}} \cdot \sqrt{m+1} \right|^2 = \frac{\varepsilon^2}{2} \cdot (m+1)$$

$$|V_{m,m-1}|^2 = \left| \frac{\varepsilon}{\sqrt{2}} \cdot \langle m | \hat{a}^+ + \cancel{\hat{a}}^{0} | m-1 \rangle \right|^2 =$$

$$= \left(\frac{\varepsilon}{\sqrt{2}} \sqrt{m} \right) = \frac{\varepsilon^2}{2} m$$

$$E_m^{(2)} = \frac{\varepsilon^2}{2} \{ -(m+1) + m \} = -\frac{\varepsilon^2}{2}$$

Дз на 08.10.14

① $\sum_{n=-\infty}^{n=+\infty} -q_n\delta(x-na)$

$\Psi=\sum_{-\infty}^{+\infty} C_n \varphi_n(x-na)$

вектр-ти после δ функц

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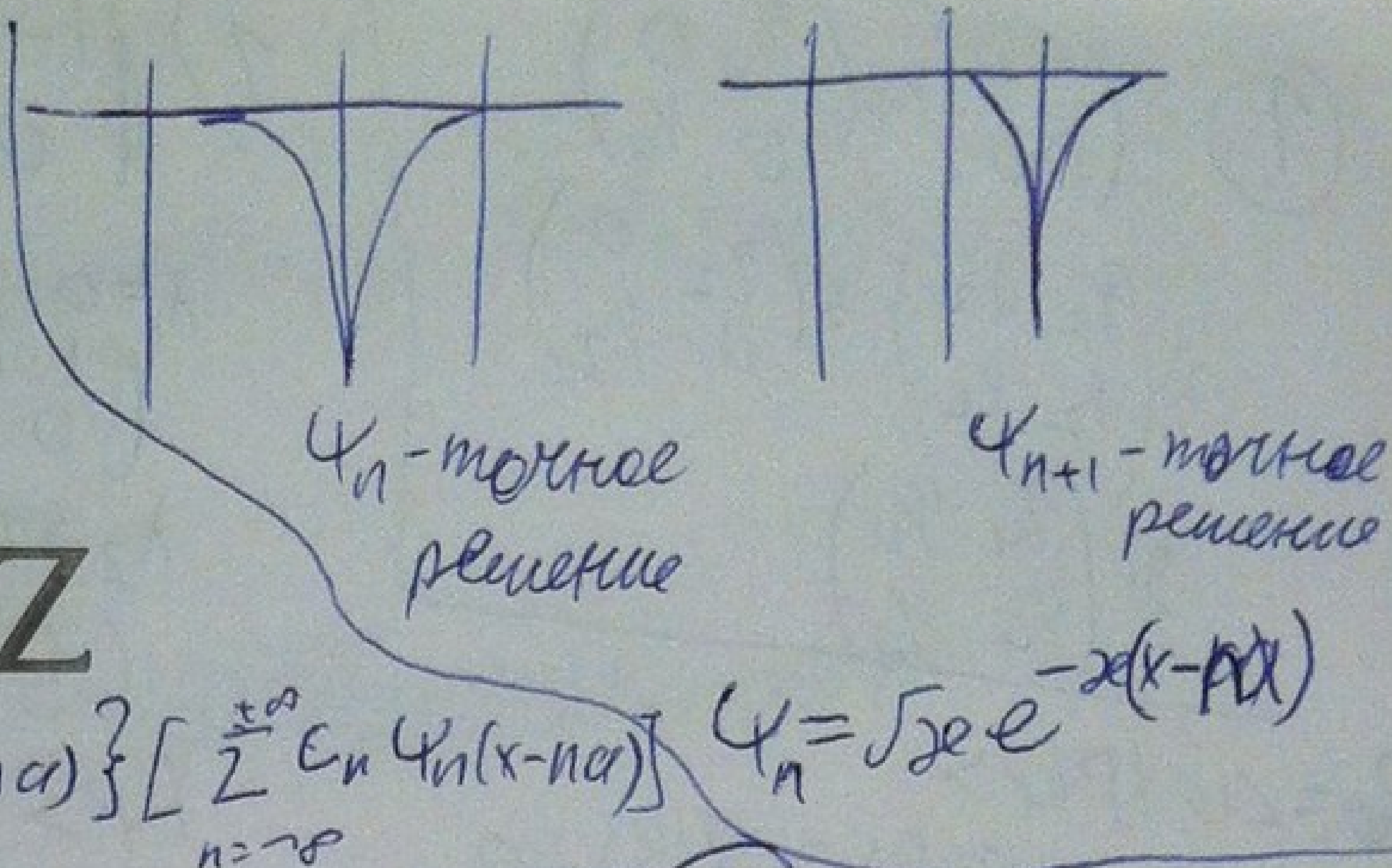

$\varphi_n=\sqrt{\varkappa}\, e^{-\varkappa|x-na|}$

$$E(C_1,C_2,\dots C_n)=\frac{\int dx\left(\sum_{n=-\infty}^{+\infty} C_n\varphi_n(x-na)\right)\left\{\frac{p^2}{2m}-\sum_{n=-\infty}^{+\infty} q_n\delta(x-na)\right\}\left[\sum_{n=-\infty}^{+\infty} C_n\varphi_n(x-na)\right]}{\left[\int dx\left(\sum_{n=-\infty}^{+\infty} C_n\varphi_n(x-na)\right)^2\right]} =$$

$$I_1=\int dx\left(\sum_{-\infty}^{+\infty} C_n\varphi_n(x-na)\right)^2=\sum_{n=-\infty}^{+\infty} C_n^2+\sum_{n,m=-\infty}^{+\infty} C_nC_m\underbrace{\int\varphi_n\varphi_m dx}_{\Delta\simeq e^{-\varkappa a}\to 0\text{ (верим Силаеву)}}=\sum_{-\infty}^{+\infty} C_n^2$$

$$I_2=\int dx\left(\sum_{-\infty}^{+\infty} C_n\varphi_n(x-na)\right)\left\{\frac{p^2}{2m}-\sum_{-\infty}^{+\infty} q_n\delta(x-na)\right\}\left(\sum_{-\infty}^{+\infty} C_n\varphi_n(x-na)\right)=$$

$$=\sum_{-\infty}^{+\infty} C_n^2E_0-\sum_{-\infty}^{+\infty} C_n^2V-\sum_{n,m=-\infty}^{+\infty} C_nC_m\cdot W+\sum_{n,m=-\infty}^{+\infty} C_nC_mE_0\cdot\Delta+\sum_{n,m} C_nC_mE_0\cdot e^{-2\varkappa a}+\dots=$$

$$=\sum_{n=-\infty}^{+\infty} C_n^2E_0-\sum_{n=-\infty}^{+\infty} C_n^2V-2\sum_{\substack{n,m=-\infty\\ m>n}}^{+\infty} C_nC_m\cdot W+2\sum_{\substack{n,m=-\infty\\ m>n}}^{\infty} C_nC_mE_0\cdot\Delta$$

$$\ominus\ \frac{I_1}{I_2}=E_0-V+\frac{-2\sum C_nC_m\cdot W+2\sum C_nC_mE_0\Delta}{\sum C_n^2}=E(C_n)$$

① 1) $\begin{pmatrix}1&0\\0&0\end{pmatrix}$ 4) $\begin{pmatrix}3/4&0\\0&1/4\end{pmatrix}$

2) $\begin{pmatrix}1/2&1/2\\1/2&1/2\end{pmatrix}$ 5) $\begin{pmatrix}1/2&0\\0&1/2\end{pmatrix}$

3) $\begin{pmatrix}3/4&1/4\\1/4&1/4\end{pmatrix}$

1) $\begin{vmatrix}1-\lambda&0\\0&-\lambda\end{vmatrix}=0 \Rightarrow -\lambda+\lambda^2=0 \Rightarrow \lambda(\lambda-1)=0 \Rightarrow \begin{cases}\lambda_1=0\\ \lambda_2=1\end{cases}$

$\lambda=0$

$\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \begin{cases}c_1=0\\c_2=c\end{cases} = c\left|\begin{matrix}0\\1\end{matrix}\right\rangle = \alpha_1$

$\lambda_2=1$

$\begin{pmatrix}0&0\\0&-1\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \begin{cases}c_2=0\\c_1=c'\end{cases} = c'\left|\begin{matrix}1\\0\end{matrix}\right\rangle = \alpha_2$

$|\psi\rangle = c\left|\begin{matrix}0\\1\end{matrix}\right\rangle + c'\left|\begin{matrix}1\\0\end{matrix}\right\rangle$

$P_{\psi} = \langle\psi|\rho|\psi\rangle$

$P_{|\psi_1\rangle} = \langle 0\,1|\begin{matrix}1&0\\0&0\end{matrix}|\begin{matrix}0\\1\end{matrix}\rangle = \langle 0\,1|\begin{matrix}0\\0\end{matrix}\rangle = 0$

$P_{|\psi_2\rangle} = \langle 1\,0|\begin{matrix}1&0\\0&0\end{matrix}|\begin{matrix}1\\0\end{matrix}\rangle = \langle 1\,0|\begin{matrix}1\\0\end{matrix}\rangle = 1$

α_2 – чистое состояние.

4) $\begin{vmatrix}3/4-\lambda&0\\0&1/4-\lambda\end{vmatrix}=0 \Rightarrow \frac{3}{16}+\lambda^2-\lambda=0 \Rightarrow 16\lambda^2-16\lambda+3=0 \Rightarrow \lambda_1=3/4,\ \lambda_2=1/4$

$\lambda_1=3/4$

$\begin{vmatrix}0&0\\0&-1/2\end{vmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \Rightarrow 0\cdot c_1 - 1/2\,c_2=0 \Rightarrow \begin{cases}c_2=0\\c_1=c\end{cases} \Rightarrow c\left|\begin{matrix}1\\0\end{matrix}\right\rangle=\alpha_1$

$\lambda_2=1/4$

$\begin{pmatrix}1/2&0\\0&0\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \Rightarrow 1/2\,c_1+0\cdot c_2=0 \Rightarrow \begin{cases}c_1=0\\c_2=c'\end{cases} \Rightarrow c'\left|\begin{matrix}0\\1\end{matrix}\right\rangle=\alpha_2$

$|\psi\rangle = c\left|\begin{matrix}1\\0\end{matrix}\right\rangle + c'\left|\begin{matrix}0\\1\end{matrix}\right\rangle$

$P_{|\psi_1\rangle} = \langle 1\,0|\begin{matrix}3/4&0\\0&1/4\end{matrix}|\begin{matrix}1\\0\end{matrix}\rangle = \langle 1\,0|\begin{matrix}3/4\\1/4\end{matrix}\rangle = 3/4$

$P_{|\psi_2\rangle} = \langle 0\,1|\begin{matrix}3/4&0\\0&1/4\end{matrix}|\begin{matrix}0\\1\end{matrix}\rangle = \langle 0\,1|\begin{matrix}0\\1/4\end{matrix}\rangle = 1/4$

2) $\begin{vmatrix}1/2-\lambda&1/2\\1/2&1/2-\lambda\end{vmatrix}=0 \Rightarrow (1/2-\lambda)^2=1/4 \Rightarrow \begin{cases}1/2-\lambda=1/2\\1/2-\lambda=-1/2\end{cases} \Rightarrow \begin{cases}\lambda_1=0\\ \lambda_2=1\end{cases}$

$\lambda_1=0$

$\begin{vmatrix}1/2&1/2\\1/2&1/2\end{vmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \Rightarrow c_1/2+c_2/2=0 \Rightarrow c_1=-c_2 \Rightarrow c\left|\begin{matrix}1\\-1\end{matrix}\right\rangle=\alpha_1$

$\lambda_2=1$

$\begin{vmatrix}-1/2&1/2\\1/2&-1/2\end{vmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \Rightarrow -c_1/2+c_2/2=0 \Rightarrow c_1=c_2 \Rightarrow c\left|\begin{matrix}1\\1\end{matrix}\right\rangle=\alpha_2$

$\alpha_1=\left|\begin{matrix}1/\sqrt2\\-1/\sqrt2\end{matrix}\right\rangle \quad \alpha_2=\left|\begin{matrix}1/\sqrt2\\1/\sqrt2\end{matrix}\right\rangle$ → чистое.

$P_{|\alpha_1\rangle} = \langle 1\,{-1}|\begin{matrix}1/2&1/2\\1/2&1/2\end{matrix}|\begin{matrix}1\\-1\end{matrix}\rangle = \langle 1\,{-1}|\begin{matrix}0\\0\end{matrix}\rangle = 0$

$P_{|\alpha_2\rangle} = \langle 1\,1|\begin{matrix}1/2&1/2\\1/2&1/2\end{matrix}|\begin{matrix}1\\1\end{matrix}\rangle = \langle 1\,1|\begin{matrix}1\\1\end{matrix}\rangle = 2$ (не нормировано)

3) $\begin{pmatrix}3/4-\lambda&1/4\\1/4&1/4-\lambda\end{pmatrix}=0 \Rightarrow 3/16+\lambda^2-\lambda-1/16=0 \Rightarrow \lambda^2-\lambda+1/8=0 \Rightarrow 8\lambda^2-8\lambda+1=0 \Rightarrow$

$\begin{cases}\lambda_1=\dfrac{2+\sqrt2}{4}\\[2mm] \lambda_2=\dfrac{2-\sqrt2}{4}\end{cases}$

$\lambda_1=\frac{2+\sqrt2}{4} \Rightarrow \begin{pmatrix}\frac{1-\sqrt2}{4}&1/4\\1/4&\frac{-1-\sqrt2}{4}\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=0 \Rightarrow \begin{cases}(1-\sqrt2)c_1+c_2=0 \Rightarrow c_2=-c_1(1-\sqrt2)\\ c_1+c_2(-1-\sqrt2)=0 \Rightarrow c_1=c_2(1+\sqrt2)\end{cases} \Rightarrow \alpha_1=c\left|\begin{matrix}1+\sqrt2\\1\end{matrix}\right\rangle$

$$\lambda_2 = \frac{2-\sqrt{2}}{4}$$

$$\begin{pmatrix} \frac{1+\sqrt{2}}{4} & \frac14 \\ \frac14 & \frac{-1+\sqrt{2}}{4} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \Rightarrow \begin{cases} c_1(1+\sqrt{2}) + c_2 = 0 \\ c_1 + c_2(-1+\sqrt{2}) = 0 \end{cases} \Rightarrow c_2 = -c_1(1+\sqrt{2}) \Rightarrow \alpha_2 = \left|\begin{matrix} 1 \\ -(1+\sqrt{2}) \end{matrix}\right\rangle$$

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~~проверка на ОН:~~

~~Ортонормировка:~~

$$\beta_1 = \frac{\alpha_1}{\|\alpha_1\|} = \frac{1}{\sqrt{2(2+\sqrt{2})}}\left|\begin{matrix} 1+\sqrt{2} \\ 1 \end{matrix}\right\rangle \qquad \beta_2 = \frac{\alpha_2}{\|\alpha_2\|} = \frac{1}{\sqrt{2(2+\sqrt{2})}}\left|\begin{matrix} 1 \\ -1-\sqrt{2} \end{matrix}\right\rangle$$

$$P_{\beta_1} = \left(\langle 1+\sqrt{2} \;\; 1 | \begin{matrix} 3/4 & 1/4 \\ 1/4 & 1/4 \end{matrix} \left| \begin{matrix} 1+\sqrt{2} \\ 1 \end{matrix}\right\rangle\right) \cdot \frac{1}{2(2+\sqrt{2})} = \frac{1}{2(2+\sqrt{2})}\langle 1+\sqrt{2} \;\; 1 \left| \begin{matrix} 1+\frac34\sqrt{2} \\ \frac12+\frac14\sqrt{2} \end{matrix}\right\rangle =$$

$$= \frac{1}{2(2+\sqrt{2})}\left((1+\sqrt{2})(1+\tfrac34\sqrt{2}) + \tfrac12 + \tfrac14\sqrt{2}\right) =$$

$$= \frac{1}{2(2+\sqrt{2})}\left(1 + \tfrac32 + \tfrac74\sqrt{2} + \tfrac12 + \tfrac14\sqrt{2}\right) = \frac{1}{2(2+\sqrt{2})}(3+2\sqrt{2})$$

$$P_{\beta_2} = \langle 1 \;\; -1-\sqrt{2} | \begin{matrix} 3/4 & 1/4 \\ 1/4 & 1/4 \end{matrix} \left| \begin{matrix} 1 \\ -1-\sqrt{2} \end{matrix}\right\rangle \cdot \frac{1}{2(2+\sqrt{2})} = \frac{1}{2(2+\sqrt{2})}\langle 1 \;\; -1-\sqrt{2} \left| \begin{matrix} \frac12 - \frac14\sqrt{2} \\ -\frac14\sqrt{2} \end{matrix}\right\rangle =$$

$$= \frac{1}{2(2+\sqrt{2})}\left(\tfrac12 - \tfrac14\sqrt{2} + \tfrac14\sqrt{2}(1+\sqrt{2})\right) = \frac{1}{2(2+\sqrt{2})} \cdot 1$$

Смешанные состояния

$$\ldots 12+2\sqrt{2} \doteq 4+2\sqrt{2} \quad 2(2+\sqrt{2})$$

5) $\begin{vmatrix} 1/2-\lambda & 0 \\ 0 & 1/2-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1/2$

$\lambda = 1/2$

$\begin{pmatrix}0&0\\0&0\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow 0\cdot c_1 + 0\cdot c_2 = 0 \Rightarrow c_1 \left|{}^1_0\right\rangle + c_2\left|{}^0_1\right\rangle$

$P_{\lambda_1} = \langle 1\ 0 | \begin{matrix}1/2&0\\0&1/2\end{matrix} \left|{}^1_0\right\rangle = \langle 1\ 0 \left| {}^{1/2}_{0} \right\rangle = 1/2$

Смешанное

$P_{\lambda_2} = \langle 0\ 1 | \begin{matrix}1/2&0\\0&1/2\end{matrix} \left|{}^0_1\right\rangle = \langle 0\ 1 \left| {}^{0}_{1/2} \right\rangle = 1/2$

---

(2) $S = -Tr(\rho \ln \rho)$

$Tr\,\rho^2$

$\rho = \begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}$ т.к. $Tr\,\rho = 1$ $\Rightarrow \rho = \begin{pmatrix}1/2+\varepsilon & 0\\ 0 & 1/2-\varepsilon\end{pmatrix} \Rightarrow \rho^2 = \begin{pmatrix}(1/2+\varepsilon)^2 & 0\\ 0 & (1/2-\varepsilon)^2\end{pmatrix}$

$\ln\rho = \begin{pmatrix}\ln(1/2+\varepsilon) & 0\\ 0 & \ln(1/2-\varepsilon)\end{pmatrix}$

$-1/2 \le \varepsilon \le 1/2$

$Tr\,\rho^2 = \left(\frac{1}{2}+\varepsilon\right)^2 + (1/2-\varepsilon)^2 = 1/4+\varepsilon+\varepsilon^2+1/4-\varepsilon+\varepsilon^2 = 1/2+2\varepsilon^2$ → растёт как степенная

$-Tr(\rho\ln\rho) = -(1/2+\varepsilon)\ln(1/2+\varepsilon) - (1/2-\varepsilon)\ln(1/2-\varepsilon) = -\left(1/2\ln(1/2+\varepsilon) + 1/2\ln(1/2-\varepsilon) + \varepsilon\ln(1/2+\varepsilon) - \varepsilon\ln(1/2-\varepsilon)\right) =$

$= -\left(1/2 \ln[(1/2+\varepsilon)(1/2-\varepsilon)] + \varepsilon\ln\left[\frac{1/2+\varepsilon}{1/2-\varepsilon}\right]\right) = -\left(1/2\ln[1/4-\varepsilon^2] + \varepsilon\ln\left[\frac{1/4-\varepsilon^2}{(1/2-\varepsilon)^2}\right]\right) =$

$= -\left(\ln\frac{(1/4-\varepsilon^2)^{1/2+\varepsilon}}{(1/2-\varepsilon)^{2\varepsilon}}\right) = -\{\ln(1/4+\varepsilon^2)^{1/2+\varepsilon} - \ln(1/2-\varepsilon)^{2\varepsilon}\} = \ln\frac{1}{(1/4+\varepsilon^2)^{1/2+\varepsilon}} + \ln(1/2-\varepsilon)^{2\varepsilon}$

при $\varepsilon \to 1/2$ бесконечно убывает.

убывает ~~[illegible]~~ быстрее чем $\left(\frac{1}{x^2}\right)$

---

(7) $|\psi\rangle = \alpha|\uparrow\uparrow\rangle + \beta|\uparrow\downarrow\rangle + \gamma|\downarrow\uparrow\rangle + \delta|\downarrow\downarrow\rangle$ $\quad \alpha \ne \alpha^*$

$\alpha^2+\beta^2+\gamma^2+\delta^2=1$

$\rho = |\psi\rangle\langle\psi| = \alpha\alpha^*|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \alpha\beta^*|\uparrow\uparrow\rangle\langle\uparrow\downarrow| + \alpha\gamma^*|\uparrow\uparrow\rangle\langle\downarrow\uparrow| + \alpha\delta^*|\uparrow\uparrow\rangle\langle\downarrow\downarrow| +$
$+ \beta\alpha^*|\uparrow\downarrow\rangle\langle\uparrow\uparrow| + \beta\beta^*|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + \beta\gamma^*|\uparrow\downarrow\rangle\langle\downarrow\uparrow| + \beta\delta^*|\uparrow\downarrow\rangle\langle\downarrow\downarrow| +$
$+ \gamma\alpha^*|\downarrow\uparrow\rangle\langle\uparrow\uparrow| + \gamma\beta^*|\downarrow\uparrow\rangle\langle\uparrow\downarrow| + \gamma\gamma^*|\downarrow\uparrow\rangle\langle\downarrow\uparrow| + \gamma\delta^*|\downarrow\uparrow\rangle\langle\downarrow\downarrow| +$
$+ \delta\alpha^*|\downarrow\downarrow\rangle\langle\uparrow\uparrow| + \delta\beta^*|\downarrow\downarrow\rangle\langle\uparrow\downarrow| + \delta\gamma^*|\downarrow\downarrow\rangle\langle\downarrow\uparrow| + \delta\delta^*|\downarrow\downarrow\rangle\langle\downarrow\downarrow|$

$\rho_1 = Tr_2\,\rho = \langle\uparrow|_2\,\rho\,|\uparrow\rangle_2 + \langle\downarrow|_2\,\rho\,|\downarrow\rangle_2 = \alpha\alpha^*|\uparrow\rangle_1\langle\uparrow|_1 + \alpha\gamma^*|\uparrow\rangle_1\langle\downarrow|_1 + \gamma\alpha^*|\downarrow\rangle_1\langle\uparrow|_1 + \gamma\gamma^*|\downarrow\rangle_1\langle\downarrow|_1 +$
$+ \beta\beta^*|\uparrow\rangle_1\langle\uparrow|_1 + \beta\delta^*|\uparrow\rangle_1\langle\downarrow|_1 + \delta\beta^*|\downarrow\rangle_1\langle\uparrow|_1 + \delta\delta^*|\downarrow\rangle_1\langle\downarrow|_1$

$\rho_2 = Tr_1\,\rho = \langle\uparrow|_1\,\rho\,|\uparrow\rangle_1 + \langle\downarrow|_1\,\rho\,|\downarrow\rangle_1 = \alpha\alpha^*|\uparrow\rangle_2\langle\uparrow|_2 + \alpha\beta^*|\uparrow\rangle_2\langle\downarrow|_2 + \beta\alpha^*|\downarrow\rangle_2\langle\uparrow|_2 + \beta\beta^*|\downarrow\rangle_2\langle\downarrow|_2 +$
$+ \gamma\gamma^*|\uparrow\rangle_2\langle\uparrow|_2 + \gamma\delta^*|\uparrow\rangle_2\langle\downarrow|_2 + \delta\gamma^*|\downarrow\rangle_2\langle\uparrow|_2 + \delta\delta^*|\downarrow\rangle_2\langle\downarrow|_2$

Если повыкидывать перекрёстные члены, то $S(\rho_1) = S(\rho_2)$

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№ 0

Дз к.а 15.10.2014

$$\frac{\hat{P}_e^2}{2m_e}+\frac{\hat{P}_p^2}{2m_p}=\hat{T}=-\frac{\hbar^2}{2m_e}\Delta-\frac{\hbar^2}{2m_p}\Delta$$

$$\hat{U}=-\frac{e^2}{r}$$

$$\hat{H}=\hat{T}+\hat{U}$$

$$\hat{H}\Psi=E_0\Psi$$

$$\left(-\frac{\hbar^2}{2}\left(\frac{1}{m_e}+\frac{1}{m_p}\right)\nabla^2-\frac{e^2}{r}\right)\Psi=E_0\Psi \qquad \vec{r}=\vec{r}_e-\vec{r}_p$$

$$\left(-\frac{\hbar^2}{2}\left(\frac{m_p+m_e}{m_e m_p}\right)\nabla^2-\frac{e^2}{r}\right)\Psi=E_0\Psi$$

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2-\frac{e^2}{r}\right)\Psi=E_0\Psi$$

$$E_n=-\frac{\mu e^4}{2\hbar^2}\frac{1}{n^2}$$

Для осн. сост. $E_0=-\frac{\mu e^4}{2\hbar^2}=-\frac{e^4}{2\hbar^2}\left(\frac{m_e m_p}{m_e+m_p}\right)=\underbrace{-\frac{e^2 m_e}{2\hbar^2}}_{Ry}\left(\frac{m_p}{m_p\left(1+\frac{m_e}{m_p}\right)}\right)$ $\frac{1}{2000}$

$\langle T_2\rangle=?$

$$E_0\approx-0{,}9995\,Ry$$

$$\left.\begin{aligned}H&=T_p+T_e+E_{\text{пе}}=E_0\\T_e&=Ry\\E_{\text{пе}}&=-2Ry\\T_p&=xRy\end{aligned}\right\}\Rightarrow xRy+Ry-2Ry=-0{,}9995\,Ry\Rightarrow xRy=0{,}0005\,Ry$$

$$x\approx0{,}0005$$

Ответ: $T_p\approx0{,}0005\,Ry$

№ 1 2 тожд. частицы

$$\Psi(x_1,x_2)=A\exp\left(-x_1^2/2-\alpha x_1x_2-x_2^2/2\right)$$

Б $\to S=0$

Ф синг $\to\frac{|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle}{\sqrt{2}}=S$

$V=?$
$w=?$

$$V(x_1)+V(x_2)+w(x_1-x_2)=U$$

$$\left(-\frac{\hbar^2}{2m_1}\frac{d^2}{dx_1^2}-\frac{\hbar^2}{2m_2}\frac{d^2}{dx_2^2}+U\right)\Psi=E\Psi \qquad \hat{H}|\Psi\rangle=E|\Psi\rangle$$

$$+A\left(-\frac{\hbar^2}{2m_1}\right)\exp(\dots)\left(-1+(x_1+\alpha x_2)^2\right)+A\left(-\frac{\hbar^2}{2m_2}\right)\exp(\dots)\left(-1+(x_2+\alpha x_1)^2\right)+\left(V(x_1)+V(x_2)+w(x_1-x_2)\right)\cdot A\exp(\dots)=$$
$$=E\cdot A\exp(\dots)$$

$$-\frac{\hbar^2}{2m_1}\left(-1+(x_1+\alpha x_2)^2\right)-\frac{\hbar^2}{2m_2}\left(-1+(x_2+\alpha x_1)^2\right)+V(x_1)+V(x_2)+w(x_1-x_2)=E$$

$$V(x_1)+V(x_2)+w(x_1-x_2)-E=+\frac{\hbar^2}{2m_1}\left(-1+x_1^2+2\alpha x_2x_1+\alpha^2x_2^2\right)+\frac{\hbar^2}{2m_2}\left(-1+x_2^2+2\alpha x_1x_2+\alpha^2x_1^2\right)$$

$$-\frac{\hbar^2}{2}\left(\frac{m_1+m_2}{m_1m_2}\right)=-E$$

$$V(x_1)=-\frac{\hbar^2}{2}\left(\frac{x_1^2}{m_1}+\frac{\alpha^2x_1^2}{m_2}\right)$$

$$V(x_2)=-\frac{\hbar^2}{2}\left(\frac{x_2^2}{m_2}+\frac{\alpha^2x_2^2}{m_1}\right)$$

$$W(x_1-x_2)=?$$

$$V(x_1)+V(x_2)-w(x_1-x_2)-E=$$
$$=\frac{\hbar^2}{2m_1}\left(-1+x_1^2+\alpha^2x_2^2-\alpha(x_1-x_2)^2+\alpha(x_1^2+x_2^2)\right)+$$
$$+\frac{\hbar^2}{2m_2}\left(-1+x_2^2+\alpha^2x_1^2-\alpha(x_2-x_1)^2+\alpha(x_2^2+x_1^2)\right)=$$
$$=-\frac{\hbar^2}{2}\left(\frac{m_1+m_2}{m_2m_1}\right)+\frac{\hbar^2}{2}x_1^2\left(\frac{1}{m_1}+\frac{\alpha}{m_1}+\frac{\alpha^2}{m_2}+\frac{\alpha}{m_2}\right)+\frac{\hbar^2}{2}x_2^2\left(\frac{1}{m_2}+\frac{\alpha}{m_2}+\frac{\alpha^2}{m_1}+\frac{\alpha}{m_1}\right)-$$
$$-\frac{\alpha\hbar^2}{2}(x_1-x_2)^2\cdot\left(\frac{m_1+m_2}{m_1m_2}\right)$$

$$2\alpha x_1x_2\to\alpha(2x_1x_2)\to\alpha\left[(x_1-x_2)^2\cdot(-1)+x_1^2+x_2^2\right]\to-\alpha(x_1-x_2)^2+\alpha(x_1^2+x_2)^2$$

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$$-E = -\frac{\hbar^2}{2}\frac{m_1+m_2}{m_1 m_2}$$

$$V(x_1) = \frac{\hbar^2}{2} x_1^2 \left(\frac{1}{m_1}+\frac{\alpha}{m_1}+\frac{\alpha^2}{m_2}+\frac{\alpha}{m_2}\right)$$

$$V(x_2) = \frac{\hbar^2}{2} x_2^2 \left(\frac{1}{m_2}+\frac{\alpha}{m_2}+\frac{\alpha^2}{m_1}+\frac{\alpha}{m_1}\right)$$

$$W(x_1-x_2) = -\frac{\hbar^2}{2}\alpha(x_1-x_2)^2\left(\frac{m_1+m_2}{m_1 m_2}\right)$$

если $m_1 = m_2 = m$

$$-E = -\frac{\hbar^2}{m}$$

$$V(x_1) = \frac{\hbar^2}{2m} x_1^2 (1+2\alpha+\alpha^2) = \frac{\hbar^2}{2m}(\alpha+1)^2 x_1^2$$

$$V(x_2) = \frac{\hbar^2}{2m} x_2^2 (1+2\alpha+\alpha^2) = \frac{\hbar^2}{2m}(\alpha+1)^2 x_2^2$$

$$W(x_1-x_2) = \frac{\hbar^2}{m}\alpha(x_1-x_2)^2\left(\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right)$$

№5 $E = cp$, l – длина ящика

$\nu(\varepsilon) = \frac{dN(\varepsilon)}{d\varepsilon}$ – плотность состояний

3D: $dN_p^{(3)}$ – число состояний в объёме $dp_x dp_y dp_z$

$$dN_p^{(3)} = 2\underbrace{\frac{L_x}{2\pi\hbar}\frac{L_y}{2\pi\hbar}\frac{L_z}{2\pi\hbar}}_{\text{обратная плотность распред. состояний}} dp_x dp_y dp_z = \frac{\overset{\text{из-за проекций спина}}{2}V}{(2\pi\hbar)^3} dp_x dp_y dp_z$$

$E = pc \Rightarrow p = \frac{E}{c} \Rightarrow dp = \frac{d\varepsilon}{c}$

$$dN_p^{(3)} = \frac{2V}{(2\pi\hbar)^3} p^2 dp \sin\theta d\theta d\varphi = \frac{2V}{(2\pi\hbar)^3}\cdot 4\pi \cdot p^2 dp$$

$$dN_\varepsilon^{(3)} = \frac{2V\cdot 4\pi}{(2\pi\hbar)^3}\cdot\frac{\varepsilon^2}{c^3}d\varepsilon = \frac{V\cdot\pi}{(\pi\hbar c)^3}\varepsilon^2 d\varepsilon$$

$$\boxed{\nu(\varepsilon) = \frac{dN(\varepsilon)}{d\varepsilon} = \frac{\pi V}{(\pi\hbar c)^3}\varepsilon^2} \qquad \nu(\varepsilon) \sim \varepsilon^2$$

2D: $\varepsilon = c(p_x+p_y) = pc$; S – площадь двумерного ящика

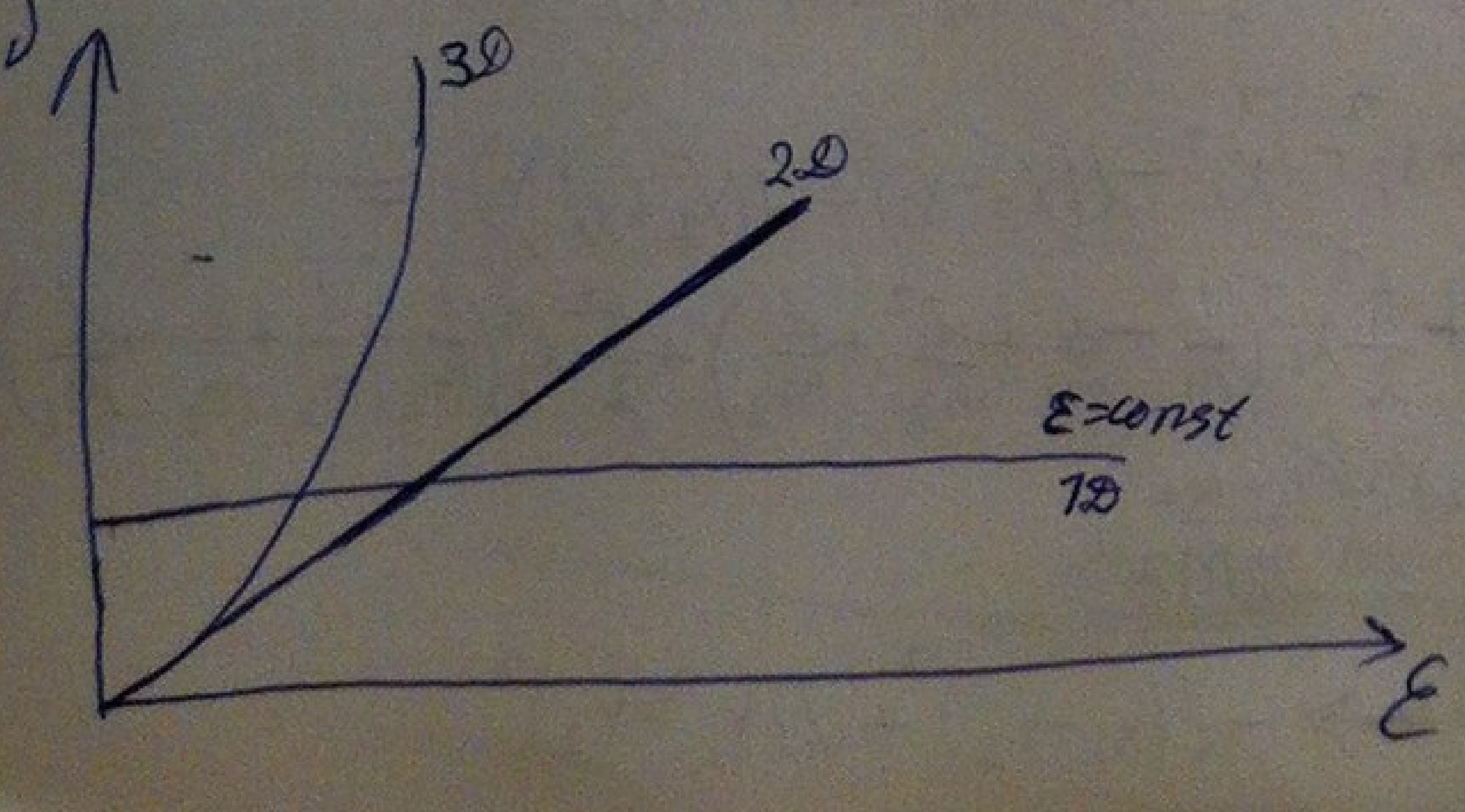
$$dN_p^{(2)} = \frac{2S}{(2\pi\hbar)^2}dp_x dp_y = dN_p^{(2)} = \frac{2S}{(2\pi\hbar)^2}\cdot\underbrace{p\,dp\,d\varphi}_{\text{полярн. коорд}} = \frac{2S\cdot 2\pi}{(2\pi\hbar)^2}p\,dp =$$

$E = pc$, $p = \frac{E}{c}$, $dp = \frac{d\varepsilon}{c}$ $\Rightarrow p\,dp = \frac{E\,dE}{c^2}$

$$= \frac{4\pi S}{(2\pi\hbar)^2}\frac{E\,dE}{c^2} \Rightarrow \boxed{\nu^{(2)}(\varepsilon) = \frac{dN(\varepsilon)}{d\varepsilon} = \frac{4\pi S}{(2\pi\hbar)^2}\frac{E}{c^2}} \Rightarrow \nu^{(2)}(\varepsilon) = \frac{\pi S E}{(\pi\hbar c)^2}$$

1D: $dN_p^{(1)} = \frac{2l}{2\pi\hbar}dp$; $dp = \frac{d\varepsilon}{c}$

$$dN_\varepsilon^{(1)} = \frac{2l}{2\pi\hbar}\frac{d\varepsilon}{c} \Rightarrow \nu^{(1)}(\varepsilon) = \frac{l}{(\pi\hbar c)}$$ – не зависит от ε



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CURTAIN STORIES

⑥ 3 частицы под окном прям поздно вечером

$$\cancel{U = \frac{kx_1^2}{2} + \frac{kx_2^2}{2} + \frac{kx_3^2}{2} + q\frac{(x_1-x_2)^2}{2} + q\frac{(x_2-x_3)^2}{2} + q\frac{(x_3-x_1)^2}{2}}$$

$$\cancel{\mathcal{L} = T - U = \frac{kx_1^2}{2} + \frac{kx_2^2}{2} + \frac{kx_3^2}{2} - q\frac{(x_1-x_2)^2}{2} - q\frac{(x_2-x_3)^2}{2} - q\frac{(x_3-x_1)^2}{2}}$$

$$U = \frac{kx_1^2}{2} + \frac{kx_2^2}{2} + \frac{kx_3^2}{2} + q\frac{(x_1-x_2)^2}{2} + q\frac{(x_2-x_3)^2}{2} + q\frac{(x_3-x_1)^2}{2}; \quad T = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \frac{P_3^2}{2m} = (\dot x_1^2 + \dot x_2^2 + \dot x_3^2)\frac{m}{2}$$

$$\mathcal{L} = T - U = \frac{1}{2}\left[-\left(kx_1^2 + kx_2^2 + kx_3^2 + q(x_1-x_2)^2 + q(x_2-x_3)^2 + q(x_3-x_1)^2\right) + (\dot x_1^2 + \dot x_2^2 + \dot x_3^2)m\right]$$ иначе никак

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot x_i} - \frac{\partial \mathcal{L}}{\partial x_i} = 0$$

m

$$\begin{cases} m\ddot x_1 + kx_1 + q(x_1-x_2) + q(x_1-x_3) = 0 \\ m\ddot x_2 + kx_2 + q(x_2-x_1) + q(x_2-x_3) = 0 \\ m\ddot x_3 + kx_3 + q(x_3-x_1) + q(x_3-x_2) = 0 \end{cases}$$

Ищем решение $x_j(t) = Re(A_j e^{i\omega t})$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = Re\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} e^{i\omega t} \Rightarrow \begin{cases} -m\omega^2 A_1 + (k+2q)A_1 - qA_2 - qA_3 = 0 \\ -m\omega^2 A_2 + (k+2q)A_2 - qA_1 - qA_3 = 0 \\ -m\omega^2 A_3 + (k+2q)A_3 - qA_1 - qA_2 = 0 \end{cases}$$

$$\begin{vmatrix} -m\omega^2 + (k+2q) & -q & -q \\ -q & -m\omega^2 + (k+2q) & -q \\ -q & -q & -m\omega^2 + (k+2q) \end{vmatrix} =$$

$$= \begin{vmatrix} -m\omega^2 + k + 3q & -q & -q \\ m\omega^2 - k - 3q & -m\omega^2 + k + 2q & -q \\ 0 & -q & -m\omega^2 + k + 2q \end{vmatrix} = \begin{vmatrix} -m\omega^2 + k + 3q & -q & -q \\ 0 & -m\omega^2 + k + q & -2q \\ 0 & -q & -m\omega^2 + k + 2q \end{vmatrix} =$$

$$= (-m\omega^2 + k + 3q)\left[-2q^2 + (-m\omega^2 + k + q)(-m\omega^2 + k + 2q)\right] = 0 \Rightarrow \left[\begin{array}{l} -m\omega^2 + k + 3q = 0 \Rightarrow \omega_{(1)}^2 = \frac{k+3q}{m} \\ -2q^2 + (-m\omega^2 + k + q)(-m\omega^2 + k + 2q) = 0 \end{array}\right.$$

$$-2q^2 + m^2\omega^4 - 2m\omega^2 k - 3m\omega^2 q + k^2 + 3kq + 2q^2 = 0$$

$$\omega^4 m^2 - \omega^2(2mk + 3mq) + k^2 + 3kq = 0$$

$$\omega_{(2,3)}^2 = \frac{+2mk + 3mq \pm m\sqrt{(2k+3q)^2 - 4(k^2+3kq)}}{2m^2} = \frac{2k + 3q \pm 3q}{2m}$$

$$\omega_{(2)}^2 = \frac{2k + 6q}{2m} = \frac{k+3q}{m} = \omega_{(1)}^2$$

$$\omega_{(3)}^2 = \frac{2k}{2m} = \frac{k}{m}$$

дальше ищем A_j

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1) $\omega_{(1)}^2 = \frac{k+3q}{m} \Rightarrow \begin{vmatrix} 0 & -q & -q \\ 0 & -2q & -2q \\ 0 & -q & -q \end{vmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow 0 \cdot A_1 - qA_2 - qA_3 = 0$

$\Rightarrow A_2 = -A_3$

$A = C_1 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\omega_{(1)}^{(2)} = \frac{k+3q}{m} \Rightarrow \begin{vmatrix} -q & -q & -q \\ -q & -q & -q \\ -q & -q & -q \end{vmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow A_1 + A_2 + A_3 = 0$

$A_1 = C_1$
$A_2 = C_2$
$A_3 = -C_1 - C_2$

$\Rightarrow A_1 = C_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + C_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$A_1 = C_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$A_{(2)} = C_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

2) $\omega_{(2)}^2 = \frac{k}{m} \Rightarrow \begin{vmatrix} 2q & -q & -q \\ -q & 2q & -q \\ -q & -q & 2q \end{vmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 2A_1 - A_2 - A_3 = 0 \\ -A_1 + 2A_2 - A_3 = 0 \\ -A_1 - A_2 + 2A_3 = 0 \end{cases}$

$\begin{vmatrix} 3q & -q & -q \\ 0 & q & -2q \\ 0 & -q & 2q \end{vmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 3A_1 - A_2 - A_3 = 0 \Rightarrow 3A_1 - 2A_3 - A_3 = 0 \Rightarrow A_1 = A_3 \\ A_2 - 2A_3 = 0 \Rightarrow A_2 = 2A_3 \end{cases}$

$A_3 = C_3 \Rightarrow A_{(3)} = C_3 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$T = \frac{1}{2} \sum_{ij} t_{ij} \dot{x}_i \dot{x}_j = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix}$

$\tilde{A}^{(1)} = C_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}; \tilde{A}^{(1)T} T \tilde{A}^{(1)} = 1 \Rightarrow C_1^2 (1\ 0\ -1) \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = C_1^2 (1\ 0\ -1) \begin{pmatrix} m \\ 0 \\ -m \end{pmatrix} = C_1^2 \cdot 2m \Rightarrow$

$\Rightarrow C_1 = \frac{1}{\sqrt{2m}} \Rightarrow \tilde{A}^{(1)} = \frac{1}{\sqrt{2m}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\tilde{A}^{(2)} = C_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}; \tilde{A}^{(2)T} T \tilde{A}^{(2)} = 1 \Rightarrow C_2^2 (0\ 1\ -1) \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = C_2^2 (0\ 1\ -1) \begin{pmatrix} 0 \\ m \\ -m \end{pmatrix} = C_2^2 \cdot 2m \Rightarrow$

$\Rightarrow C_2 = \frac{1}{\sqrt{2m}} \Rightarrow \tilde{A}^{(2)} = \frac{1}{\sqrt{2m}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$\tilde{A}^{(3)} = C_3 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}; \tilde{A}^{(3)T} T \tilde{A}^{(3)} = 1 \Rightarrow C_3^2 (1\ 2\ 1) \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = C_3^2 (1\ 2\ 1) \begin{pmatrix} m \\ 2m \\ m \end{pmatrix} = C_3^2 \cdot 6m \Rightarrow$

$\Rightarrow C_3 = \frac{1}{\sqrt{6m}} \Rightarrow \tilde{A}^{(3)} = \frac{1}{\sqrt{6m}} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\omega_{(1)}^2 = \frac{k+3q}{m} \longrightarrow \begin{cases} A^{(1)} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \\ A^{(2)} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \end{cases}$

$\omega_{(2)}^2 = \frac{k}{m} \to A^{3} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$A = \begin{pmatrix} \frac{1}{\sqrt{2m}} & 0 & \frac{1}{\sqrt{6m}} \\ 0 & \frac{1}{\sqrt{2m}} & \frac{2}{\sqrt{6m}} \\ -\frac{1}{\sqrt{2m}} & -\frac{1}{\sqrt{2m}} & \frac{1}{\sqrt{6m}} \end{pmatrix}$

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} \Rightarrow \begin{cases} x_1 = \frac{1}{\sqrt{2m}} \xi_1 + \frac{1}{\sqrt{6m}} \xi_3 \\ x_2 = \frac{1}{\sqrt{2m}} \xi_2 + \frac{2}{\sqrt{6m}} \xi_3 \\ x_3 = -\frac{1}{\sqrt{2m}} \xi_1 - \frac{1}{\sqrt{2m}} \xi_2 + \frac{1}{\sqrt{6m}} \xi_3 \end{cases}$

$p = m\dot{\xi}$ — нормальный импульс (размерность другая)

$\mathcal{L} = \frac{1}{2}(\dot{\xi}_1^2 - \omega_{(1)}^2 \xi_1^2) + \frac{1}{2}(\dot{\xi}_2^2 - \omega_{(1)}^2 \xi_2^2) + \frac{1}{2}(\dot{\xi}_3^2 - \omega_{(2)}^2 \xi_3^2)$

$H = \frac{1}{2}\left(\frac{p_1^2}{m^2} + \omega_{(1)}^2 \xi_1^2\right) + \frac{1}{2}\left(\frac{p_2^2}{m^2} + \omega_{(1)}^2 \xi_2^2\right) + \frac{1}{2}\left(\frac{p_3^2}{m^2} + \omega_{(2)}^2 \xi_3^2\right)$

$H = \frac{1}{2m}\left(\frac{p_1^2}{m} + (k+3q)\xi_1^2\right) + \frac{1}{2m}\left(\frac{p_2^2}{m} + (k+3q)\xi_2^2\right) + \frac{1}{2m}\left(\frac{p_3^2}{m} + k\xi_3^2\right)$

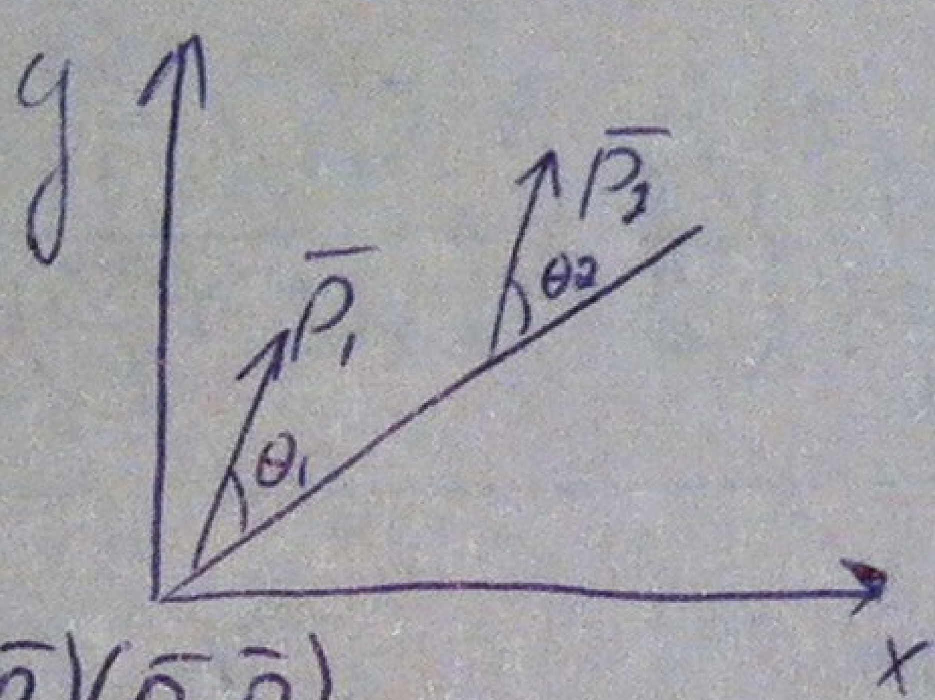
$H = \left[\frac{1}{2m}\left(\frac{p_1^2}{m} + (k+3q)\xi_1^2\right) + \frac{1}{2m}\left(\frac{p_2^2}{m} + (k+3q)\xi_2^2\right)\right] \cdot \frac{|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle}{\sqrt{2}} + \frac{1}{2m}\left(\frac{p_3^2}{m} + k\xi_3^2\right) \left\{ |\uparrow\rangle_3 - |\downarrow\rangle_3 \right.$ для фермионов

$H = \left[\frac{1}{2m}\left(\frac{p_1^2}{m} + (k+3q)\xi_1^2\right) + \frac{1}{2m}\left(\frac{p_2^2}{m} + (k+3q)\xi_2^2\right)\right] \begin{bmatrix} |\uparrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \\ \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}} \end{bmatrix} + \frac{1}{2m}\left(\frac{p_3^2}{m} + k\xi_3^2\right) \left\{ |\uparrow\rangle + |\downarrow\rangle \right.$ для бозонов

$W_{(n\to m)} \ll 1$

---

(5) 2 атома H в осн. сост., $R \gg a_Б$



$\bar{P}_1 = e\bar{r}_1$

$\bar{P}_2 = e\bar{r}_2$

$$\mathcal{E}_{12} = q_2\varphi_1(\bar{r}_2) - (\bar{d}_2, \bar{E}_1(r_1)) =$$

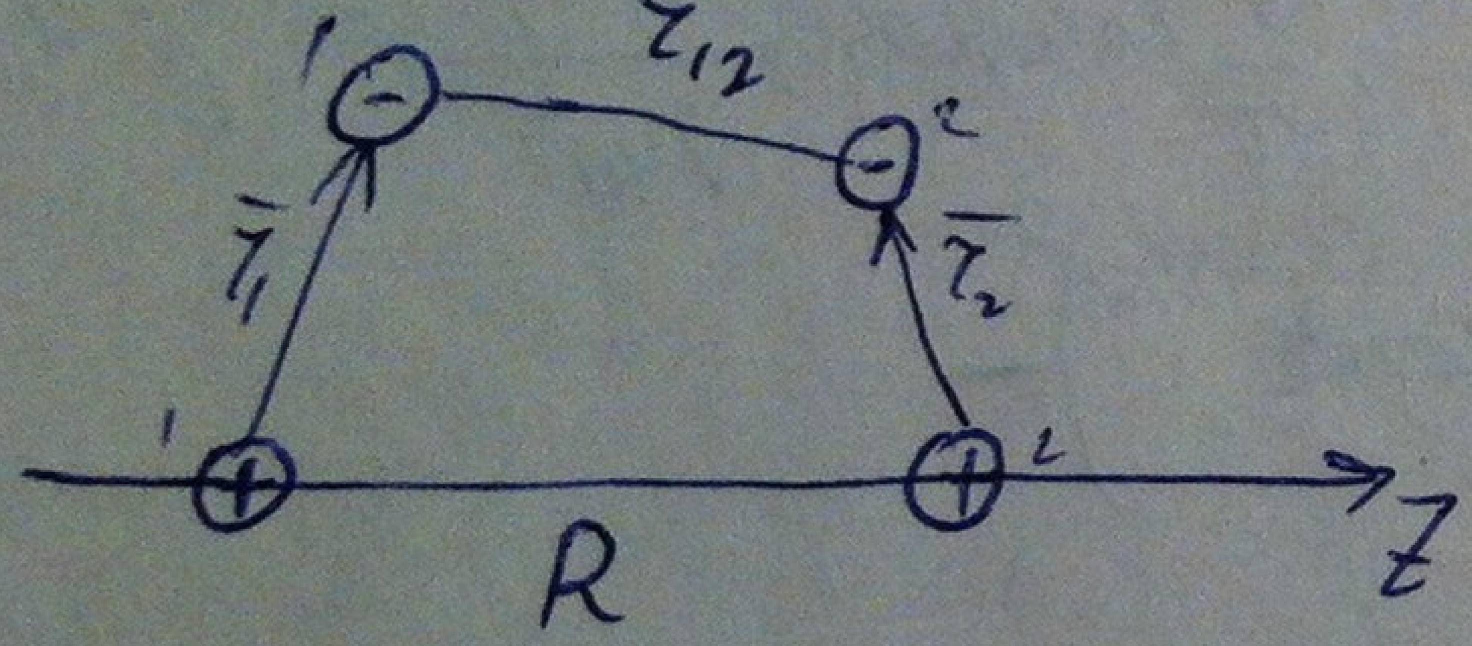
$$= 0\left(\frac{\bar{P}_1\bar{r}}{r^3}\right) - \left(\bar{P}_2, \frac{3(\bar{P}_1\bar{R})\bar{R}}{R^5} - \frac{\bar{P}_1}{R^3}\right) = \frac{(\bar{P}_1, \bar{P}_2)}{R^3} - 3\frac{(\bar{P}_1\bar{R})(\bar{P}_2\bar{R})}{R^5}$$

Энергия взаимодействия диполей

$$\psi_1 = \psi_{100} = \frac{1}{\sqrt{4\pi a^3}} e^{-r_1/a}$$

$$\psi_2 = \psi_{100} = \frac{1}{\sqrt{4\pi a^3}} e^{-r_2/a}$$

$$\mathcal{E}_{12} = \frac{e^2}{R^3}(x_1x_2 + y_1y_2 - 2z_1z_2)$$



ВФ системы $\Psi = \psi_1(r_1)\psi_2(r_2)$

Гамильтониан системы $H = H_0 + \mathcal{E}_{12}$

$$H_0 = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r_1^2} + \frac{\partial^2}{\partial r_2^2}\right) - \frac{e^2}{r_1} - \frac{e^2}{r_2}$$

$$E^{(1)} = \langle \Psi | \mathcal{E}_{12} | \Psi \rangle = \int d^3r_1\, d^3r_2 \frac{e^2}{R^3}(x_1x_2 + y_1y_2 - 2z_1z_2)\frac{1}{4\pi a^3}e^{-2r_1/a}\frac{1}{4\pi a^3}e^{-2r_2/a} =$$

$$= \frac{e^2}{R^3}\left[\left(\int d^3r\, x\frac{1}{4\pi a^3}e^{-2r/a}\right)^2 + \left(\int d^3r\, y\frac{1}{4\pi a^3}e^{-2r/a}\right)^2 - 2\left(\int d^3r\, z\frac{1}{4\pi a^3}e^{-2r/a}\right)^2\right] = 0$$

$$\int_0^{2\pi}\cos\varphi\, d\varphi = 0 \qquad \int_0^{2\pi}\sin\varphi\, d\varphi = 0 \qquad \int_0^{\pi}\sin\theta\cdot\cos\theta\, d\theta = 0$$

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$$E^{(2)} = \sum_n \frac{|\langle 0|\mathcal{E}_{12}|n\rangle|^2}{E_0 - E_n}; \quad \left.\begin{array}{l} \text{т.к. } E_n > E_0 \\ \mathcal{E}_{12} \sim \frac{1}{R^3} \end{array}\right\} \Longrightarrow E^{(2)} = -\frac{C}{R^6}, \; C > 0$$

$$F_{вдв} = -\frac{\partial E}{\partial R} = \frac{C_1}{R^7}$$

Сила притяжения Ван-дер-Ваальса обратнопропорциональна седьмой степени расстояния между атомами.

$$E^{(2)} = \frac{|\langle \psi^{(0)}_{n_1 l_1 m_1} |\mathcal{E}_{12}| \psi^{(0)}_{n_2 l_2 m_2}\rangle|^2}{E^{(0)}_{(2)} - E^{(0)}_{(1)}}, \quad \psi^{(0)}_{n_1 l_1 m_1} = \psi^{(0)}_{210} = \sqrt{\frac{3}{4\pi}}\cos\theta \frac{r}{2\sqrt{6a^5}} e^{-r/2a}$$

$$\psi^{(0)}_{n_2 l_2 m_2} = \psi^{(0)}_{100} = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

$$\langle \psi_{210}|\mathcal{E}_{12}|\psi_{100}\rangle = \frac{e^2}{R^3}\left[\int d^3r_1 d^3r_2 (x_1x_2 + y_1y_2 - 2z_1z_2)\frac{1}{\pi a^3} e^{-r_1/a} e^{-r_2/a} \cdot \frac{3}{4\pi}\cos\theta_1\cos\theta_2 \frac{r_1 r_2}{24a^5} e^{-r_1/2a} e^{-r_2/2a}\right]$$

$$= \frac{e^2}{R^3}\left[\left(\int d^3r\, x \cdot \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \cdot \sqrt{\frac{3}{4\pi}}\frac{r}{2\sqrt{6a^5}}\cos\theta e^{-r/2a}\right)^2 + \left(\int d^3r\, y \frac{1}{\sqrt{\pi a^3}} e^{-r/a}\sqrt{\frac{3}{4\pi}}\frac{r}{2\sqrt{6a^5}}\cos\theta e^{-r/2a}\right)^2 - \right.$$

$$\left. - 2\left(\int d^3r\, z \frac{1}{\sqrt{\pi a^3}} e^{-r/a}\sqrt{\frac{3}{4\pi}}\frac{r}{2\sqrt{6a^5}}\cos\theta e^{-r/2a}\right)^2\right]$$

$$I_1 = \int r\sin\theta\cos\varphi \frac{1}{\sqrt{\pi a^3}} e^{-r/a}\sqrt{\frac{3}{4\pi}}\frac{r}{2\sqrt{6a^5}}\cos\theta e^{-r/2a} \cdot r^2 dr \cdot \sin\theta d\theta d\varphi = 0, \text{ т.к. } \int_0^{2\pi}\cos\varphi d\varphi = 0$$

$$I_2 = \int r\sin\theta\sin\varphi \frac{1}{\sqrt{\pi a^3}} e^{-r/a}\sqrt{\frac{3}{4\pi}}\frac{r}{2\sqrt{6a^5}}\cos\theta e^{-r/2a} r^2 dr \sin\theta d\theta d\varphi = 0, \text{ т.к. } \int_0^{2\pi}\sin\varphi d\varphi = 0$$

$$I_3 = \frac{\sqrt{3}}{\sqrt{4\pi a^4}\cdot 2\sqrt{6}}\int_0^\infty r^4 \exp(-r\cdot 3/2a) dr \underbrace{\int_0^\pi \cos^2\theta\cdot\sin\theta d\theta}_{2/3}\underbrace{\int_0^{2\pi} d\varphi}_{2\pi} = \frac{2}{3}\cdot 2\pi \frac{1}{\sqrt{4\pi a^4}\cdot 2\sqrt{2}}\int_0^\infty r^4 \exp(-r\cdot 3/2a) dr =$$

$$= \frac{1}{3\sqrt{2}a^4}\cdot\frac{24 a^5}{(3/2)^5} = \frac{a}{\sqrt{2}}\frac{8\cdot 2^5}{3^5} = \frac{a 2^7\sqrt{2}}{3^5}$$

$$\langle \psi^{(0)}_{210} | \varepsilon_{12} | \psi^{(0)}_{100} \rangle = \frac{e^2}{R^3}\left[-2 I_3^2\right] = \frac{-2e^2}{R^3} \cdot \frac{a^2 2^{15}}{3^{10}} = -\frac{a^2 e^2}{R^3} \frac{2^{16}}{3^{10}}$$

$$|\langle \psi^{(0)}_{210} | \varepsilon_{12} | \psi^{(0)}_{100} \rangle|^2 = \frac{a^4 e^4}{R^6} \frac{2^{32}}{3^{20}}$$

$$E_2 - E_1 = -Ry - \left(-\frac{Ry}{4}\right) = -\frac{3}{4} Ry$$

$$E^{(2)} = \frac{|\langle \psi^{(0)}_{210} | \varepsilon_{12} | \psi^{(0)}_{100} \rangle|^2}{E^{(0)}_{(2)} - E^{(0)}_{(1)}} = \frac{a^4 e^4}{R^6} \frac{2^{32}}{3^{20}} \cdot \frac{-4}{3\, Ry} = -\frac{a^4 e^4}{R^6 Ry} \frac{2^{34}}{3^{21}}$$

$$F_{\text{В-дВ}} = -\frac{\partial E}{\partial R} = \frac{6 a^4 e^4}{R^7 Ry} \frac{2^{34}}{3^{21}} = \frac{a^4 e^4}{R^7 Ry} \frac{2^{35}}{3^{20}}$$

$\vee\ \psi_{211}, \psi_{21-1}$

$$\psi_{211} = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\varphi} \cdot \frac{r}{2\sqrt{6a^5}} e^{-r/2a}$$

$$\psi_{21-1} = \sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\varphi} \frac{r}{2\sqrt{6a^5}} e^{-r/2a}$$

$$\langle \psi_{211} | \varepsilon_{12} | \psi_{100} \rangle = \frac{e^2}{R^3}\left[\int d^3 r_1 d^3 r_2 (x_1 x_2 + y_1 y_2 - 2 z_1 z_2) \left(\frac{1}{\sqrt{4\pi a^3}}\right)^2 e^{-r_1/a} e^{-r_2/a} \left(-\sqrt{\frac{3}{8\pi}}\right)^2 \sin\theta_1 \sin\theta_2 e^{-i\varphi_1} e^{-i\varphi_2} \frac{r_1 r_2 e^{\frac{-r_1 - r_2}{2a}}}{(2\sqrt{6a^5})^2}\right]$$

$$I_1 = \int r \sin\theta \cos\varphi \frac{1}{\sqrt{4\pi a^3}} e^{-r/a} \left(-\sqrt{\frac{3}{8\pi}}\right) \sin\theta e^{-i\varphi} \frac{r}{2\sqrt{6a^5}} e^{-r/2a} r^2 dr \sin\theta d\theta d\varphi = \frac{-1}{8\pi a^4} \times$$

$$\times \int_0^\infty r^4 \exp(-r \cdot 3/2a) dr \cdot \int_0^\pi \sin^3\theta d\theta \int_0^{2\pi} \cos\varphi e^{-i\varphi} d\varphi = \frac{-1}{8\pi a^4} \frac{24 a^5}{(3/2)^5} \frac{4}{3} \pi = -\frac{a\, 12 \cdot 2^5}{3^6} = -\frac{a \cdot 2^7}{3^5}$$

$$I_2 = \int r \sin\theta \sin\varphi \frac{1}{\sqrt{4\pi a^3}} e^{-r/a} \left(-\sqrt{\frac{3}{8\pi}}\right) \sin\theta e^{-i\varphi} \frac{r}{2\sqrt{6a^5}} e^{-r/2a} r^2 dr \sin\theta d\theta d\varphi = \frac{-1}{8\pi a^4} \times$$

$$\times \int_0^\infty r^4 \exp(-r \cdot 3/2a) dr \int_0^\pi \sin^3\theta d\theta \int_0^{2\pi} \sin\varphi e^{-i\varphi} d\varphi = \frac{-1}{8\pi a^4} \frac{24 a^5}{(3/2)^5} \frac{4}{3} \frac{+\pi}{i} = \frac{-1}{i} \frac{a\, 12 \cdot 2^5}{3^6} = \frac{-a 2^7}{i\, 3^5}$$

$$I_3 = \int r \cos\theta \frac{1}{\sqrt{4\pi a^3}} e^{-r/a} \sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\varphi} \frac{r}{2\sqrt{6a^5}} e^{-r/2a} r^2 dr \sin\theta d\theta d\varphi = \frac{1}{8\pi a^4} \times$$

$$\times \int_0^\infty r^4 \exp(-r \cdot 3/2a) dr \int_0^\pi \cos\theta \sin^2\theta d\theta \int_0^{2\pi} e^{-i\varphi} d\varphi = \frac{1}{8\pi a^4} \frac{24 a^5}{(3/2)^5} 0 = 0$$

$$\langle \psi_{211} | \varepsilon_{12} | \psi_{100} \rangle = \frac{e^2}{R^3}\left[\frac{a^2 2^{14}}{3^{10}} + \frac{a^2 2^{14}}{3^{10}}\right] = \frac{e^2 a^2}{R^3} \frac{2^{15}}{3^{10}}$$

$$E^{(2)} = \frac{|\langle \psi^{(0)}_{211} | \varepsilon_{12} | \psi^{(0)}_{100} \rangle|^2}{E_2^{(0)} - E_1^{(0)}} = \frac{e^4 a^4}{R^6} \frac{2^{30}}{3^{20}} \frac{-4}{3\, Ry} = -\frac{e^4 a^4}{R^6} \frac{2^{32}}{3^{21} \cdot Ry}$$

$$F_{\text{ВдВ}} = -\frac{\partial E}{\partial R} = \frac{6 \cdot a^4 e^4}{R^7} \frac{2^{32}}{3^{21} Ry} = \frac{a^4 e^4}{R^7 Ry} \frac{2^{33}}{3^{20}}$$

аналогично для ψ_{21-1}

$$\frac{|\langle \dots \rangle|^2}{E_2^{(0)}-E_1^{(0)}}=0$$

$$E^{(2)}=\frac{|\langle \psi_{300}|\varepsilon_{12}|\psi_{100}\rangle|^2}{E_3^{(0)}-E_1^{(0)}}=0$$

$$E^{(2)}=\frac{|\langle \psi_{31-1}|\varepsilon_{12}|\psi_{100}\rangle|^2}{E_3^{(0)}-E_1^{(0)}}=0$$

$$E^{(2)}=\frac{|\langle \psi_{311}|\varepsilon_{12}|\psi_{100}\rangle|^2}{E_3^{(0)}-E_1^{(0)}}=0$$

$$E^{(2)}=\frac{|\langle \psi_{310}|\varepsilon_{12}|\psi_{100}\rangle|^2}{E_3^{(0)}-E_1^{(0)}}$$

$$\psi_{100}=\frac{1}{\sqrt{\pi a^3}}e^{-r/a}$$

$$\psi_{310}=\sqrt{\frac{3}{4\pi}}\cos\theta\,\frac{8\,r}{27\sqrt{6a_0^5}}\left(1-\frac{r}{6a_0}\right)e^{-r/3a_0}$$

$$\langle \psi_{310}|\varepsilon_{12}|\psi_{100}\rangle=\frac{e^2}{R^3}\left[\int d^3r_1 d^3r_2 (x_1x_2+y_1y_2-2z_1z_2)\frac{1}{\pi a^3}e^{-r_1/a-r_2/a}\frac{3}{4\pi}\cos\theta_1\cos\theta_2\left(\frac{8}{27\sqrt{6a^5}}\right)^2 r_1r_2\left(1-\frac{r_1}{6a_0}\right)\left(1-\frac{r_2}{6a_0}\right)e^{-\frac{r_1-r_2}{3a_0}}\right]=$$

$$=\frac{e^2}{R^3}\left[\int d^3r\, x\frac{1}{\sqrt{\pi a^3}}e^{-r/a}\sqrt{\frac{3}{4\pi}}\cos\theta\frac{8}{27\sqrt{6a^5}}r\left(1-\frac{r}{6a_0}\right)e^{-r/3a}\right]^2+\dots$$

$$I_1=\int r\sin\theta\cos\varphi\frac{1}{\sqrt{\pi a^3}}e^{-r/a}\sqrt{\frac{3}{4\pi}}\cos\theta\frac{8}{27\sqrt{6a^5}}r\left(1-\frac{r}{6a_0}\right)e^{-r/3a}r^2dr\sin\theta d\theta d\varphi=0,\ \text{т.к.}\int_0^{2\pi}\cos\varphi\, d\varphi=0$$

$$I_2=\int r\sin\theta\sin\varphi\frac{1}{\sqrt{\pi a^3}}e^{-r/a}\sqrt{\frac{3}{4\pi}}\cos\theta\frac{8}{27\sqrt{6a^5}}r\left(1-\frac{r}{6a}\right)e^{-r/3a}r^2dr\sin\theta d\theta d\varphi=0,\ \text{т.к.}\int_0^{2\pi}\sin\varphi\, d\varphi=0$$

$$I_3=\int_0^\infty r^4\left(1-\frac{r}{6a}\right)\exp(-r\cdot 4/3a)\,dr\underbrace{\int_0^\pi\cos^2\theta\sin\theta d\theta}_{2/3}\underbrace{\int_0^{2\pi}d\varphi}_{2\pi}\cdot\frac{1}{\pi a^4}\frac{\sqrt{8}}{27}=\frac{2\sqrt{2}}{\pi a^4\cdot 27}\cdot\frac{2}{3}\cdot 2\pi\int_0^\infty r^4(1-r/6a)\exp(-r\cdot 4/3a)\,dr$$

$$=\frac{8\sqrt{2}}{a^4\cdot 3^4}\frac{a^5\cdot 3^7}{2^{10}}=\frac{a\,3^3}{2^6\sqrt{2}}$$

$$\langle \psi_{310}|\varepsilon_{12}|\psi_{100}\rangle=\frac{e^2}{R^3}\left[-2I_3^2\right]=\frac{e^2}{R^3}\frac{-2\cdot a^2 3^6}{2^{13}}=-\frac{e^2a^2}{R^3}\frac{3^6}{2^{14}}$$

$$|\langle \psi_{310}|\varepsilon_{12}|\psi_{100}\rangle|^2=\frac{e^4a^4}{R^6}\frac{3^{12}}{2^{28}};\quad E_3-E_1=\frac{Ry}{9}-\frac{Ry}{1}=-\frac{8Ry}{9}$$

$$E^{(2)} = \frac{|\langle\psi_{310}^{(0)}|\varepsilon_{12}|\psi_{100}^{(0)}\rangle|^2}{E_{(3)}^{(0)} - E_{(1)}^{(0)}} = \frac{a^4e^4}{R^6}\frac{(3)12}{(2)28}\left(-\frac{9}{8Ry}\right) = -\frac{a^4e^4}{R^6 Ry}\frac{3^{14}}{2^{31}}$$

$$F_{B\text{-}g\text{-}B} = -\frac{\partial E^{(2)}}{\partial R} = \frac{6a^4e^4}{R^7 Ry}\frac{3^{14}}{2^{31}} = \frac{a^4e^4 3^{15}}{R^7 Ry\, 2^{30}}$$

$\langle\psi_{320}|\varepsilon_{12}|\psi_{100}\rangle$: $\psi_{320} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)\frac{4}{81\sqrt{30a^7}}r^2 e^{-r/a\cdot 3}$

$\int\psi_{320}\, x\, \psi_{100} \sim \int(3\cos^2\theta - 1)\cdot\sin\theta\cos\varphi\cdot\sin\theta d\theta d\varphi = 0$, т.к. $\int_0^{2\pi}\cos\varphi d\varphi = 0$

$\int\psi_{320}\, y\, \psi_{100} \sim \int(3\cos^2\theta - 1)\cdot\sin\theta\sin\varphi\,\sin\theta d\theta d\varphi = 0$. т.к. $\int_0^{2\pi}\sin\varphi d\varphi = 0$

$\int\psi_{320}\, z\, \psi_{100} \sim \int(3\cos^2\theta - 1)\cos\theta\cdot\sin\theta d\theta d\varphi \sim \underbrace{\int_{-1}^{1}\cos^3 d\cos\theta}_{0} - \underbrace{\int_0^{\pi}\cos\theta\sin\theta d\theta}_{0} = 0$

$\langle\psi_{320}|\varepsilon_{12}|\psi_{100}\rangle = 0$

$$E^{(2)} = \frac{|\langle\psi_{320}|\varepsilon_{12}|\psi_{100}\rangle|^2}{E_3^{(0)} - E_1^{(0)}} = 0$$

$\langle\psi_{321}|\varepsilon_{12}|\psi_{100}\rangle$: $\psi_{321} = \frac{4 r^2 e^{-r/3a}}{81\sqrt{30a^7}}\left(\sqrt{\frac{15}{8\pi}}\cos\theta\sin\theta e^{i\varphi}\right)$

$$\langle\psi_{321}|\varepsilon_{12}|\psi_{100}\rangle = \frac{e^2}{R^3}\int dr_1^3 dr_2^3 (x_1x_2 + y_1y_2 + z_1z_2)\frac{1}{\pi a^3}e^{\frac{-r_1-r_2}{a}}\left(\frac{4}{81\sqrt{30a^7}}\right)^2 r_1^2 r_2^2 e^{\frac{-r_1-r_2}{3a}}\left(\frac{15}{8\pi}\cos\theta_1\cos\theta_2\sin\theta_1\sin\theta_2 e^{i\varphi_1 i\varphi_2}\right)$$

$I_1 = \int\psi_{321}\, x\, \psi_{100} \sim \int\sin\theta\cos\varphi\cos\theta\sin\theta\,\sin\theta d\theta d\varphi = 0$, т.к. $\int_0^{2\pi}\cos\varphi d\varphi = 0$

$I_2 = \int\psi_{321}\, y\, \psi_{100} \sim \int\sin\theta\sin\varphi\cos\theta\sin\theta\,\sin\theta d\theta d\varphi = 0$, т.к. $\int_0^{2\pi}\sin\varphi d\varphi = 0$

$$I_3 = \int\psi_{321}\, z\, \psi_{100} = \frac{1}{\pi a^5}\frac{4}{81}\frac{1}{4}\int_0^\infty r^5\exp(-r\cdot 4/3a)dr\underbrace{\int_0^{\pi}\cos\theta\cos\theta\sin\theta\sin\theta d\theta}_{\frac{\pi}{8}}\underbrace{\int_0^{2\pi}e^{i\varphi}d\varphi}_{0} =$$

$$= \frac{1}{\pi a^5}\frac{1}{81}\cdot\frac{a^6 5\cdot 3^7}{2^9}\cdot\frac{\pi}{8}\cdot 0 = 0$$

$\langle\psi_{321}|\varepsilon_{12}|\psi_{100}\rangle = 0$

$\langle\psi_{32-1}|\varepsilon_{12}|\psi_{100}\rangle = 0$

$\langle\psi_{322}|\varepsilon_{12}|\psi_{100}\rangle$: $\psi_{322} = \frac{4 r^2 e^{-r/3a}}{81\sqrt{30a^7}}\left(\sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}\right)$

$I_1 = \int\psi_{322}\, x\, \psi_{100} \sim \int\sin\theta\cos\varphi\sin^2\theta e^{2i\varphi}\sin\theta d\theta d\varphi = 0$, т.к. $\int_0^{2\pi}\cos\varphi\, e^{2i\varphi} = 0$

$I_2 = \int\psi_{322}\, y\, \psi_{100} \sim \int\sin\theta\sin\varphi\sin^2\theta e^{2i\varphi}\sin\theta d\theta d\varphi = 0$, т.к. $\int_0^{2\pi}\sin\varphi\, e^{2i\varphi} = 0$

$I_3 = \int\psi_{322}\, z\, \psi_{100} \sim \int\cos\theta\sin^2\theta e^{2i\varphi}\sin\theta d\theta d\varphi = 0$, т.к. $\int_0^{\pi}\cos\theta\cdot\sin^3\theta d\theta = 0$

$\langle\psi_{322}|\varepsilon_{12}|\psi_{100}\rangle = \langle\psi_{32-2}|\varepsilon_{12}|\psi_{100}\rangle = 0$

$$F_{B\text{-}AB} = \frac{a^4e^4}{R^7 Ry}\left(\underbrace{\frac{2^{35}}{3^{21}}}_{\approx 3{,}28} + \underbrace{\frac{2^{33}}{3^{20}}}_{\approx 2{,}46} + \underbrace{\frac{2^{33}}{3^{20}}}_{\approx 2{,}46} + \underbrace{\frac{3^{15}}{2^{30}}}_{\approx 0{,}013}\right)$$

ДЗ на 22.10.14.

№ 01

NБ $S=0$

NФ $S=1/2$

$\Delta x = ?$

$\psi_0 = ?$

$$U = \sum_{i<j} \frac{q(x_i - x_j)^2}{2}$$

BEZMEZ
CURTAIN STORIES

$$H = \begin{pmatrix} q(N-1) & & 0 \\ & q(N-1) & \\ 0 & & q(N-1) \\ & & & \ddots \end{pmatrix} + \begin{pmatrix} 0 & -q & -q & -q & \dots \\ -q & 0 & & & \\ -q & & 0 & & \\ -q & & & \ddots & \\ -q & -q & \dots & & \end{pmatrix} =$$

$$= \begin{pmatrix} qN & & 0 \\ & qN & \\ 0 & & qN \\ & & & \ddots \end{pmatrix} + \begin{pmatrix} -q & \dots \\ \vdots & \ddots \end{pmatrix} =$$

$$= \begin{pmatrix} qN & & 0 \\ & qN & \\ 0 & & qN \\ & & & \ddots \end{pmatrix} - q \begin{pmatrix} 1 & \dots \\ \vdots & \ddots \end{pmatrix}$$

$$\det \begin{vmatrix} 1-\lambda & 1 & \dots \\ 1 & 1-\lambda & \\ & & \ddots \end{vmatrix} = 0 \Rightarrow \left[\begin{array}{l} \lambda_1 = \lambda_2 = \dots = \lambda_{N-1} = 0 \\ \lambda_N = N \end{array} \right.$$

$$\begin{pmatrix} qN & & 0 \\ & qN & \\ 0 & & qN \\ & & & \ddots \end{pmatrix} - q \begin{pmatrix} N & & 0 \\ & 0 & \\ 0 & & 0 \\ & & & \ddots \end{pmatrix} = \begin{pmatrix} 0 & & 0 \\ & qN & \\ 0 & & qN \\ & & & qN \end{pmatrix} = \begin{pmatrix} \omega_1^2 & & 0 \\ & \omega_2^2 & \\ 0 & & \omega_3^2 \\ & & & \ddots \end{pmatrix}$$

$$\hat{U}_{\ell k} = \omega_k^2 \delta^{\ell k}$$

для $\omega_1^2 = 0$

$\ddot{x} + \omega_1^2 x = 0 \Rightarrow \ddot{x} = 0$

$X(t) = C_1 t + C_2$

$X(t) = \frac{p}{m} t + C_2$

$p \neq p(t)$

прямолин. движ.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & \dots \\ 1 & -1 & 0 & 0 & 0 & \dots \\ 1 & 1 & -2 & 0 & \dots & \\ 1 & 1 & 1 & -3 & 0 & \dots \\ 1 & 1 & 1 & \dots & \dots & -(N-1) \end{pmatrix}^T$$

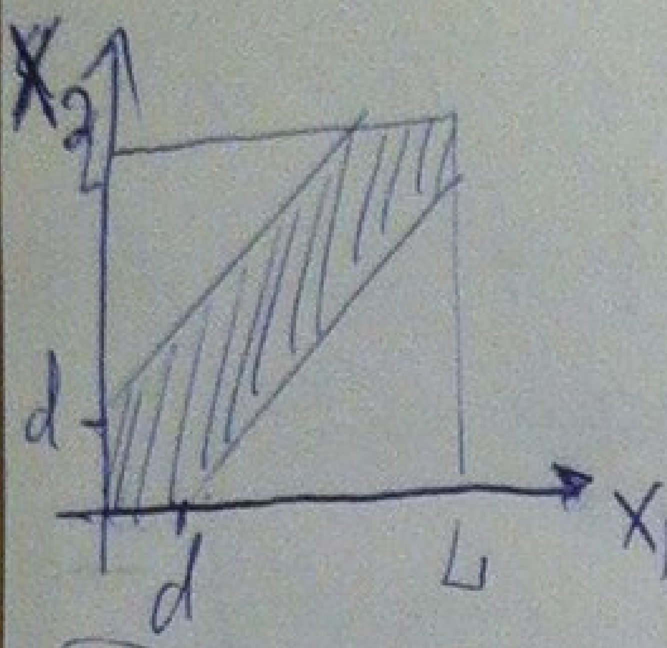
– ортогональные столбцы.

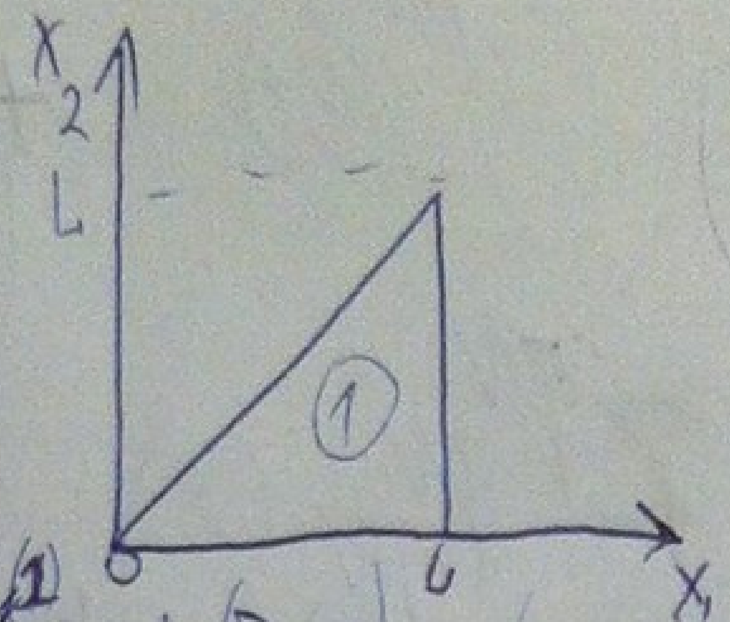
$$\begin{cases} x_1 + x_2 + x_3 + \dots + x_N = \tilde{x}_1 \\ x_1 - x_2 = \tilde{x}_2 \\ x_1 + x_2 - 2x_3 = \tilde{x}_3 \\ \vdots \\ x_1 + x_2 + x_3 + \dots - (N-1) x_N = \tilde{x}_N \end{cases}$$

прямолин. движ.

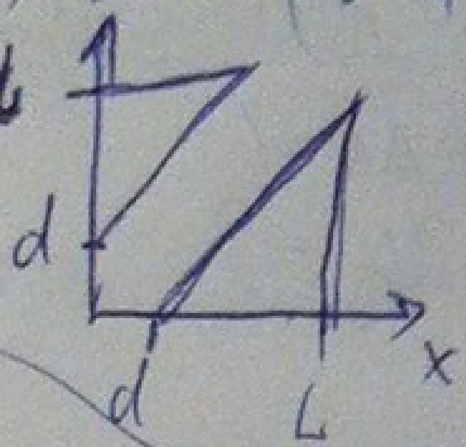
$$\psi_{000\dots0_N} = e^{-a_1 \tilde{x}_1^2} \cdot e^{-a_2 \tilde{x}_2^2} \cdot e^{-a_3 \tilde{x}_3^2} \cdot \ldots \cdot e^{-a_N \tilde{x}_N^2}$$

№ 02 $W(x_1-x_2)=\begin{cases}\infty, & |x_1-x_2|<d\\ 0, & |x_1-x_2|>d\end{cases}$ **2Б**



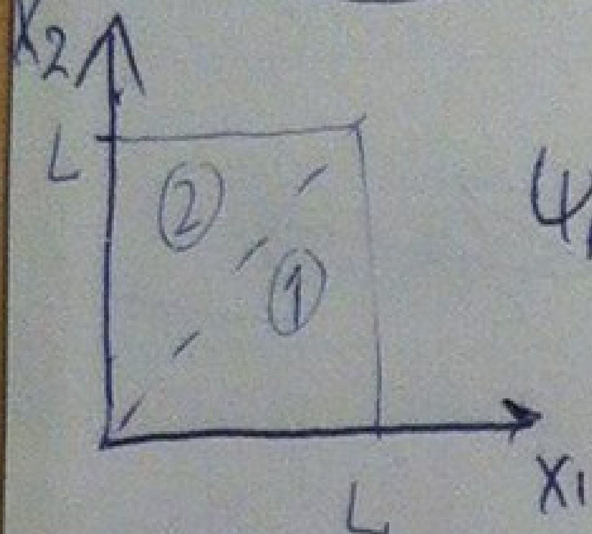


$\cancel{\Psi_{n_1n_2}=\sin\frac{\pi n_1 x_1}{L-d}\cdot\sin\frac{\pi n_2 x_2}{L-d}-\sin\frac{\pi n_1 x_2}{L-d}\sin\frac{\pi n_2 x_1}{L-d}}$



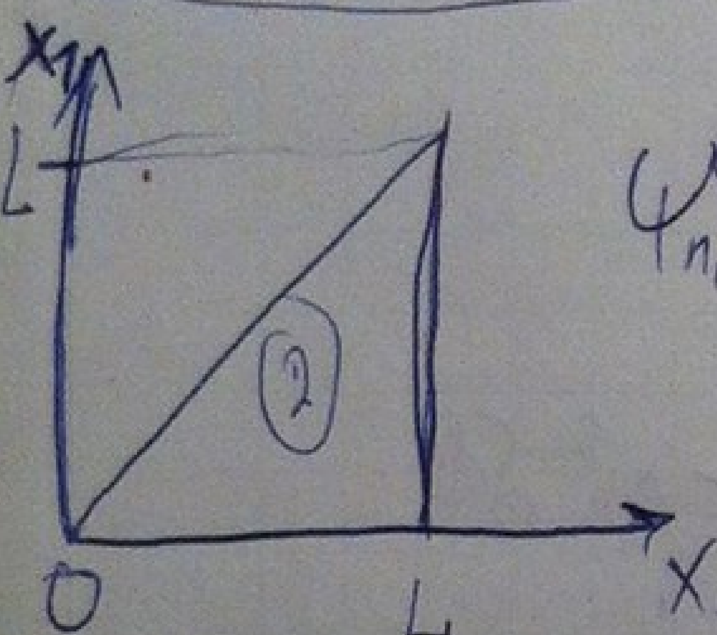
$\Psi=\Psi^{(1)}\cdot\Psi^{(2)}$

$\Psi^{(1)}_{n_1n_2}=\sin\left(\frac{\pi n_1 x_1}{L}\right)\sin\frac{\pi n_2 x_2}{L}-\sin\left(\frac{\pi n_1 x_2}{L}\right)\sin\left(\frac{\pi n_2 x_1}{L}\right)$



$\Psi_{n_1n_2}=\sin\left(\frac{\pi n_1 x_1}{L}\right)\cdot\sin\left(\frac{\pi n_2 x_2}{L}\right)$

$\begin{cases}L\to L-d\\ x_1\to\tilde{x}_1+d\\ x_2\to\tilde{x}_2+d\end{cases}$ $0<\tilde{x}_1<L-d$, $0<\tilde{x}_2<L-d$



$\Psi^{(2)}_{n_1n_2}=\sin\left(\frac{\pi n_2 x_2}{L}\right)\cdot\sin\left(\frac{\pi n_1 x_1}{L}\right)-\sin\left(\frac{\pi n_2 x_1}{L}\right)\sin\left(\frac{\pi n_1 x_2}{L}\right)=\Psi^{(1)}_{n_1n_2}$

$\Psi_{n_1n_2}=\left[\sin\left(\frac{\pi n_1(\tilde{x}_1+d)}{L-d}\right)\sin\left(\frac{\pi n_2(\tilde{x}_2+d)}{L-d}\right)-\sin\left(\frac{\pi n_2(\tilde{x}_1+d)}{L-d}\right)\sin\left(\frac{\pi n_1(\tilde{x}_2+d)}{L-d}\right)\right]^2$ — симметричн.

$\hat{H}=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}$ $\hat{H}\Psi=E\Psi$

№ 1 Оценить среднюю скорость нуклонов в ядре.

r_0, $R_{яд}$, N_p, N_n, r_f
$\langle v\rangle=?$

Энергия ферми $E_F=\frac{P_F^2}{2M}$ P_F — граничный импульс, ниже которого все состояния заняты

Объём фазового пространства $V_\phi=\frac{4\pi P_0^3}{3}$

$\frac{V\phi}{(2\pi\hbar)^3}$ — число состояний, в которых может быть по 2 протона ↑↓ и 2 нейтрона ↑↓, итого целых 4 частицы.

$4\cdot\frac{V\phi}{(2\pi\hbar)^3}=4\cdot\frac{4\pi}{3}\left(\frac{P_0}{2\pi\hbar}\right)^3=\frac{A}{V_я}$ A — массовое число; $A=N_p+N_n$
$V_я$ — объём ядра; $R_я=r_0\cdot A^{1/3}$, $r_0=1{,}2$ fm
$V_я=\frac{4\pi}{3}R_я^3=\frac{4\pi}{3}r_0^3A$

$\cancel{\left(\frac{P_0}{2\pi\hbar}\right)^3=\frac{3}{16\pi}\cdot\frac{A}{r_0A^{1/3}}=\frac{3}{16\pi}\frac{A^{2/3}}{r_0}\Rightarrow P_0=2\pi\hbar\ldots}$

$\frac{2\pi}{3}\left(\frac{P_0}{\pi\hbar}\right)^3=\frac{2}{3}\frac{P_0^3}{\pi^2\hbar^3}=\frac{A}{V_я}\Rightarrow P_0^3=\frac{3\pi^2\hbar^3}{2}\frac{A}{V_я}\Rightarrow P_0=\hbar\left(\frac{3\pi^2A}{2V_я}\right)^{1/3}=\hbar\left(\frac{3\pi^2A}{2\cdot\frac{4\pi}{3}r_0^3A}\right)^{1/3}=\frac{\hbar}{r_0}\left(\frac{9\pi}{8}\right)^{1/3}\approx$

$\approx1{,}52\frac{\hbar}{r_0}\approx244\,\frac{МэВ}{c}$

$P_F=\frac{mv_F}{\sqrt{1-v^2/c^2}}\Rightarrow v^2=\frac{P^2/m^2}{\frac{P^2}{m^2c^2}+1}$. С учетом единиц измерения $v=c\sqrt{\frac{\frac{P^2c^2}{m^2c^4}}{\frac{P^2c^2}{m^2c^4}+1}}$

$v=c\cdot\sqrt{\frac{244^2/938^2}{244^2/938^2+1}}\approx0{,}2517c$

② $E_F = \frac{p_F^2}{2m_e}$, $p_F \to mc$

$$N = \underset{\uparrow \text{спин}}{2} \cdot V \cdot \frac{d^3p}{(2\pi\hbar)^3} = \frac{2V}{(2\pi\hbar)^3}\int_0^{p_F} 4\pi p_F^2 dp_F = \frac{2V \cdot \frac{4}{3}\pi p_F^3}{(2\pi\hbar)^3} \Rightarrow \frac{N}{V} = n = \frac{1}{3}\frac{\pi p_F^3}{(\pi\hbar)^3} \Rightarrow$$

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$$\Rightarrow p_F = \hbar(3\pi^2 n)^{1/3}$$

если $p_F = mc$, то $n = \frac{1}{3}\frac{\pi m^3 c^3}{\pi^3 \hbar^3} = \frac{1}{3\pi^2}\left(\frac{mc}{\hbar}\right)^3$

Ответ: $n \simeq \frac{1}{3\pi^2}\left(\frac{m_e c}{\hbar}\right)^3 \simeq \frac{1}{3\pi^2}\frac{m_e^3 c^3}{\hbar^3} = \frac{1}{3\pi^2}\frac{(m_e c^2)^3}{(\hbar c)^3} = \frac{1}{3\pi^2}\frac{(0{,}51\,\text{МэВ})^3}{(200\,\text{МэВ}\cdot\text{фм})^3} =$

$\simeq 5{,}63 \cdot 10^{-10}\ \text{фм}^{-3} \simeq 5{,}63 \cdot 10^{35}\ \frac{1}{\text{м}^3}$

---

③ $\frac{Z}{A} = \eta$, $Z = N_p$, $A = N_p + N_n$

n – суммарная концентрация нуклонов в ядре

$n_p = \eta \cdot n$

$n_n = (1-\eta)\cdot n$

$E_F^p = \frac{\hbar^2}{2m}(3\pi^2 n_p)^{2/3} = \eta^{2/3} \cdot E_F^0 + E_c$

$E_F^n = (1-\eta)^{2/3} \cdot E_F^0$

$R = R_0 A^{1/3}$

$R_0 = 1{,}25 \cdot 10^{-13}$ см

$E_c = \frac{U}{Z}$, U – энергия заряж. шара (заряж. ядро)

$$U = \frac{3}{5}\frac{Q^2}{RZ} = \frac{3}{5}\frac{e^2(\eta A)^2}{RZ} = \frac{3}{5}\frac{e^2(\eta A)^2}{\eta A R_0 \cdot A^{1/3}} = \underbrace{\frac{3}{5}\cdot\frac{e^2}{R_0}}_{\varepsilon}\eta A^{2/3}$$

условие устойчивости

$E_F^p = \eta^{2/3} E_F^0 + E_c = E_F^n$

$E_F^0 = (3\pi^2 n)^{2/3}\frac{\hbar^2}{2m}$

$\eta^{2/3}E_F^0 + \eta\varepsilon A^{2/3} = (1-\eta)^{2/3}E_F^0$

$\eta = Z/A$

---

⑤ Идеальный ферми-газ. вычислить модуль всестороннего сжатия $B = V\frac{\partial^2 E}{\partial V^2}\Big|_N$

$E_0 = \frac{3}{5}N \cdot E_F = \frac{3}{5}\cdot N \cdot \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$, где $n = \frac{N}{V}$

$$B = V\frac{\partial P}{\partial V} = V\left(\frac{\partial^2 E_0}{\partial V^2}\right)\Big|_N = \frac{3}{5}VN\left(\frac{\partial^2 E_F}{\partial V^2}\right)\Big|_N = \frac{3}{5}\cdot V \cdot N \cdot C \cdot \left(\frac{\partial^2}{\partial V^2}\left(\frac{1}{V}\right)^{2/3}\right),$$

$\left\{C = \frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}\right\}$; $\frac{\partial^2}{\partial V^2}\left(\frac{1}{V}\right)^{2/3} = \frac{\partial^2}{\partial V^2}\left(V^{-2/3}\right) = \frac{\partial}{\partial V}\left(-\frac{2}{3}V^{-5/3}\right) = \frac{10}{9}V^{-8/3} = \frac{10}{9}\left(\frac{1}{V^8}\right)^{1/3}$

$$B = \frac{3}{5}\cdot\frac{N}{V}\cdot V^2 \cdot \frac{\hbar^2}{2m}\cdot(3\pi^2 N)^{2/3}\cdot\frac{10}{9}\left(\frac{1}{V^8}\right)^{1/3} = \frac{3}{5}\cdot\frac{N}{V}\cdot\frac{10}{9}\cdot\left(3\pi^2 N \cdot \frac{V^3}{V^4}\right)^{2/3} = \frac{3}{5}\cdot\frac{10}{9}\,nE_F$$

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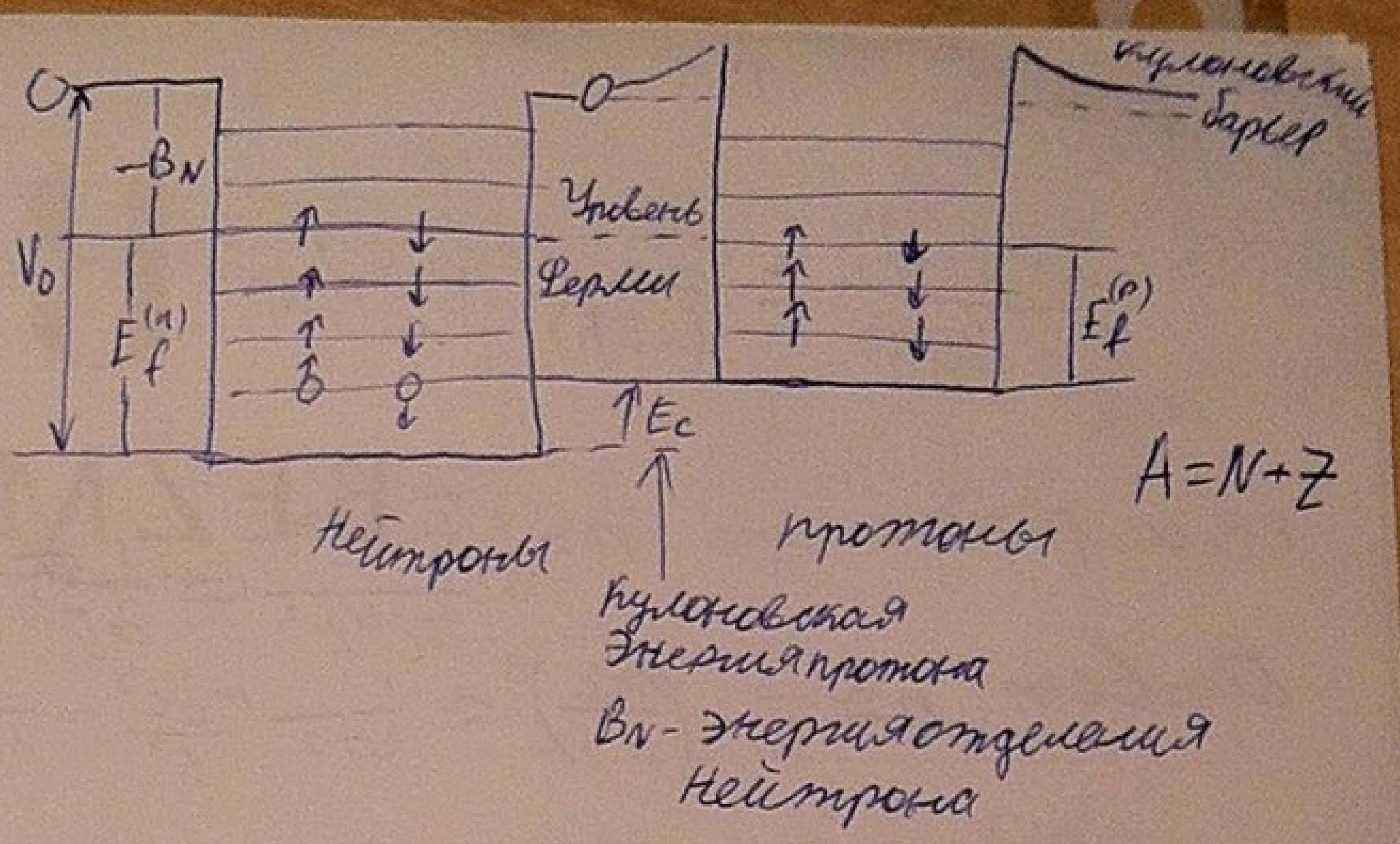
№4 $E_f^{(n)} = \frac{(p_f^{(n)})^2}{2m}$; $E_f^{(p)} = \frac{(p_f^{(p)})^2}{2m}$



$A = N + Z$

$$dN = \frac{2\cdot V}{(2\pi\hbar)^3}\cdot d^3p = \frac{2V\cdot 4\pi}{(2\pi\hbar)^3}p^2dp = \left(\frac{8\pi V p^2}{h^3}\right)dp$$

$$N = 2\left(\frac{4\pi V}{(2\pi\hbar)^3}\right)\int_0^{p_f^{(n)}} p^2 dp \qquad Z = \frac{2\cdot 4\pi V}{(2\pi\hbar)^3}\int_0^{p_f^{(p)}} p^2 dp$$

число нейтронов; число протонов

$$p_f^{(n)} = \left(3\pi^2\hbar^3\frac{N}{V}\right)^{1/3} \qquad p_f^{(p)} = \left(3\pi^2\hbar^3\frac{Z}{V}\right)^{1/3}$$

$$E_{пол} = \underbrace{\int_0^{p_f^{(p)}}\frac{p^2}{2m}dZ}_{протоны} + \underbrace{\int_0^{p_f^{(n)}}\frac{p^2}{2m}dN}_{нейтроны} = \int_0^{p_f^{(p)}}\frac{p^2}{2m}\frac{2V\cdot 4\pi}{(2\pi\hbar)^3}p^2dp + \int_0^{p_f^{(n)}}\frac{p^2}{2m}\frac{2V\cdot 4\pi}{(2\pi\hbar)^3}p^2dp =$$

полная кинетич. энергия ферми газа

$$= \frac{V\cdot(p_f^{(p)})^5}{5\cdot m\pi^2\hbar^3} + \frac{V(p_f^{(n)})^5}{5\cdot m\pi^2\hbar^3} = \frac{3}{5}\left(ZE_f^{(p)} + NE_f^{(n)}\right)$$

$$E_{пол} = \frac{3}{5}\left(ZE_f^{(p)} + (A-Z)E_f^{(n)}\right)$$

Макс $p = \frac{\hbar}{r_0}\left(\frac{9\pi}{8}\right)^{1/3}$ → не зависит от N, Z, A. $\Rightarrow E_{f\,max}^{(p)} = E_{f\,max}^{(n)}$

$E_{пол} = \frac{3}{5}E_{f\,max}(Z + A - Z) = \frac{3}{5}E_f\cdot A \Rightarrow$ средняя кин. энергия нуклонов в ядре $E_{ср} = \frac{E_{пол}}{A} \approx 20$ МэВ

$\Rightarrow N = A/2 = Z$ — мин. энергия

Если $\left.\begin{array}{l}\frac{Z}{A} = \eta \\ \frac{N}{A} = 1-\eta\end{array}\right\} \Rightarrow \begin{array}{l}Z = \eta A \\ N = (1-\eta)A\end{array}$, то $E_{пол} = \frac{3}{5}AE_f^0\left[\eta^{5/3} + (1-\eta)^{5/3}\right]$

, где $E_f^0 = \frac{\left(3\pi^2\hbar^3\frac{A}{V}\right)^{2/3}}{2m} = 49$ МэВ

т.к. $N \approx Z \approx \frac{A}{2}$, то $\eta = \frac{1}{2} - \Delta$; Δ мало

$$E_{пол} = \frac{3}{5}AE_f^0\left(\left(\tfrac{1}{2}-\Delta\right)^{5/3} + \left(\tfrac{1}{2}+\Delta\right)^{5/3}\right) \underset{\text{в ряд}}{=} \frac{3}{5}E_f^0A\Big\{\underbrace{\left(\tfrac{1}{2}\right)^{5/3}}_{f_1(x_0)} + \underbrace{\tfrac{5}{3}\cdot\left(\tfrac{1}{2}\right)^{2/3}\cdot\Delta}_{f_1'(x_0)(x-x_0)} + \underbrace{\tfrac{5}{3}\cdot\tfrac{2}{3}\cdot\left(\tfrac{1}{2}\right)^{-1/3}\cdot\tfrac{1}{2!}\Delta^2}_{\frac{f_1''(x_0)(x-x_0)^2}{2!}} +$$

$$+ \underbrace{\left(\tfrac{1}{2}\right)^{5/3}}_{f_2(x_0)} - \underbrace{\tfrac{5}{3}\cdot\left(\tfrac{1}{2}\right)^{2/3}\cdot\Delta}_{f'(x_0)(x-x_0)} + \underbrace{\tfrac{5}{3}\cdot\tfrac{2}{3}\cdot\left(\tfrac{1}{2}\right)^{-1/3}\cdot\tfrac{1}{2!}}_{\frac{f''(x_0)(x-x_0)^2}{2!}}\Big\} = \frac{3}{5}E_f^0A\left\{2\left(\tfrac{1}{2}\right)^{5/3} + 2\cdot\tfrac{10}{9}\cdot\left(\tfrac{1}{2}\right)^{2/3}\cdot\Delta^2\right\}$$

$$T = -\varepsilon\frac{(A-2Z)^2}{A} = -\varepsilon\frac{(A-2\eta A)^2}{A} = -\varepsilon A(1-2\eta)^2 = -\varepsilon A\left(1 - 2\left(\tfrac{1}{2}-\Delta\right)\right)^2 = -\varepsilon A(1-1+2\Delta)^2 = -4\varepsilon A\Delta^2$$
$$= \frac{3}{5}E_f^0A\left(2\left(\tfrac{1}{2}\right)^{5/3} + 2\cdot\tfrac{10}{9}\left(\tfrac{1}{2}\right)^{2/3}\Delta^2\right)$$

$$-4\varepsilon A\Delta^2 = \frac{3}{5}E_f^0A\cdot\frac{20}{9}\left(\tfrac{1}{2}\right)^{2/3}\Delta^2 \Rightarrow \varepsilon = -\frac{1}{6}\sqrt[3]{2}\,E_f^0 = 10{,}29\text{ МэВ} \approx 10{,}3\text{ МэВ}.$$

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Дз на 29.10.14

$a(z) = \frac{\hbar^2}{Ze^2 m}$

① $H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} - \frac{Ze^2}{r_1} - \frac{Ze^2}{r_2} + \frac{e^2}{|\vec r_1 - \vec r_2|}$

$\Psi = N e^{-r/a(z)}$

$\psi = N e^{-r_1/\alpha} e^{-r_2/\alpha}$ $\quad \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$

$Ry \cdot Z_{eff}^2 = \left(\frac{\partial^2}{\partial r_1^2} + \frac{2}{r_1}\frac{\partial}{\partial r_1}\right) e^{-r_1/\alpha}$

$$E(\alpha) = \frac{\int \sin\theta_1 d\theta_1 r_1^2 dr_1 \sin\theta_2 d\theta_2 r_2^2 dr_2 \psi(r_1,r_2)\Big(\underbrace{-\frac{\hbar^2}{2m}\Delta_1 \psi(r_1,r_2)}_{} \underbrace{- \frac{\hbar^2}{2m}\Delta_2 \psi(r_1,r_2)}_{Ry \cdot Z_{eff}^2} \underbrace{- \frac{Ze^2}{r_1}\psi(r_1,r_2)}_{-2Ry Z_{eff} \cdot \frac{Z}{Z_{eff}}} - \frac{Ze^2}{r_2}\psi(r_1,r_2)\Big)}{\int 4\pi r_1^2 dr_1 \, 4\pi r_2^2 dr_2 \, \psi^2(r_1,r_2)} +$$

$$+ \underbrace{\int \sin\theta_1 d\theta_1 r_1^2 dr_1 e^{-2r_1/\alpha} d\varphi_1 \int \sin\theta_2 d\theta_2 r_2^2 dr_2 e^{-2r_2/\alpha} \frac{q^2}{|r_1 - r_2|} d\varphi_2}_{= J}$$

$\frac{1}{\alpha} = \frac{Z_{eff}}{a_Б}$ $\quad \langle T \rangle = Ry \cdot Z_{eff}^2$

$\langle U \rangle = -2Ry \, Z_{eff}^2$

$\int d\varphi \sin\theta d\theta \, P_l(\cos\theta) P_{l'}(\cos\theta) = \delta_{ll'} \cdot 2\pi \cdot \frac{2}{2l+1}$

$$\frac{1}{|\vec r_1 - \vec r_2|} = \frac{1}{\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_{12}}} = \Sigma \begin{cases} r_1 > r_2 & \frac{1}{r_1}\sum_{l=0}^{\infty}\left(\frac{r_2}{r_1}\right)^l P_l(\cos\theta) \\ r_1 < r_2 & \frac{1}{r_2}\sum_{l=0}^{\infty}\left(\frac{r_1}{r_2}\right)^l P_l(\cos\theta) \end{cases}$$

$\uparrow^{r_1} \uparrow^{r}$

$$\underbrace{\int_0^\infty d\varphi_1 \sin\theta_1 d\theta_1}_{4\pi} r_1^2 dr_1 e^{-2r_1/\alpha} \int_0^{r_1} d\varphi_2 \sin\theta_2 d\theta_2 r_2^2 dr_2 \frac{1}{r_1}\sum_{l=0}^{\infty}\left(\frac{r_2}{r_1}\right)^l P_l(\cos\theta_{12}) e^{-2r_2/\alpha} q^2 =$$

$$= q^2 \int_0^\infty 4\pi r_1^2 dr_1 e^{-2r_1/\alpha} \int_0^{r_1} r_2^2 dr_2 \frac{1}{r_1} 4\pi e^{-2r_2/\alpha} = \quad \Big| \times 2 \text{ для случая } r_1 < r_2$$

$$= q^2 32\pi^2 \int_0^\infty r_1^2 dr_1 e^{-2r_1/\alpha} \cdot \frac{1}{r_1} \int_0^{r_1} r_2^2 dr_2 \cdot e^{-2r_2/\alpha} = q^2 32\pi^2 \int_0^\infty r_1 dr_1 e^{-2r_1/\alpha} \cdot \frac{1}{4}\alpha\left(\alpha^2 - e^{-2r_1/\alpha}(\alpha^2 + 2\alpha r_1 + 2r_1^2)\right) =$$

$$= q^2 8\pi^2 \alpha \left[\int_0^\infty r_1 dr_1 e^{-2r_1/\alpha} \cdot \alpha^2 - \int_0^\infty r_1 dr_1 e^{-4r_1/\alpha} \cdot \alpha^2 - \int_0^\infty r_1^2 dr_1 \cdot 2\alpha e^{-4r_1/\alpha} - \int_0^\infty r_1^3 dr_1 e^{-4r_1/\alpha} \cdot 2\right] =$$

$$= q^2 8\pi^2 \alpha \left[\alpha^2 \cdot \frac{\alpha^2}{4} - \alpha^2 \cdot \frac{\alpha^2}{16} - \frac{\alpha^4}{16} - \frac{3\alpha^4}{64}\right] = q^2 8\pi^2 \alpha^5 \cdot \frac{5}{64} = \frac{5}{8}\pi^2 \alpha^5 q^2 \quad \Big| \quad \frac{1}{\alpha} = \frac{Z_{eff}}{a_Б}$$

$$\int_0^\infty 4\pi r_1^2 dr_1 e^{-2r_1/\alpha} \int_0^\infty 4\pi r_2^2 dr_2 e^{-2r_2/\alpha} = 16\pi^2 \left(\frac{\alpha^3}{4} \cdot \frac{\alpha^3}{4}\right) = \alpha^6 \pi^2 \cdot N^2 \Rightarrow N = \frac{1}{\pi \alpha^3}$$

$$J = q^2 \frac{5}{8}\pi^2 \alpha^5 \cdot \frac{1}{\alpha^6 \pi^2} = \frac{1}{\alpha}\frac{5}{8} = \frac{5}{8}\frac{Z_{eff} q^2}{a_Б} = \frac{5}{4} Z_{eff} \underbrace{\frac{e^2}{2a_Б}}_{Ry} = \frac{5}{4} Z_{eff} \cdot Ry$$

$$E = -2Ry \, Z_{eff}^2 - 4Ry \, Z_{eff} \cdot Z + \frac{5}{4} Z_{eff} \cdot Ry$$

$$\frac{\partial E}{\partial Z_{eff}} = 0 = -4Z_{eff} - 4Z + \frac{5}{4} = 0 \Rightarrow \boxed{Z_{eff} = Z - \frac{5}{16}}$$

Если $Z_{eff} = Z - \sigma$, то $\sigma = \frac{5}{16}$

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$$\Psi(r_1,r_2)=\left(e^{-\alpha r_1-\beta r_2}+e^{-\alpha r_2-\beta r_1}\right)\frac{|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle}{\sqrt2}$$

$$\int_0^\infty 4\pi r_1^2dr_1\int_0^\infty 4\pi r_2^2dr_2\left(e^{-\alpha r_1-\beta r_2}+e^{-\alpha r_2-\beta r_1}\right)^2=\int_0^\infty 4\pi r_1^2dr_1\int_0^\infty 4\pi r_2^2dr_2\Big(e^{-2\alpha r_1-2\beta r_2}+$$

$$+2e^{-r_1(\alpha+\beta)-r_2(\alpha+\beta)}+e^{-2\alpha r_2-2\beta r_1}\Big)=16\pi^2\int_0^\infty r_1^2dr_1\left(\frac{e^{-2\alpha r_1}}{4\beta^3}+\frac{4e^{-r_1(\alpha+\beta)}}{(\alpha+\beta)^3}+\frac{e^{-2\beta r_1}}{4\alpha^3}\right)=$$

$$=16\pi^2\left(\frac{1}{16\beta^3\alpha^3}+\frac{8}{(\alpha+\beta)^6}+\frac{1}{16\beta^3\alpha^3}\right)=2\pi^2\left(\frac{1}{\beta^3\alpha^3}+\frac{64}{(\alpha+\beta)^6}\right)=\frac{2\pi^2}{\beta^3\alpha^3}\left(1+\frac{64\alpha^3\beta^3}{(\alpha+\beta)^6}\right)=\frac{1}{N^2}$$

$$\Rightarrow N^2=\frac{\beta^3\alpha^3}{2\pi^2}\left(1+\frac{64\alpha^3\beta^3}{(\alpha+\beta)^6}\right)^{-1}\qquad \text{если } N^2=1\quad \frac{2\pi^2}{\beta^3\alpha^3}=\left(1+\frac{64\alpha^3\beta^3}{(\alpha+\beta)^6}\right)^{-1}=2C^2$$

$$E(\alpha,\beta)=\frac{1}{N^2}\Big[\int 4\pi r_1^2dr_1\int 4\pi r_2^2dr_2\,\Psi(r_1,r_2)\Big(-\frac{\hbar^2}{2m}\Delta_1\Psi(r_1,r_2)-\frac{\hbar^2}{2m}\Delta_2\Psi(r_1,r_2)-\frac{Ze^2}{r_1}\Psi(r_1,r_2)-\frac{Ze^2}{r_2}\Psi(r_1,r_2)\Big)\Big]+$$

$$+\frac{1}{N^2}\Big[\int\sin\theta_1d\theta_1\,r_1^2dr_1d\varphi_1\int\sin\theta_2d\theta_2d\varphi_2\,r_2^2dr_2\,|\Psi(r_1,r_2)|^2\frac{e^2}{|r_1-r_2|}\Big]$$

1) $$\int 4\pi r_1^2dr_1\int 4\pi r_2^2dr_2\cdot\frac{Ze^2}{r_1}\cdot\left(e^{-2\alpha r_1-2\beta r_2}+2e^{-r_1(\alpha+\beta)-r_2(\alpha+\beta)}+e^{-2\alpha r_2-2\beta r_1}\right)=$$

$$=\Big[\int 4\pi r_1dr_1e^{-2\alpha r_1}\int 4\pi r_2^2dr_2e^{-2\beta r_2}+2\int 4\pi r_1dr_1e^{-r_1(\alpha+\beta)}\cdot\int 4\pi r_2^2dr_2e^{-r_2(\alpha+\beta)}+\int 4\pi r_1dr_1e^{-2\beta r_1}\cdot\int 4\pi r_2^2dr_2e^{-2\alpha r_2}\Big](-Ze^2)$$

$$=\Big[16\pi^2\frac{1}{4\alpha^2}\frac{1}{4\beta^3}+16\pi^2\cdot\frac{1}{(\alpha+\beta)^2}\frac{2}{(\alpha+\beta)^3}+16\pi^2\frac{1}{4\beta^2}\frac{1}{4\alpha^3}\Big](-Ze^2)=$$

$$=\Big[\frac{1}{\alpha^2\beta^3}+\frac{1}{\alpha^3\beta^2}+\frac{64}{(\alpha+\beta)^5}\Big](-Ze^2)\pi^2$$

2) $$\int 4\pi r_1^2dr_1\int 4\pi r_2^2dr_2\cdot\frac{-Ze^2}{r_2}|\Psi(r_1,r_2)|^2=(-Ze^2)\Big[\frac{1}{\beta^2\alpha^3}+\frac{1}{\alpha^2\beta^3}+\frac{64}{(\alpha+\beta)^5}\Big]\cdot\pi^2$$

1)+2) $$=2(-Ze^2)\Big[\frac{1}{\beta^2\alpha^3}+\frac{1}{\alpha^2\beta^3}+\frac{64}{(\alpha+\beta)^5}\Big]\pi^2=2(-Ze^2\pi^2)\cdot\frac{1}{\beta^3\alpha^3}\left(\beta+\alpha+\frac{64\alpha^3\beta^3}{(\alpha+\beta)^5}\right)=$$

$$=(-Ze^2\pi^2 2)\cdot\frac{(\alpha+\beta)}{\beta^3\alpha^3}\left(1+\frac{64\alpha^3\beta^3}{(\alpha+\beta)^6}\right)=2C^2\Big[-Ze^2(\alpha+\beta)-Ze^2\cdot\frac{64\alpha^3\beta^3}{(\alpha+\beta)^6}(\alpha+\beta)\Big]$$

3) $$\int dv_1\int dv_2\,\Psi(r_1,r_2)\Delta_1\Psi(r_1,r_2)=\int dv_1\int dv_2\left(e^{-\alpha r_1-\beta r_2}+e^{-\alpha r_2-\beta r_1}\right)\cdot\Delta_1\left(e^{-\alpha r_1-\beta r_2}+e^{-\alpha r_2-\beta r_1}\right)=$$

$$=\int dv_1\int dv_2\Big(e^{-2\beta r_2}\big(e^{-\alpha r_1}\Delta_1e^{-\alpha r_1}\big)+e^{-\beta r_2-\alpha r_2}\cdot\big(e^{-\alpha r_1}\Delta_1e^{-\beta r_1}\big)+$$

$$+e^{-r_2(\alpha+\beta)}\cdot\big(e^{-\beta r_1}\Delta_1e^{-\alpha r_1}\big)+e^{-2\alpha r_2}\big(e^{-\beta r_1}\Delta_1e^{-\beta r_1}\big)\Big)=-\int dv_1\int dv_2\Big(e^{-2\beta r_2}(\nabla e^{-\alpha r_1})(\nabla e^{-\alpha r_1})+$$

$$+e^{-r_2(\alpha+\beta)}(\nabla e^{-\alpha r_1})(\nabla e^{-\beta r_1})+e^{-r_2(\alpha+\beta)}(\nabla e^{-\beta r_1})(\nabla e^{-\alpha r_1})+e^{-2\alpha r_2}(\nabla e^{-\beta r_1})(\nabla e^{-\beta r_1})\Big)=$$

$$=-\int dv_1\int dv_2\left(e^{-2\beta r_2}\cdot\alpha^2e^{-2\alpha r_1}+2e^{-r_2(\alpha+\beta)}\cdot\alpha\beta e^{-(\alpha+\beta)r_1}+e^{-2\alpha r_2}\cdot\beta^2e^{-2\beta r_1}\right)=$$

$$=-16\pi^2\left(\alpha^2\cdot\frac{1}{4\beta^3}\frac{1}{4\alpha^3}+2\frac{2}{(\alpha+\beta)^3}\cdot\frac{2}{(\alpha+\beta)^3}\cdot\alpha\beta+\beta^2\frac{1}{4\alpha^3}\frac{1}{4\beta^3}\right)=-\pi^2\left(\frac{1}{\beta^3\alpha}+\frac{16\cdot 8\alpha\beta}{(\alpha+\beta)^6}+\frac{1}{\alpha^3\beta}\right)$$

4) $$\int dv_1\int dv_2\,\Psi(r_1,r_2)\Delta_2\Psi(r_1,r_2)=-\pi^2\left(\frac{1}{\alpha^3\beta}+\frac{1}{\beta^3\alpha}+\frac{16\cdot 8\alpha\beta}{(\alpha+\beta)^6}\right)$$

3)+4) $= \frac{-2\pi^2}{\alpha^3\beta^3}\left(\beta^2+\alpha^2+\frac{16\cdot 8\cdot\alpha^4\beta^4}{(\alpha+\beta)^6}\right) =$ $\left|x-\frac{h^2}{2m}\right| = 2C^2\left(\frac{\alpha^2+\beta^2}{2}+\frac{64\alpha\beta\cdot\alpha^3\beta^3}{(\alpha+\beta)^6}\right)\frac{h^2}{m}$

5) $\int dv_1\int dv_2\frac{e^2}{|r_1-r_2|}\cdot\left(e^{-2\alpha r_1-2\beta r_2}+2e^{-r_1(\alpha+\beta)}e^{-r_2(\alpha+\beta)}+e^{-2\alpha r_2-2\beta r_1}\right)=?$

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$\int dv_1\int dv_2\frac{e^2}{|r_1-r_2|}e^{-\alpha r_1-\beta r_2}$ = рассуждения из пробника номера $= e^2\int_0^\infty 4\pi r_1^2dr_1 e^{-\alpha r_1}\int_0^{r_1}4\pi\frac{1}{r_1}r_2^2dr_2e^{-\beta r_2} =$

$= e^2 16\pi^2\int_0^\infty r_1dr_1e^{-\alpha r_1}\int_0^{r_1}r_2^2dr_2e^{-\beta r_2} = e^2 16\pi^2\int_0^\infty\frac{2+e^{-\beta r_1}(-2-2\beta r_1-\beta^2r_1^2)}{\beta^3}\cdot r_1e^{-\alpha r_1}dr_1 =$

$= e^2\frac{16\pi^2}{\beta^3}\left(\int_0^\infty\left(2r_1e^{-\alpha r_1}-2r_1e^{-r_1(\alpha+\beta)}-2\beta r_1^2e^{-r_1(\alpha+\beta)}-\beta^2r_1^3e^{-r_1(\alpha+\beta)}\right)dr_1\right) =$

$= e^2\frac{16\pi^2}{\beta^3}\left(\frac{2}{\alpha^2}-\frac{2}{(\alpha+\beta)^2}-\frac{4\beta}{(\alpha+\beta)^3}-\frac{6\beta^2}{(\alpha+\beta)^4}\right) = e^2 16\pi^2\left(\frac{2}{\alpha^2\beta^3}-\frac{2}{\beta^3(\alpha+\beta)^2}-\frac{4}{(\alpha+\beta)^3\beta^2}-\frac{6}{\beta(\alpha+\beta)^4}\right) =$

$= e^2 16\pi^2\left(\frac{1}{\beta}\left(\frac{2(\alpha+\beta)^3}{\alpha^2\beta^2(\alpha+\beta)^3}-\frac{2(\alpha+\beta)\cdot\alpha^2}{\alpha^2\beta^2(\alpha+\beta)^3}\right)-\frac{4\alpha^2}{(\alpha+\beta)^3\beta^2\alpha^2}-\frac{6}{\beta(\alpha+\beta)^4}\right) =$

$= e^2\cdot 16\pi^2\left(\frac{1}{\beta}\frac{2(\alpha+\beta)}{\alpha^2\beta^2(\alpha+\beta)^3}\left((\alpha+\beta)^2-\alpha^2\right)-\frac{4\alpha^2}{(\alpha+\beta)^3\beta^2\alpha^2}-\frac{6}{\beta(\alpha+\beta)^4}\right) = e^2\cdot 16\pi^2\left(\frac{2\alpha\beta+\beta^2}{\beta^3}\cdot\frac{2(\alpha+\beta)}{\alpha^2\beta^2(\alpha+\beta)^3}-\frac{4\alpha^2}{(\alpha+\beta)^3\beta^2\alpha^2}-\frac{6}{\beta(\alpha+\beta)^4}\right)$

$= e^2\cdot 16\pi^2\left(\frac{(2\alpha+\beta)\cdot 2\cdot(\alpha+\beta)}{\alpha^2\beta^2(\alpha+\beta)^3}-\frac{4\alpha^2}{\alpha^2\beta^2(\alpha+\beta)^3}-\frac{6}{\beta(\alpha+\beta)^4}\right) = e^2\cdot 16\pi^2\left(\frac{2}{\alpha^2\beta^2(\alpha+\beta)^3}\left(\underbrace{(2\alpha+\beta)(\alpha+\beta)-\alpha^2}_{2\alpha^2+3\alpha\beta+\beta^2-\alpha^2}\right)-\frac{6}{\beta(\alpha+\beta)^4}\right) =$

$= \frac{2(\alpha^2+\beta^2)}{\alpha^2\beta^2(\alpha+\beta)^3}+\frac{6\alpha\beta}{\alpha^2\beta^2(\alpha+\beta)^3}-\frac{6}{\beta(\alpha+\beta)^4} = \frac{2(\alpha^2+\beta^2)}{\alpha^2\beta^2(\alpha+\beta)^3}+\frac{6}{\beta(\alpha+\beta)^3}\left(\frac{1}{\alpha}-\frac{1}{\alpha+\beta}\right) = \frac{2(\alpha^2+\beta^2)}{\alpha^2\beta^2(\alpha+\beta)^3}+\frac{6}{\beta(\alpha+\beta)^3}\cdot\frac{\alpha+\beta-\alpha}{\alpha(\alpha+\beta)} =$

$= \frac{2(\alpha^2+\beta^2)}{\alpha^2\beta^2(\alpha+\beta)^3}+\frac{6}{\alpha(\alpha+\beta)^4}$ — это для $r_1>r_2$, для $r_1<r_2$ симметрично относительно α и β

$(r_1>r_2)+(r_1<r_2) = \frac{4(\alpha^2+\beta^2)}{\alpha^2\beta^2(\alpha+\beta)^3}+\frac{6}{\alpha(\alpha+\beta)^4}+\frac{6}{\beta(\alpha+\beta)^4} = \left[\frac{4(\alpha^2+\beta^2)}{\alpha^2\beta^2(\alpha+\beta)^3}+\frac{6}{\alpha\beta(\alpha+\beta)^3}\right]e^2 16\pi^2$

должно быть 2.

$\int dv_1dv_2\frac{e^2}{|r_1-r_2|}e^{-\alpha r_1-\beta r_2} = \left[\frac{2(\alpha^2+\beta^2)}{\alpha^2\beta^2(\alpha+\beta)^3}+\frac{6}{\alpha\beta(\alpha+\beta)^3}\right]e^2 16\pi^2$

поэтому

I $\int dv_1dv_2\frac{e^2}{|r_1-r_2|}e^{-2\alpha r_1-2\beta r_2} = \left[\frac{2(4\alpha^2+4\beta^2)}{4\alpha^2\cdot 4\beta^2(\alpha+\beta)^3\cdot 8}+\frac{6}{2\alpha\cdot 2\beta(\alpha+\beta)^3\cdot 8}\right]e^2\cdot 16\pi^2 = e^2\cdot\pi^2\left[\frac{\alpha^2+\beta^2}{\alpha^2\beta^2(\alpha+\beta)^3}+\frac{3}{\alpha\beta(\alpha+\beta)^3}\right]$

II $\int dv_1dv_2\frac{e^2}{|r_1-r_2|}e^{-2\alpha r_2-2\beta r_1} = e^2\pi^2\left[\frac{\alpha^2+\beta^2}{\alpha^2\beta^2(\alpha+\beta)^3}+\frac{3}{\alpha\beta(\alpha+\beta)^3}\right]$

III $\int dv_1dv_2\frac{e^2}{|r_1-r_2|}\cdot 2e^{-r_1(\alpha+\beta)-r_2(\alpha+\beta)} = e^2\cdot 16\pi^2\cdot 2\left[\frac{2\cdot 2C^2}{C^4\cdot 8C^3}+\frac{6}{C^2\cdot 8C^3}\right] = \frac{e^2\cdot 16\pi^2\cdot 2}{C^5}\left[\frac{4+6}{8}\right] = \frac{e^2\cdot 4\pi^2\cdot 10}{(\alpha+\beta)^5}$

$= e^2\cdot\frac{2\pi^2}{\alpha^3\beta^3}\cdot\frac{20\alpha^3\beta^3}{(\alpha+\beta)^5} = e^2\cdot 2C\cdot\frac{20\alpha^3\beta^2}{(\alpha+\beta)^5}$

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T: +90 224 411 08 00 F: +90 224 411 08 09 E: bezmez@bezmez.net
www.bezmez.net

$$I + II = e^2 2\pi^2\left(\frac{\alpha^2+\beta^2}{\alpha^2\beta^2(\alpha+\beta)^3} + \frac{3}{\alpha\beta(\alpha+\beta)^3}\right) = e^2\cdot\frac{2\pi^2}{\alpha^3\beta^3}\left(\frac{(\alpha^2+\beta^2)\alpha\beta}{(\alpha+\beta)^3} + \frac{3\cdot\alpha^2\beta^2}{(\alpha+\beta)^3}\right)$$

$$E(\alpha,\beta) = 2C^2\left\{-Z(\alpha+\beta) + \frac{\alpha^2+\beta^2}{2} + \frac{(\alpha^2+\beta^2)\alpha\beta}{(\alpha+\beta)^3} + \frac{3\alpha^3\beta^2}{(\alpha+\beta)^3} + \frac{20\alpha^3\beta^3}{(\alpha+\beta)^5} + 64\alpha^3\beta^3\frac{\alpha\beta - Z(\alpha+\beta)}{(\alpha+\beta)^6}\right\}$$

Ответ в учебнике

$$E(\alpha,\beta) = 2C^2\left\{-Z(\alpha+\beta) + \frac{\alpha^2+\beta^2}{2} + \frac{\alpha\beta}{(\alpha+\beta)} + \frac{\alpha^2\beta^2}{(\alpha+\beta)^3} + \frac{20\alpha^3\beta^3}{(\alpha+\beta)^5} + 64\alpha^3\beta^3\frac{\alpha\beta - Z(\alpha+\beta)}{(\alpha+\beta)^6}\right\}$$

если $Z=1$, $\alpha=1$, $\beta=1/4$, $E(\alpha\beta) = -0{,}512$ Ry $\Rightarrow$ H^- устойчив

$E_{осн} = 0{,}5$ Ry

③ ^4_2He осн. сост. $l=0$, $s=0$ $\quad \psi(r_1,r_2) = \frac{(Z')^3}{\pi a^3} e^{-\frac{Z'(r_1+r_2)}{a}}$, $Z' = \frac{27}{16}$

$H = \frac{1}{2m}\left(\bar{p} - \frac{e}{c}\bar{A}\right)^2 + e\varphi(r) + V(r)$ — в общем случае

$$H = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{e}{c}\bar{A}\right)^2 + e\varphi + V(r) = \frac{(-1)}{2m}\hbar^2\nabla^2 - \frac{1}{2m}\frac{e}{c}\frac{\hbar}{i}(\nabla\bar{A} + \bar{A}\nabla) + \frac{e^2}{2mc^2}A^2 + e\varphi + V$$

т.к. $\nabla(\bar{A}\psi) = \psi\,\mathrm{div}\bar{A} + (\bar{A}\nabla)\psi$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{e\hbar}{mc}i\left(\bar{A}\nabla + \frac{1}{2}\mathrm{div}\bar{A}\right) + \frac{e^2}{2mc^2}A^2 + e\varphi + V = H_0 + \frac{e\hbar i}{mc}\left(\bar{A}\nabla + \frac{1}{2}\mathrm{div}\bar{A}\right) + \frac{e^2}{2mc^2}A^2 + \varphi\cdot e$$

для $\mathcal{H} = const$; $\mathcal{H}\parallel z$

$H = H_0 - \frac{e\hbar}{mc}i\bar{A}\nabla + \frac{e^2}{2mc^2}A^2$ $\quad |\ \mathrm{div}A = 0$; $A_x = -\frac{1}{2}\mathcal{H}y$, $A_y = \frac{1}{2}\mathcal{H}x$, $A_z = 0$ $\quad |\ H = H_0 + \frac{e\hbar}{mc}i\frac{\mathcal{H}}{2}\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right) + \frac{e^2}{2mc^2}\frac{\mathcal{H}^2}{4}(x^2+y^2)$

$$H = H_0 + \frac{e\mathcal{H}}{2mc}L_z + \frac{e^2\mathcal{H}^2}{8mc^2}r^2\sin^2\theta$$

$$\Delta E_n = \langle n|\overbrace{\frac{e\mathcal{H}}{2mc}L_z}^{E^{(1)}} + \overbrace{\frac{e^2\mathcal{H}^2}{8mc^2}r^2\sin^2\theta}^{E^{(2)}}|n\rangle + \sum_{n'}\frac{|\langle n'|\frac{e\mathcal{H}}{2mc}L_z|n\rangle|^2}{E_n - E_{n'}}$$

Осн. сост; $\hbar l$, $\hbar m_l$

$$E^{(1)} = \frac{e\mathcal{H}}{2mc}\hbar m_l\ ;\ \overline{E^{(1)}} = \frac{e\hbar}{2mc}\mathcal{H}\frac{\sum_{m_l=-l}^{m_l=l} m_l e^{-\alpha m_l}}{\sum_{m_l=-l}^{m_l=l} e^{-\alpha m_l}} = \frac{e\hbar}{2mc}\mathcal{H}\frac{(m_l - \alpha m_l^2 + \frac{1}{2}\alpha^2 m_l^3 + \ldots)}{\sum(1 - \alpha m_l + \frac{1}{2}\alpha^2 m_l^2 + \ldots)}$$

$$\alpha = \frac{e\hbar}{2mc}\frac{\mathcal{H}}{kT} \ll 1 \qquad \frac{-\alpha\sum m_l^2 + O(\alpha^3)}{\sum 1 + O(\alpha^2)} = -\alpha\frac{\frac{1}{6}l(l+1)(2l+1) + \ldots}{(2l+1) + \ldots}$$

$$\overline{E^{(1)}}N = -\frac{Ne\hbar}{2mc}\mathcal{H}\left(\frac{e\hbar}{2mc}\frac{\mathcal{H}}{kT}\right)\frac{1}{6}l(l+1)$$

$E^{(2)} = \frac{e^2\mathcal{H}^2}{8mc^2}\langle n|r^2\sin^2\theta|n\rangle$ $\quad$ для ^4_2He $l=0 \Rightarrow E^{(1)} = 0$

$$\frac{1}{4\pi}\oint\sin^2\theta\, d\Omega = \frac{1}{4\pi}\oint\sin^2\theta\cdot\sin\theta\, d\theta\, d\varphi = \frac{1}{2}\oint\sin^3\theta\, d\theta = \frac{2}{3}$$

$$E^{(2)} = \frac{e^2\mathcal{H}^2}{12mc^2}\underbrace{\langle 0|r^2|0\rangle}_{\text{средн. знач. } r^2}^{\text{осн. сост.}}$$

$$\left.\begin{array}{l}\delta E = -M\delta\mathcal{H}\\ M = \chi\mathcal{H}\end{array}\right\} \Rightarrow \delta E - \chi\mathcal{H}\delta\mathcal{H} \Rightarrow \left.\begin{array}{l}E = -\frac{1}{2}\chi\mathcal{H}^2\\ E = \frac{e^2\mathcal{H}^2}{12mc^2}\cdot N\langle 0|r^2|0\rangle\end{array}\right\} \Rightarrow \chi_{диа} = -\frac{Ne^2}{6mc^2}\langle 0|r^2|0\rangle$$

для $\psi(r_1 r_2) = \frac{(z')^3}{\pi a^3} e^{-\frac{z'(r_1+r_2)}{a}}$: $\chi = -\frac{e^3 N_A}{6mc^2}(\overline{r_1^2} + \overline{r_2^2})$

$\overline{r_1^2} + \overline{r_2^2} = \int (r_1^2 + r_2^2)\psi^2 dV_1 dV_2$

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$\int (r_1^2 + r_2^2) \cdot \frac{(z')^6}{\pi^2 a^6} \exp\left(\frac{z' r_1}{a}\right) \cdot \exp\left(\frac{-2z' r_2}{a}\right) \cdot 4\pi r_1^2 dr_1 \cdot 4\pi r_2^2 dr_2 = \frac{(z'^6)}{\pi^2 a^6} \cdot 16\pi^2 \times$

одинаковы $\Rightarrow \times 2$

$\times \left\{ \int r_1^4 \exp\left(\frac{-2z' r_1}{a}\right) \cdot r_2^2 \exp\left(\frac{-2z' r_2}{a}\right) dV_1 dV_2 + r_2^4 \exp\left(\frac{-2z' r_2}{a}\right) r_1^2 \exp\left(\frac{-2z' r_1}{a}\right) \right\} =$

$= \frac{(z')^6}{\pi^2 a^6} \cdot 16\pi^2 \cdot 2 \cdot \frac{3a^5}{4(z')^5} \cdot \frac{a^3}{4(z')^3} = \frac{a^2 \cdot 6}{(z')^2}$

$\chi = -\frac{e^2}{mc^2} \cdot \frac{N_A}{6} \cdot \frac{a^2}{(z')^2} = -\frac{2{,}82 \cdot 10^{-13} \text{см} \cdot 6{,}02 \cdot 10^{23} \cdot (0{,}529 \cdot 10^{-8})^2 \text{см}^2}{\left(\frac{27}{16}\right)^2} = -1{,}67 \cdot 10^{-6}$

(4) ^2_4He $\quad l=0, \; S=0$ $\quad \psi(r_1 r_2) = \frac{(z')^3}{\pi a^3} e^{-\frac{z'(r_1+r_2)}{a}}$, $z' = 27/16$

$H = \mathrm{rot} A = 0$

$E = -\mathrm{grad}\, \varphi - \frac{1}{c}\frac{\partial A}{\partial t}$

---

(5) Орто и парагелий $\quad 1s \quad nl$

$[\psi_{100}(\bar{x}_1)\psi_{nlm}(\bar{x}_2) \pm \psi_{100}(\bar{x}_2)\psi_{nlm}(\bar{x}_1)]$

$\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$ парагелий

$|\uparrow\uparrow\rangle; |\downarrow\downarrow\rangle; \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$

$z=1$

$E = \left[-\frac{Z^2}{2} - \frac{(Z-1)^2}{2n^2}\right]\frac{e^2}{a_0} = [-Z^2 - (Z-1)^2]\,Ry$

---

(6) $-Ry = -\frac{me^4 z^2}{2\hbar^2 n^2}$ для 1s: $1^{ый}\, e \to -4Ry$, $2^{ой}\, e \to -Ry$; для 2s: $1^{ый}\, e : -Ry$, $2^{ой}\, e : -Ry/4$

U+T без учёта кул. отталк.

кул. отталк (из $1^{ого}$ номера) $\sim \frac{5}{4} Z_{eff} \cdot Ry = \frac{5}{4} Ry\left(Z - \frac{5}{16}\right)$ для $2^{ого}\, e$ $Z=1$ $\Rightarrow Z_{eff} = \frac{11}{16}$

$\Rightarrow E^{2ого\, e} = -Ry/4 + \frac{5}{4} \cdot \frac{11}{16} Ry = \frac{Ry}{4}\left(-1 + \frac{55}{16}\right) > 0 \Rightarrow$ $2^{ой}\, e$ свободен, связанного состояния нет.

---

(7) $J \sim 1$ эВ $\quad k_B T_k \sim J \Rightarrow T_k \sim J/k_B = \frac{1\text{эВ}}{10^{-5} \cdot 8{,}6173324\ \text{эВ/К}} = 0{,}116 \cdot 10^5 =$

$T_k = ?$

$= 11604$ К

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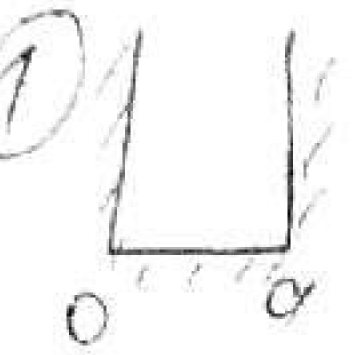
T : +90 224 411 08 00 F : +90 224 411 08 09 E : bezmez@bezmez.net
www.bezmez.net

Д/з на 12.11.14. 11.13 → 8.23

$\Psi(t=-\infty)=\Psi_n=\sin\frac{\pi n x}{a}=\sin\frac{\pi(n+1)x}{a}$

$P_m(t=+\infty)-?$

физически $n=0$.

① 

$H_I(t)=-xF_0\begin{cases}e^{-|t|/\tau}\\ e^{-t^2/\tau^2}\\ 1/1+t^2/\tau^2\end{cases}$

$\omega_{mn}=\frac{E_m-E_n}{\hbar}=\frac{\hbar\pi^2 n(n+2)}{2ma^2}$

$$V_{n,0}(t)=-\frac{2}{a}F_0 f(t)\int_0^a x\underbrace{\sin\frac{(n+1)\pi x}{a}}_{n\neq0}\underbrace{\sin\frac{\pi x}{a}}_{n=0}dx=-\frac{2}{a}F_0 f(t)\int_0^a\frac{x}{2}\left(\cos\left(\frac{\pi n x}{a}\right)-\cos\left(\frac{\pi x(n+2)}{a}\right)\right)dx=$$

$$=-\frac{1}{a}F_0 f(t)\left[\frac{x\sin\left(\frac{\pi n x}{a}\right)}{(\pi n/a)}\Big|_0^a+\frac{\cos\left(\frac{\pi n x}{a}\right)}{(\pi n/a)^2}\Big|_0^a-\frac{x\cdot\sin\frac{\pi(n+2)x}{a}}{\pi(n+2)/a}\Big|_0^a-\frac{\cos\frac{\pi(n+2)x}{a}}{\pi^2(n+2)^2/a^2}\right]=$$

$$\cancel{=-\frac{1}{a}F_0 f(t)\left[\frac{a\sin\pi n}{\pi n/a}-\frac{a\sin\pi(n+2)}{\pi(n+2)/a}+\frac{\cos\pi n}{(\pi n/a)^2}-\frac{\cos\pi(n+2)}{(\pi(n+2)/a)^2}\right]}\ \begin{cases}n\text{-чётн},\ 0\\ n\text{-нечётн},\end{cases}$$

$\cos\pi n\cos 2\pi-\sin\pi n\sin 2\pi$

$$=-\frac{1}{a}F_0 f(t)\left[\frac{a\sin\pi n}{\pi n/a}-\frac{a\sin\pi(n+2)}{\pi(n+2)/a}+\frac{\cos\pi n}{(\pi n/a)^2}-\frac{\cos\pi(n+2)}{(\pi(n+2)/a)^2}-0+0-\frac{1}{(\pi n/a)^2}+\frac{1}{(\pi(n+2)/a)^2}\right]=$$

$$=-\frac{1}{a}F_0 f(t)\left[\frac{\cos\pi n}{(\pi n/a)^2}-\frac{\cos\pi n}{(\pi(n+2)/a)^2}-\left(\frac{1}{(\pi n/a)^2}-\frac{1}{(\pi(n+2)/a)^2}\right)\right]=-\frac{aF_0 f(t)(\cos\pi n-1)((n+2)^2-n^2)}{\pi^2 n^2(n+2)^2}=$$

$$=-\frac{aF_0 f(t)\cdot 4(\cos\pi n-1)(n+1)}{\pi^2 n^2(n+2)^2}\Rightarrow\begin{cases}0,\ \text{при } n=\text{чёт}\\ +\frac{8aF_0 f(t)(n+1)}{\pi^2 n^2(n+2)^2},\ \text{при } n=\text{нечётн.}\end{cases}$$

а) $f(t)=e^{-t^2/\tau^2}$

$$I_a=\int_{-\infty}^{+\infty}e^{-t^2/\tau^2}e^{i\omega_{mn}t}dt=\int_{-\infty}^{+\infty}\exp(-t^2/\tau^2+i\omega t)dt=\left[-t^2/\tau^2+i\omega t=\underbrace{-t^2/\tau^2+2\frac{t}{\tau}\cdot\frac{\tau}{2}\cdot i\omega-\frac{i^2\omega^2\tau^2}{4}}+\frac{i^2\omega^2\tau^2}{4}=-\left(t/\tau-\frac{i\omega\tau}{2}\right)^2-\frac{\omega^2\tau^2}{4}\right]=$$

$$=\tau\int_{-\infty}^{+\infty}\exp\left[-\left(t/\tau-\frac{i\omega\tau}{2}\right)^2-\frac{\omega^2\tau^2}{4}\right]d\left(t/\tau-\frac{i\omega\tau}{2}\right)=\tau\sqrt{\pi}\cdot\exp\left(-\frac{\omega^2\tau^2}{4}\right)$$

б) $f(t)=e^{-|t|/\tau}$

$$I_б=\int_{-\infty}^{0}e^{+t/\tau+i\omega t}dt+\int_0^{\infty}e^{-t/\tau+i\omega t}dt=\int_{-\infty}^{0}e^{t(1/\tau+i\omega)}dt+\int_0^{+\infty}e^{-t(1/\tau-i\omega)}dt=\frac{1}{1/\tau+i\omega}+\frac{1}{1/\tau-i\omega}=$$

$$=\frac{1/\tau-i\omega+1/\tau+i\omega}{1/\tau^2+\omega^2}=\frac{2}{\tau}\frac{1}{1/\tau^2+\omega^2}=\frac{2}{\tau}\frac{\tau^2}{1+\omega^2\tau^2}=\frac{2\tau}{1+\omega^2\tau^2}$$

в) $f(t)=\frac{1}{1+t^2/\tau^2}$

$$I_в=\int_{-\infty}^{+\infty}\frac{1}{1+t^2/\tau^2}\exp^{i\omega t}dt=\pi\tau\exp(-\omega\tau)$$

Усл. применимости $ma^3F_0\ll\hbar^2\pi^2$

при $t\to\infty$ $\quad W'(0\to n)=\begin{cases}0,\ n=\text{чёт}\\ \frac{64a^2F_0^2(n+1)^2}{\pi^4 n^4(n+2)^4}\bar{I}_i^2,\ n=\text{нечёт}\end{cases}$

② 11.1.4 → 8.24

Г.О. $E(t) = \begin{cases} E_0 \exp(-t^2/\tau^2) \\ E_0 \exp(-|t|/\tau) \end{cases} \Rightarrow \hat{V} = \begin{cases} -x E_0 \exp(-t^2/\tau^2) \\ -x E_0 \exp(-|t|/\tau) \end{cases}$

$\hat{V}_{kn} = -\frac{eaE(t)}{\sqrt{2}} \begin{cases} \sqrt{n+1} & k=n+1 \\ \sqrt{n} & k=n-1 \\ 0 & \text{ост.} \end{cases} \Rightarrow W^{(1)}_{n\to k} = \frac{e^2a^2|I(t)|^2}{2\hbar^2} \begin{cases} n+1 & k=n+1 \\ n & k=n \\ 0 & \text{else} \end{cases}$

$I = \int\limits_{-\infty}^{+\infty} E(t) \exp(\pm i\omega t)\,dt$ $\quad {}^{k=n+1}_{k=n-1}$

$eaE_0\sqrt{n+1} \ll \hbar\omega$

Из прошлой задачи $I_1 = E_0 \sqrt{\pi}\tau \exp\left(-\frac{\omega^2\tau^2}{4}\right)$.

$I_2 = \frac{E_0 \cdot 2\tau}{1+\omega^2\tau^2}$

---

④ 11.1.6 → 8.26

$\hat{V} = -d\mathcal{E}(t)\cos\varphi$; $\cos\varphi = (e^{i\varphi} + e^{-i\varphi})/2 \Rightarrow V_{nk} = \begin{cases} -d\mathcal{E}_0/2, & k=n\pm1 \\ 0, & \text{else} \end{cases}$

$V_{mm'}(t) = \begin{cases} -d\mathcal{E}(t)/2, & m'=m\pm1 \\ 0, & \text{else} \end{cases}$

$W^{(1)}(m\to m') = \frac{d^2}{4\hbar^2} \left| \int\limits_{-\infty}^{+\infty} \mathcal{E}(t) \exp(i\omega_{mm'}t)\,dt \right|^2$

$\mathcal{E}$ те же: $I_1 = E_0\sqrt{\pi}\tau \exp\left(-\frac{\omega^2\tau^2}{4}\right)$, $\quad \omega_{mm'} = \frac{E_{m'}-E_m}{\hbar} = \frac{\frac{\hbar^2}{2I}(m\pm1)^2 - \frac{\hbar^2}{2I}m^2}{\hbar} = \frac{\hbar(1\pm2m)}{2I}$

$I_2 = \frac{E_0 \cdot 2\tau}{1+\omega^2\tau^2}$

$d\mathcal{E}_0 \ll \hbar\omega = \frac{\hbar^2(1\pm2m)}{I}$

---

③ 11.1.7 → 8.30 $\quad P_m - ?$ $\quad n \to n\pm1 \to n\pm2$

Подсказка $\int\limits_{-\infty}^{+\infty} dt\, f(t) \int\limits_{-\infty}^{t} dt' f(t') = \int\limits_{-\infty}^{+\infty} dt' f(t') \int\limits_{0}^{t'} dt\, f(t) \Rightarrow \frac{1}{2}\int\limits_{-\infty}^{+\infty} dt\, f(t) \int\limits_{-\infty}^{+\infty} dt' f(t')$

$\hat{V}_{kn} = -\frac{eaE(t)}{\sqrt{2}} \begin{cases} \sqrt{(n+1)(n+2)} & k=n+2 \\ \sqrt{n(n-1)} & k=n-2 \\ 0 & \text{else} \end{cases} \Rightarrow$

$a^{(2)}_{n+2}(+\infty) =$

$= -\frac{e^2a^2\sqrt{n+1}\sqrt{n+2}}{4\hbar^2} \int\limits_{-\infty}^{+\infty} \mathcal{E}(t) e^{i\omega t} \left(\int\limits_{-\infty}^{t} \mathcal{E}(t') e^{i\omega t'} dt' \right) dt = -\frac{e^2a^2\sqrt{(n+1)(n+2)}}{4\hbar^2} \left[\int\limits_{-\infty}^{+\infty} \mathcal{E}(t) e^{i\omega t} dt \right]^2$

$a^{(2)}_{n-2,n}(+\infty) = -\frac{e^2a^2\sqrt{n(n-1)}}{4\hbar^2} \left[\int\limits_{-\infty}^{+\infty} \mathcal{E}(t) e^{i\omega t} dt \right]^2$

$W^{(2)}(n\to m) = |a^{(2)}_{nm}|^2 = \frac{e^4a^4}{16\hbar^4} \begin{cases} (n+1)(n+2)|I(\omega)|^4, & m=n+2 \\ n(n-1)|I(-\omega)|^4, & m=n-2 \end{cases}$

$\frac{W^{(2)}}{W^{(1)}} \simeq \frac{e^2a^2}{\hbar^2}|I|^2(n+1) \lesssim \frac{e^2a^2\mathcal{E}_0(n+1)}{\hbar^2\omega^2} \ll 1$

Дз на 19.11.14.

(1) $-q_1\delta(x) \to -q_2\delta(x)$

$\psi_0=\sqrt{\varkappa_1}e^{-\varkappa_1|x|}$ $\quad\psi_0'=\sqrt{\varkappa_2}e^{-\varkappa_2|x|}$

$\varkappa_1=\frac{mq_1}{\hbar^2}$ $\quad\varkappa_2=\frac{mq_2}{\hbar^2}$

$$P=|\langle\psi_0'|\psi_0\rangle|^2=\left|\int(\psi_0')^*\psi_0\,dx\right|^2=$$
$$=\left|\int dx\sqrt{\varkappa_2}e^{-\varkappa_2|x|}\sqrt{\varkappa_1}e^{-\varkappa_1|x|}\right|^2=\left|\sqrt{\varkappa_1\varkappa_2}\int_{-\infty}^{\infty}dx\,e^{-|x|(\varkappa_1+\varkappa_2)}\right|^2=$$
$$=\left|\frac{2\sqrt{\varkappa_1\varkappa_2}}{\varkappa_1+\varkappa_2}\int_0^\infty e^{-x(\varkappa_1+\varkappa_2)}d(x(\varkappa_1+\varkappa_2))\right|^2=$$
$$=\left|+\frac{2\sqrt{\varkappa_1\varkappa_2}}{\varkappa_1+\varkappa_2}\right|^2=\frac{4\varkappa_1\varkappa_2}{(\varkappa_1+\varkappa_2)^2}=\frac{4q_1q_2}{(q_1+q_2)^2}=4\frac{q_2/q_1}{(1+q_2/q_1)^2}$$

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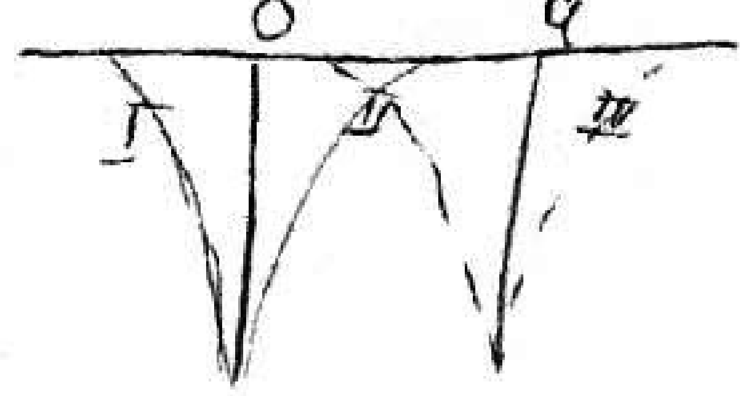
(2) $-q_1\delta(x) \to -q_1\delta(x-a)$

$q_1\delta(x)$ $\quad q_1\delta(x-a)$



$\psi_0=\sqrt{\varkappa_1}e^{-\varkappa_1|x|}$ $\quad\psi_0'=\sqrt{\varkappa_1}e^{-\varkappa_1|x-a|}$

$$P=|\langle\psi_0'|\psi_0\rangle|^2=\left|\int(\psi_0')^*\psi_0dx\right|^2=\left|\int_{-\infty}^{+\infty}dx\sqrt{\varkappa_1}e^{-\varkappa_1|x-a|}\sqrt{\varkappa_1}e^{-\varkappa_1|x|}dx\right|^2=\varkappa_1^2\left|\int_{-\infty}^0 e^{\varkappa_1(x-a)+\varkappa_1x}dx+\int_0^a e^{\varkappa_1(x-a)-\varkappa_1x}dx+\right.$$
$$\left.+\int_a^\infty e^{-\varkappa_1(x-a)-\varkappa_1x}dx\right|^2=\varkappa_1^2\left|\int_{-\infty}^0 e^{+2\varkappa_1x}e^{-\varkappa a}dx+\int_0^a e^{-\varkappa_1a}dx+\int_a^{+\infty}e^{-2\varkappa_1x}e^{+\varkappa a}dx\right|^2=$$
$$=\varkappa_1^2\left|e^{-\varkappa a}\cdot\frac{1}{2\varkappa_1}+a\cdot e^{-\varkappa_1a}+e^{+\varkappa_1a}\frac{1}{2\varkappa_1}e^{-\varkappa\cdot2a}\right|^2=\varkappa_1^2e^{-2\varkappa_1a}\left|\frac{1}{2\varkappa_1}+a+\frac{1}{2\varkappa_1}\right|^2=\varkappa_1^2e^{-2\varkappa_1a}\left|\frac{1}{\varkappa_1}+a\right|^2=$$
$$=e^{-2\varkappa a}(1+\varkappa a)^2$$

---

(3) тритий → гелий 3 иониз.

$^3H\to{}^3He^+$

$Z=1$ $\quad Z=2$

$1S$ $\quad 1S';2S'$

$$\psi_H=\psi_{100}=R_{10}(r)Y_{00}(\theta,\varphi)=2(Z/a_0)^{3/2}\exp(-Zr/a_0)\cdot\frac{1}{\sqrt{4\pi}}=$$
$$=\frac{1}{\sqrt{\pi a_0^3}}\exp(-r/a_0)$$
$$\psi_{He_1}=\psi_{100}=R_{10}(r)Y_{00}(\theta,\varphi)=2(Z/a_0)^{3/2}\exp(-Zr/a_0)\cdot\frac{1}{\sqrt{4\pi}}=$$
$$=\sqrt{\frac{8}{\pi a_0^3}}\exp(-2r/a_0)$$
$$\psi_{He}(2S')=\psi_{200}=R_{20}(r)Y_{00}(\theta,\varphi)=2(Z/2a_0)^{3/2}(1-Zr/2a_0)\exp(-Zr/2a_0)\cdot\frac{1}{\sqrt{4\pi}}=$$
$$=\sqrt{\frac{1}{\pi a_0^3}}(1-Zr/a_0)\exp(-r/a_0)$$

$$P_{1S'}=|\langle\psi_0'|\psi_0\rangle|^2=\left|\int(\psi_0')^*\psi_0d^3r\right|^2=\left|\int d^3r\cdot\frac{1}{\sqrt{\pi a_0^3}}\exp(-r/a_0)\cdot\sqrt{\frac{8}{\pi a_0^3}}\exp(-2r/a_0)\right|^2=$$
$$=\left(4\pi\cdot\frac{\sqrt8}{\pi a_0^3}\right)^2\left|\int_0^\infty r^2dr\exp(-3r/a_0)\right|^2=\left(4\pi\frac{\sqrt8}{\pi a_0^3}\right)^2\left|\int_0^\infty\left(\frac{3r}{a_0}\right)^2d\left(\frac{3r}{a_0}\right)\exp(-3r/a_0)\cdot\left(\frac{a_0}{3}\right)^3\right|^2=$$
$$=\frac{16\cdot8}{a_0^6}\left|2\cdot\frac{a_0^3}{3^3}\right|^2=\frac{16\cdot32}{3^6}=\frac{2^9}{3^6}\approx0{,}7$$

$$\left\{\begin{aligned}&\int_0^\infty\exp^{-kx}=\frac1k\quad\Big|\frac{\partial}{\partial k}\\&\int_0^\infty -x\exp(-kx)=-\frac{1}{k^2}\Big|\frac{\partial}{\partial k}\\&\int_0^\infty x^2\exp(-kx)=\frac{2}{k^3}\end{aligned}\right\}\quad\left\{\int_0^\infty -x^3\exp(-kx)=\frac{-6}{k^4}\right\}$$

$$P_{2s'}=|\langle\psi_0'|\psi_0\rangle|^2=\left|\int(\psi_0')^*\psi_0 d^3r\right|^2=\left|\int d^3r\frac{1}{\sqrt{\pi a_0^3}}(1-r/a_0)\exp(-r/a_0)\cdot\frac{1}{\sqrt{\pi a_0^3}}\exp(-r/a_0)\right|^2=$$

$$=\left(4\pi\cdot\frac{1}{\pi a_0^3}\right)^2\left|\int_0^\infty r^2dr(1-r/a_0)\exp(-2r/a_0)\right|^2=\frac{16}{a_0^6}\left|\int_0^\infty r^2dr\exp(-2r/a_0)-\int_0^\infty\frac{r^3}{a_0}dr\exp(-2r/a_0)\right|^2=$$

$$=\frac{16}{a_0^6}\left|\frac{2}{(2/a_0)^3}-\frac{1}{a_0}\frac{6}{(2/a_0)^4}\right|^2=16\left|\underbrace{\frac{2}{2^3}}_{1/4}-\underbrace{\frac{6}{2^4}}_{3/8}\right|^2=\frac{16}{64}\cdot\left(\frac{1}{8}\right)^2=\frac{16}{64}=\frac{1}{4}=0,25$$

11.2.13 – 8.46

(4) Г.О. $\frac{kx^2}{2}$ $\quad H_I=-Fx \quad |0\rangle\to|n'\rangle \quad P_{|n'\rangle}=?$

$$H=\frac{kx^2}{2}-Fx=\frac{k}{2}\left(x^2-2x\frac{F}{k}\right)=\frac{k}{2}\left(x^2-2x\frac{F}{k}+\left(\frac{F}{k}\right)^2\right)-\frac{F^2}{k^2}\cdot\frac{k}{2}=\frac{k}{2}\left(x-\frac{F}{k}\right)^2-\frac{F^2}{2k}=\frac{k}{2}(x-x_0)^2-\frac{F^2}{2k}$$

$\tilde{x}=x-x_0$, где $x_0=\frac{F}{k}$ $\qquad \psi(x)=\left(\frac{1}{\pi a^2}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}e^{-x^2/2a^2}H_n\left(\frac{x}{a}\right) \qquad a=\sqrt{\hbar/m\omega}$

$\tilde{H}=\frac{k\tilde{x}^2}{2}-\frac{F^2}{2k}$ ← просто добавка

$$P_{\langle n'|0\rangle}=|\langle n'|0\rangle|^2=\left|\int_{-\infty}^{+\infty}dx\,\psi^*(\tilde{x})\psi(x)dx\right|^2=\left|\int_{-\infty}^{+\infty}dx\underbrace{\left(\frac{1}{\pi a^2}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}\exp\left(-\frac{(x-x_0)^2}{2a^2}\right)H_n\left(\frac{x-x_0}{a}\right)}_{\psi_n(\tilde{x})}\cdot\underbrace{\left(\frac{1}{\pi a^2}\right)^{1/4}\cdot 1\cdot e^{-x^2/2a^2}\cdot 1}_{\psi_0(x)}\right|^2=$$

$$=\frac{1}{2^n n!\cdot\pi}\left|\int_{-\infty}^{+\infty}H_n\left(\frac{x-x_0}{a}\right)\cdot\exp\left(-\frac{(x-x_0)^2}{2a^2}-\frac{x^2}{2a^2}\right)\frac{dx}{a}\right|^2=\frac{1}{2^n n!\,\pi}\left|\int_{-\infty}^{+\infty}H_n\left(\frac{\tilde{x}}{a}\right)\exp\left(-\frac{\tilde{x}^2+(\tilde{x}+x_0)^2}{2a^2}\right)\frac{d\tilde{x}}{a}\right|^2=$$

$$=\frac{1}{2^n n!\,\pi}\left|\int_{-\infty}^{+\infty}H_n(y)\exp\left(-y^2+\frac{2yx_0}{2a^2}-\frac{x_0^2}{2a^2}\right)dy\right|^2=$$

произв. ф-я

$$\exp(-t^2+2xt)=\sum_{n=0}^\infty H_n(x)\frac{t^n}{n!} \qquad \left|\int_{-\infty}^{+\infty}dx\exp\left(-x^2+\frac{xk}{a}\right)\right.\Rightarrow$$

$$\Rightarrow\int_{-\infty}^{+\infty}\sum H_n(x)\frac{t^n}{n!}\exp\left(-x^2+\frac{xk}{a}\right)dx=\int_{-\infty}^{+\infty}\exp\left(-t^2+2xt-x^2+\frac{xk}{a}\right)dx$$

$$=\sqrt{\pi}\exp\left(\frac{k(k+4at)}{4a^2}\right)=\sqrt{\pi}\exp\left(\frac{k^2}{4a^2}\right)\exp\left(\frac{kt}{a}\right)=$$

$$=\sqrt{\pi}\exp\left(\left(\frac{k}{2a}\right)^2\right)\exp\left(t\frac{k}{a}\right)=\sqrt{\pi}\exp\left(\left(\frac{k}{2a}\right)^2\right)\sum_{n=0}^\infty\frac{1}{n!}\left(\frac{k}{a}\right)^n t^n$$

$$\int_{-\infty}^{+\infty}\sum H_n(x)\frac{t^n}{n!}\exp\left(-x^2+\frac{xk}{a}\right)dx=\sqrt{\pi}\exp\left(\frac{k^2}{4a^2}\right)\sum_{n=0}^\infty\frac{1}{n!}\left(\frac{k}{a}\right)^n t^n$$

$$t^n:\int_{-\infty}^{+\infty}H_n(x)\frac{1}{n!}\exp\left(-x^2+\frac{xk}{a}\right)dx=\sqrt{\pi}\exp\left(\frac{k^2}{4a^2}\right)\left(\frac{k}{a}\right)^n\frac{1}{n!}$$

Замена $x\to y$

$k\to x_0$

$$=\frac{1}{2^n n!\,\pi}\left|\sqrt{\pi}\exp\left(-\frac{x_0^2}{4a^2}\right)\left(\frac{x_0}{a}\right)^n\right|^2=$$

$$=\frac{1}{2^n n!\,\pi}\pi\exp\left(-\frac{x_0^2}{2a^2}\right)\left(\frac{x_0^2}{a^2}\right)^n=$$

$$=\exp\left(-\frac{x_0^2}{2a^2}\right)\cdot\frac{1}{n!}\left(\frac{x_0^2}{2a^2}\right)^n$$

(5) 

a) $b > a$

11.2.10 → 8.41

$$P_{\langle\psi_n'|\psi_0\rangle} = |\langle\psi_n'|\psi_0\rangle|^2 = \left|\int_{-\infty}^{+\infty} dx \sqrt{\tfrac{2}{a}}\sin\frac{\pi x}{a}\sqrt{\tfrac{2}{b}}\sin\frac{\pi(n+1)x}{b}\right|^2 =$$

$$= \frac{4}{ab}\left|\int_0^a \sin\frac{\pi x}{a}\sin\frac{\pi(n+1)x}{b}\,dx\right|^2 =$$

$$= \frac{4}{ab}\left|\int_0^a \frac{1}{2}\left(\cos\pi x\left(\frac{n+1}{b}-\frac{1}{a}\right)-\cos\pi x\left(\frac{n+1}{b}+\frac{1}{a}\right)\right)dx\right|^2 =$$

$$\psi_0 = \sin\frac{\pi x}{a}\cdot\sqrt{\tfrac{2}{a}} \qquad \psi_n' = \sin\frac{\pi(n+1)x}{b}\cdot\sqrt{\tfrac{2}{b}}$$

$$= \frac{1}{ab}\left|\frac{\sin\pi x\left(\frac{n+1}{b}-\frac{1}{a}\right)}{\pi\left(\frac{n+1}{b}-\frac{1}{a}\right)} - \frac{\sin\pi x\left(\frac{n+1}{b}+\frac{1}{a}\right)}{\left(\frac{n+1}{b}-\frac{1}{a}\right)\pi}\right|_0^2 = \frac{a^2b^2}{\pi^2 ab}\left|\frac{\sin\pi x\frac{an+a-b}{ab}}{(an+a-b)} - \frac{\sin\pi x\frac{an+a+b}{ab}}{(an+a+b)}\right|^2\Bigg|_0^a =$$

$$= \frac{ab}{\pi}\left|\frac{\sin\pi\frac{an+a-b}{b}}{an+a-b} - \frac{\sin\frac{an+a+b}{b}\pi}{an+a+b}\right|^2 = \frac{ab}{\pi}\left|\frac{\sin\frac{an+a}{b}(an+a+b-an-a+b)}{(an+a-b)(an+a+b)}\right|^2 =$$

$\sin(-\pi+\alpha) = -\sin\alpha$

$\sin(\pi+\alpha) = -\sin\alpha$

$$= \frac{4ab^2}{\pi}\left(\frac{\sin\frac{an+a}{b}\pi}{(an+a-b)(an+a+b)}\right)^2$$

b) $b = 2a$ $\qquad P = \frac{16a^3}{\pi}\left(\frac{\sin\frac{n+1}{2}\pi}{a(n-1)a(n+3)}\right)^2$

---

(6) $H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2\end{pmatrix}$ $\quad \psi_0\begin{pmatrix}1\\0\end{pmatrix}$

$H_I = \begin{pmatrix} 0 & V \\ V & 0\end{pmatrix}$ $\quad$ a) $P_2(t)$ $\quad$ б) $P_2'(t)$

a) $\det\begin{pmatrix} E_1-\lambda & 0 \\ 0 & E_2-\lambda\end{pmatrix} = 0 \Rightarrow (E_1-\lambda)(E_2-\lambda) = 0 \Rightarrow \begin{cases}\lambda_1 = E_1\\ \lambda_2 = E_2\end{cases}$

$\lambda_1 = E_1$

$\begin{pmatrix} 0 & 0 \\ 0 & E_2-E_1\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow 0\cdot a + \underbrace{(E_2-E_1)}_{\neq 0}\cdot b = 0 \Rightarrow b = 0$ $\qquad |\alpha_1\rangle = \begin{pmatrix}A\\0\end{pmatrix} = A\begin{pmatrix}1\\0\end{pmatrix}$

$\lambda_2 = E_2$

$\begin{pmatrix} E_1-E_2 & 0 \\ 0 & 0\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow (E_1-E_2)a + 0\cdot b = 0 \Rightarrow a = 0$ $\qquad |\alpha_2\rangle = \begin{pmatrix}0\\B\end{pmatrix} = B\begin{pmatrix}0\\1\end{pmatrix}$

$A^2 + B^2 = 1$

$$P_2(t) = |\langle\alpha_2|\psi\rangle|^2 = \left|B(0\ \ 1)\begin{pmatrix}1\\0\end{pmatrix}\right|^2 = 0$$

б) $H = \begin{pmatrix} E_1 & V \\ V & E_2\end{pmatrix}$ $\quad \det\begin{vmatrix} E_1-\lambda & V \\ V & E_2-\lambda\end{vmatrix} = 0 \Rightarrow (E_1-\lambda)(E_2-\lambda) = V^2 \Rightarrow E_1E_2 - \lambda(E_1+E_2) + \lambda^2 - V^2 = 0$

$\lambda^2 - \lambda(E_1+E_2) + E_1E_2 - V^2 = 0$

$$\lambda_{1,2} = \frac{(E_1+E_2) \pm (E_1-E_2) \pm \frac{2V^2}{(E_1-E_2)}}{2}$$

$$\lambda = \frac{E_1+E_2 \pm\sqrt{(E_1+E_2)^2 - 4(E_1E_2-V^2)}}{2}$$

$$\begin{vmatrix} \frac{V^2}{E_1-E_2} & V \\ V & (E_2-E_1) - \frac{V^2}{E_1-E_2}\end{vmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \begin{cases} -\frac{V^2}{E_1-E_2}a + Vb = 0 \\ Va + \left((E_2-E_1) - \frac{V^2}{E_1+E_2}\right)b = 0\end{cases} \Bigg| + \frac{+V}{E_1-E_2}$$

$$\lambda_{1,2} = \frac{E_1+E_2 \pm\sqrt{(E_1-E_2)^2 + 4V^2}}{2}$$

$$\Downarrow\ b = \frac{V}{E_1-E_2}a$$

$$\lambda_{1,2} = \frac{E_1+E_2 \pm (E_1-E_2)\sqrt{1+\frac{4V^2}{(E_1-E_2)^2}}}{2}$$

$$b\left(V - (E_1-E_2)\cdot\frac{V}{E_1-E_2} - \frac{V^2}{E_1-E_2}\cdot\frac{V}{E_1-E_2}\right) = 0$$

$$|\alpha_1\rangle = A\begin{pmatrix}1\\ \frac{V}{E_1+E_2}\end{pmatrix}$$

$$b\left(\frac{V^3}{(E_1-E_2)^2}\right) = 0$$

$$\lambda_{1,2} \simeq \frac{(E_1+E_2) \pm (E_1-E_2)\left(1+\frac{2V^2}{(E_1-E_2)^2}\right)}{2}$$

$$\begin{pmatrix} (E_1-E_2)+\frac{V^2}{E_1-E_2} & V \\ V & \frac{V^2}{E_1-E_2}\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix} \quad \left.\begin{aligned} &\left((E_1-E_2)+\frac{V^2}{E_1-E_2}\right)a+Vb=0\\ &Va+\frac{V^2}{E_1-E_2}b=0\end{aligned}\right\} \quad \begin{aligned} &a=-b\frac{V}{E_1-E_2}\\ &|\alpha_2\rangle=B\begin{pmatrix}-\frac{V}{E_1-E_2}\\ 1\end{pmatrix}\end{aligned}$$

$\langle\alpha_1|\alpha_2\rangle=0$

$V_{11}=V_{22}=0 \qquad \psi(0)=\psi_1=\begin{pmatrix}1\\ 0\end{pmatrix}$ $\qquad \omega_1=\frac{E_1}{\hbar}=E_1/\hbar$

$V_{12}=V_{21}=V \qquad \psi(t)=a(t)e^{-i\omega_1 t}\psi_1+b(t)e^{-i\omega_2 t}\psi_2$ $\qquad \omega_2=E_2/\hbar$

$\Omega=V/\hbar$

$\hat H=\hat H_0+\hat V \; ; \; i\hbar\frac{\partial\psi}{\partial t}=\hat H\psi$

$i\hbar\left(\frac{\partial a}{\partial t}e^{-i\omega_1 t}\psi_1-i\omega_1 a e^{-i\omega_1 t}\psi_1+\frac{\partial b}{\partial t}e^{-i\omega_2 t}\psi_2-i\omega_2 b e^{-i\omega_2 t}\psi_2\right)=$

$=a(t)e^{-i\omega_1 t}E_1\psi_1+b(t)e^{-i\omega_2 t}E_2\psi_2+a(t)e^{-i\omega_1 t}V\psi_1+b(t)e^{-i\omega_2 t}V\psi_2$

$$\left.\begin{aligned} &i\hbar\frac{\partial a}{\partial t}e^{-i\omega_1 t}=b(t)e^{-i\omega_2 t}\cdot V \;\; |\cdot\psi_1\\ &i\hbar\frac{\partial b}{\partial t}e^{-i\omega_2 t}=a(t)\cdot e^{-i\omega_1 t}\cdot V \;\; |\psi_2\end{aligned}\right\}$$

$\omega=\omega_1-\omega_2$

$$\left.\begin{aligned} &\dot a=-i\Omega e^{-i\omega t}b\\ &\dot b=-i\Omega^* e^{i\omega t}a\end{aligned}\right\} \qquad \begin{aligned} &a(0)=1\\ &b(0)=0\end{aligned} \qquad \begin{aligned} &b=-\frac{\dot a}{i\Omega e^{-i\omega t}}=i\frac{\dot a}{\Omega e^{-i\omega t}}\\ &-i\Omega^* e^{i\omega t}a=i\frac{\ddot a+i\omega\dot a}{\Omega e^{-i\omega t}}\\ &\ddot a+i\omega\dot a+\Omega^2 a=0\end{aligned}$$

$a=Ae^{i\varkappa t}$

$-\varkappa^2 A+i\omega(i\varkappa)+\Omega^2=0$

$\varkappa^2+\omega\varkappa-\Omega^2=0$

$\varkappa_{1,2}=\frac{-\omega\pm\sqrt{\omega^2+4\Omega^2}}{2}$

$a=A\exp\left(-i\frac{\omega+\sqrt{\omega^2+4\Omega^2}}{2}t\right)+B\exp\left(-i\frac{\omega-\sqrt{\omega^2+4\Omega^2}}{2}t\right)$

$b=\dot a e^{i\omega t}\cdot\frac{i}{\Omega}$

$b=i\frac{1}{\Omega}e^{i\omega t/2}\cdot C\cdot\sin\frac{\sqrt{\omega^2+4\Omega^2}}{2}t$

$a=\frac{+i}{\Omega^*}e^{-i\omega t}\dot b=\frac{i}{\Omega^*}e^{-i\omega t}\frac{i}{\Omega}C\left(e^{i\frac{\omega+\sqrt{\omega^2+4\Omega^2}}{2}t}-e^{i\frac{\omega-\sqrt{\omega^2+4\Omega^2}}{2}t}\right)\cdot 2i=$

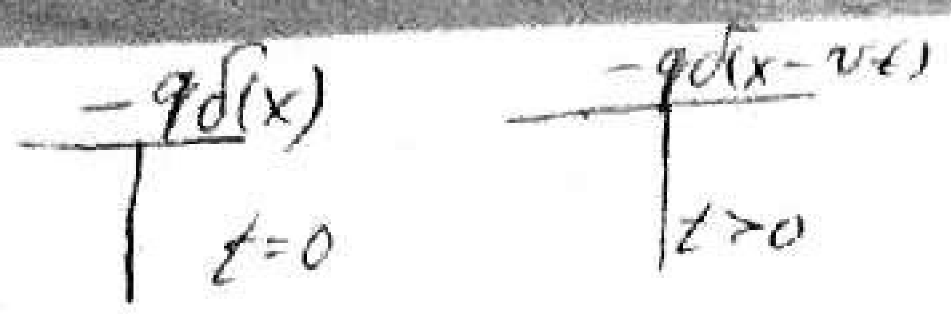
$=-\frac{2i}{\Omega^2}\cdot C\cdot e^{-i\omega t}\left(i\frac{\omega+\sqrt{\omega^2+4\Omega^2}}{2}\cdot e^{i\frac{\omega+\sqrt{\omega^2+4\Omega^2}}{2}}-i\frac{\omega-\sqrt{\omega^2+4\Omega^2}}{2}e^{i\frac{\omega-\sqrt{\omega^2+4\Omega^2}}{2}}\right)=$

$=+\frac{2}{\Omega^2}Ce^{-i\omega t/2}\sqrt{\omega^2+4\Omega^2}\cos\frac{\sqrt{\omega^2+4\Omega^2}t}{2} \Rightarrow C=\frac{\Omega^2}{2\sqrt{\omega^2+4\Omega^2}}$

$b(t)=-\frac{2i\Omega^* e^{i\omega t/2}}{\sqrt{\omega^2+4\Omega^2}}\cdot\sin\frac{\sqrt{\omega^2+4\Omega^2}}{2}t$

$W=|b(t)|^2=\frac{4\Omega^2}{\omega^2+4|\Omega|^2}\cdot\sin^2\left(\frac{\sqrt{\omega^2+4\Omega^2}}{2}t\right)$

⑨ 11.2.3. → 8.45



$\psi_0(x) = \sqrt{\varkappa}\exp(-\varkappa|x|)$ $\quad$ $\cancel{\psi'_0(x',t) = \psi_0(x-vt)} = \sqrt{\varkappa}\exp(-\varkappa|x-vt|)$

$\varkappa = \frac{mq}{\hbar^2}$

Преобразования Галилея 7.28

$U'(x',t') \equiv U'(x-Vt, t) = U(x,t)$

$$\begin{cases} i\hbar\frac{\partial}{\partial t}\psi(x,t) = \hat{H}\psi = [\frac{p^2}{2m} + U(x,t)]\psi(x,t) & : \psi(x,t) \\ i\hbar\frac{\partial}{\partial t}\psi'(x,t) = \hat{H}'\psi' = [\frac{p'^2}{2m} + U'(x',t)]\psi'(x',t) & ; \psi'(x',t) = \hat{U}\psi(x,t) \\ |\psi'(x',t)|^2 = |\psi'(x-Vt,t)|^2 = |\psi(x,t)|^2 \end{cases}$$

$\hat{U} = \exp\{iS(x,t)\} \Rightarrow \psi'(x',t) = \exp(iS(x,t))\psi(x,t)$

$$\begin{cases} i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + i\hbar\left(-\frac{\hbar}{m}\frac{\partial S}{\partial x} - V\right)\frac{\partial\psi}{\partial x} + \left[U(x,t) - \frac{i\hbar^2}{2m}\frac{\partial^2 S}{\partial x^2} + \frac{\hbar^2}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + \hbar V\frac{\partial S}{\partial x} + \hbar\frac{\partial S}{\partial t}\right]\psi(x,t) \\ i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[\frac{p^2}{2m} + U(x,t)\right]\psi(x,t) \end{cases}$$

$$\begin{cases} \frac{\hbar}{m}\frac{\partial S}{\partial x} + V = 0 \\ -\frac{i\hbar^2}{2m}\frac{\partial^2 S}{\partial x^2} + \frac{\hbar^2}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + \hbar V\frac{\partial S}{\partial x} + \hbar\frac{\partial S}{\partial t} = 0 \end{cases} \Rightarrow S(x,t) = -\frac{mVx}{\hbar} + f(t)$$

$S(x,t) = -\frac{mVx}{\hbar} + \frac{mV^2t}{2\hbar} + C$

$\Downarrow$

$\cancel{\varphi(p',t) = \varphi(p = p'+mv,t)\exp\left(-\frac{imv^2t}{2\hbar} + \frac{ipVt}{\hbar}\right)}$

$\psi'(x',t) = \exp\left(i\left(-\frac{mVx}{\hbar} + \frac{mV^2t}{2\hbar}\right)\right)\psi(x,t)$

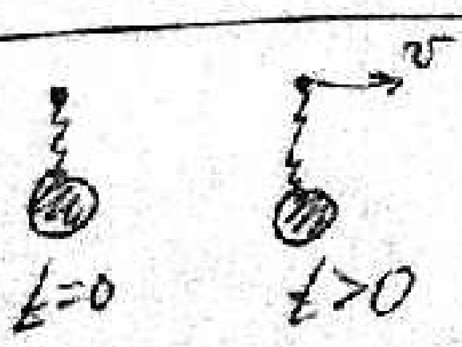
при $t = t' = 0$ $\quad \psi'(x',t) = \exp\left(-i\frac{mVx}{\hbar}\right)\psi_0(x,t)$

$$P = |\langle\psi_0^*|\psi'(x',t)\rangle|^2 = \left|\int_{-\infty}^{+\infty}\sqrt{\varkappa}\exp(-\varkappa|x|)\sqrt{\varkappa}\exp\left(-\frac{imVx}{\hbar}\right)\exp(-\varkappa|x|)dx\right|^2 =$$

$$= \varkappa^2\left|\int_{-\infty}^{0}\exp\left(2\varkappa x - \frac{imVx}{\hbar}\right)dx + \int_0^{+\infty}\exp\left(-2\varkappa x - \frac{imVx}{\hbar}\right)dx\right|^2 = \varkappa^2\left|\frac{1}{2\varkappa - imV/\hbar} + \frac{1}{2\varkappa + imV/\hbar}\right|^2 =$$

$$= \varkappa^2\left|\frac{4\varkappa}{4\varkappa^2 + m^2V^2/\hbar^2}\right|^2 = \left[\frac{1}{1 + \frac{m^2V^2}{\hbar^2 \cdot 4\varkappa^2}}\right]^2$$

⑩ Г.О.



$P_{|n'\rangle} = ?$ $\quad F = -mv\delta(t)$

$H = -x(-vm)\delta(t) = xvm\delta(t)$

$\psi'(x',t) = \exp\left(-\frac{imVx}{\hbar}\right)\psi_0(x,t)$

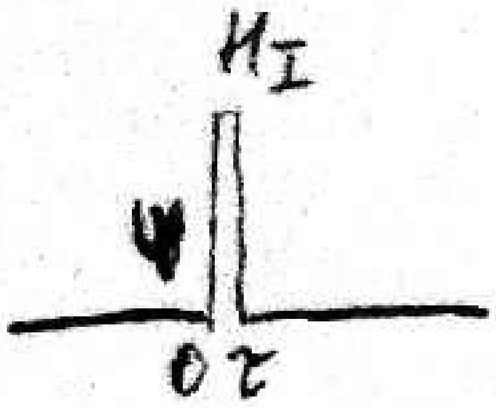
$$P = |\langle\psi_n|\hat{U}\,\psi_0(x,t)\rangle|^2 = \left|\int_{-\infty}^{+\infty}\psi_n^*(x)\exp\left(-\frac{imvx}{\hbar}\right)\cdot\psi_0(x)dx\right|^2 = \text{аналогично } (9) =$$

$$= \exp\left(-\frac{m^2V^2a^2}{2\hbar^2}\right)\frac{1}{n!}\left(\frac{m^2V^2a^2}{2\hbar^2}\right)^n$$

19.11.14.

$$H_I = W\delta(t)$$

$$C_m(t) = -\frac{i}{\hbar}\int_{-\infty}^{t} e^{i\omega_{mn}t} \langle \psi_m | H_I | \psi_n \rangle dt = \langle \psi_m | W | \psi_n \rangle \left(-\frac{i}{\hbar}\right) \begin{cases} 0 \\ 1 \end{cases}$$



$$|\psi(\tau)\rangle = e^{-\frac{i}{\hbar}(H_0 + H_I)\tau} |\psi(0)\rangle$$

$$\tau \to 0$$

$$W = H_I \tau$$

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}W} |\psi(0)\rangle = \left(1 - \frac{i}{\hbar}W\right)|\psi(0)\rangle$$

$$C_m = \langle \psi_m | \psi(t) \rangle \underbrace{\langle \psi_m | 1 | \psi_n \rangle}_{=0} - \frac{i}{\hbar}\langle \psi_m | W | \psi_n \rangle$$

$$|\psi(0)\rangle = |\psi_n\rangle$$

---

(4) $|\alpha\rangle = e^{-\frac{\alpha\alpha^*}{2}} e^{\alpha a^+} |0\rangle$

$$e^{\alpha a^+} = e^{\alpha\left(\frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{x_0} - i\frac{\hat{p}}{p_0}\right)\right)} = e^{\frac{\alpha}{\sqrt{2}}\frac{x}{x_0}} e^{-i\frac{\alpha}{\sqrt{2}}\frac{p}{p_0}} e^{[\,,\,]}$$

трансляция в импульс. пр-ве — трансляция в коорд пр-ве.

$$\langle \alpha | \hat{x} | \alpha \rangle = \frac{x_0}{\sqrt{2}}(\alpha + \alpha^*)$$

$$D_x = x_0^2/2$$

$$\langle \alpha | \hat{p} | \alpha \rangle = \frac{p_0}{\sqrt{2}}\left(\frac{\alpha - \alpha^*}{i}\right)$$

$$D_p = p_0^2/2$$

$$\langle \alpha | 0 \rangle$$

$$\langle \alpha | n \rangle$$

$$e^{-\frac{\alpha\alpha^*}{2}} e^{\alpha a^+} |0\rangle = e^{-\frac{\alpha\alpha^*}{2}} \sum_n \frac{\alpha^n}{n!} \underbrace{(a^+)^n |0\rangle}_{\sqrt{n!}\,|n\rangle}$$

$$a = \frac{F}{k} = \frac{x_0\, 2\,\mathrm{Re}\,\alpha}{\sqrt{2}}$$

$$|\alpha\rangle = e^{\frac{i}{\hbar}\hat{p}a} |0\rangle$$

$$\mathrm{Im}\,\alpha = 0$$

---

(10) $e^{-\frac{i}{\hbar}W}|0\rangle = e^{-\frac{i}{\hbar}(-xF)}|0\rangle$

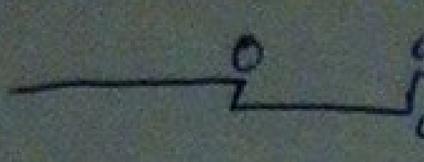
$$\mathrm{Re}\,\alpha = 0$$

$$\mathrm{Im}\,\alpha \cdot \frac{p_0}{\sqrt{2}} = F$$

Дз на 6.11.14

Мелкая яма

(0) → 11.2.11. → 8.42



а) $a \to b$ $P_{|\psi'>} = ?$

б) $u_1 \to u_2$ $P_{|\psi'>} = ?$

$$\psi(x) = \begin{cases} A\cos\left[\sqrt{2m(u_0-|E|)/\hbar^2}\,x\right] & |x|\le a \\ B\exp\left[-\sqrt{2mE/\hbar^2}\,|x|\right] & |x|>a \end{cases}$$

чётные уровни

$\Rightarrow \sqrt{u_0-|E|}\,\mathrm{tg}\sqrt{2m(u_0-|E|)a^2/\hbar^2} = \sqrt{|E|}$

т.к. $ma^2u_0/\hbar^2 \ll 1$; $\mathrm{tg}\,x \simeq x$

$\sqrt{u_0-|E_0|}\sqrt{2m(u_0-|E_0|)a^2/\hbar^2} \simeq \sqrt{|E_0|}$

т.к. $\sqrt{2m(u_0-|E_0|)a^2/\hbar^2} \ll 1 \Rightarrow |E_0| \ll u_0 \Rightarrow$

$\Rightarrow \sqrt{u_0}\sqrt{2mu_0a^2/\hbar^2} \simeq \sqrt{|E_0|} \Rightarrow |E_0| \simeq \frac{2mu_0a^2}{\hbar^2}u_0 \ll u_0$

$\psi_0(x) \simeq B\exp\left[-\sqrt{\frac{m\cdot 2|E_0|}{\hbar^2}}|x|\right]$, где $B = \left(\frac{2m|E_0|}{\hbar^2}\right)^{1/4}$

$\psi_0(x) \simeq \sqrt{C}\exp(-C|x|) \to$ похоже на ψ δ-потенциала: $\psi = \sqrt{\varkappa}\exp(-\varkappa|x|)$

а) $C_1 = \sqrt{\frac{2m|E_0|}{\hbar^2}} = \sqrt{\frac{2m}{\hbar^2}\cdot\frac{2mu_0^2a^2}{\hbar^2}} = \frac{2m}{\hbar^2}u_0a$

$C_2 = \sqrt{\frac{2m|E_0'|}{\hbar^2}} = \sqrt{\frac{2m}{\hbar^2}\frac{2mu_0^2b^2}{\hbar^2}} = \frac{2m}{\hbar^2}u_0b$

$P_{|\psi'>} = |\langle\psi'|\psi\rangle|^2 = \left|\int_{-\infty}^{+\infty}\sqrt{C_2}e^{-C_2|x|}\sqrt{C_1}e^{-C_1|x|}dx\right|^2 = C_1C_2\left|\int_{-\infty}^{+\infty}e^{-(C_1+C_2)|x|}dx\right|^2 = C_1C_2\left|2\int_0^{+\infty}e^{-(C_1+C_2)x}dx\right|^2 =$

$= 4C_1C_2\left|\frac{1}{C_1+C_2}\right|^2 = \frac{4C_1C_2}{(C_1+C_2)^2} = 4\frac{\frac{2m}{\hbar^2}u_0a\,\frac{2m}{\hbar^2}u_0b}{\left(\frac{2m}{\hbar^2}u_0a+\frac{2m}{\hbar^2}u_0b\right)^2} = \frac{4ab}{(a+b)^2}$

б) $C_1 = \frac{2mu_1a}{\hbar^2}$

$C_2 = \frac{2mu_2a}{\hbar^2}$ $P_{|\psi'>} = |\langle\psi'|\psi\rangle|^2 = \left|\int_{-\infty}^{+\infty}\sqrt{C_1C_2}\exp(-(C_1+C_2)|x|)dx\right|^2 = \frac{4C_1C_2}{(C_1+C_2)^2} = \frac{4u_1u_2}{(u_1+u_2)^2}$

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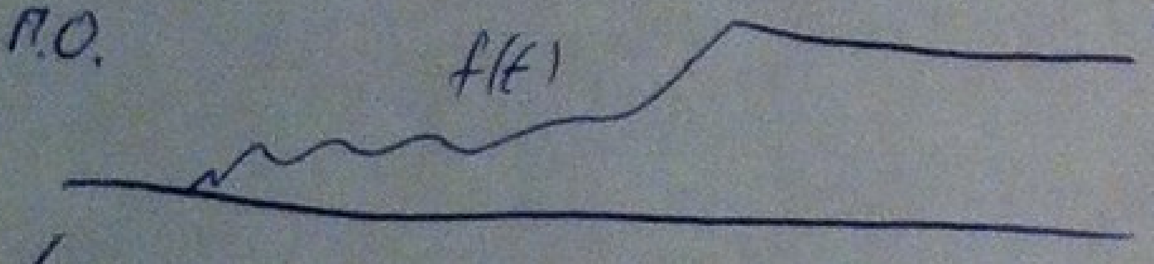
(1) П.О.



f_0 $t=+\infty$ $a = \frac{F_0}{k}$ $|n'\rangle = e^{i/\hbar\,\hat{p}a}|n\rangle$

$H_I = -f(t)\cdot\hat{x}$ $|\psi(t)\rangle = e^{\mu(t)}e^{\vartheta(t)a^+}|0\rangle$

$|\psi(-\infty)\rangle = |0\rangle$ $\alpha = \vartheta(t)$

$H_0 = \hbar\omega(a^+a + 1/2)$

$H_I = -f(t)\frac{x_0}{\sqrt{2}}(a+a^+)$

$[A,B] = \lambda$

$[A,f(B)] = \lambda f'(B)$

$[a, e^{\vartheta(t)a^+}] = \vartheta e^{\vartheta a^+}$

$i\hbar\frac{d}{dt}|\psi(t)\rangle = (H_0 + H_I(t))|\psi(t)\rangle$

$a|\psi(t)\rangle = ae^{\mu(t)}e^{\vartheta(t)a^+}|0\rangle =$

$= e^{\mu(t)}[a, e^{\vartheta(t)a^+}]|0\rangle + e^{\mu(t)}e^{\vartheta(t)a^+}\overset{0}{a|0\rangle} = e^{\mu(t)}\vartheta(t)e^{\vartheta(t)a^+}|0\rangle = \vartheta(t)|\psi(t)\rangle$

$i\hbar\dot{\mu}|\psi(t)\rangle + i\hbar\dot{\vartheta}(t)a^+|\psi(t)\rangle = \hbar\omega a^+\underbrace{a|\psi(t)\rangle}_{\vartheta|\psi(t)\rangle} + \frac{\hbar\omega}{2}|\psi(t)\rangle - f(t)\frac{x_0}{\sqrt{2}}\underbrace{a|\psi(t)\rangle}_{\vartheta|\psi(t)\rangle} - f(t)\frac{x_0}{\sqrt{2}}a^+|\psi(t)\rangle$

$$\left.\begin{aligned} i\hbar\dot{\mu} &= \hbar\frac{\omega}{2} - f(t)\frac{x_0}{\sqrt{2}}\vartheta(t) \\ i\hbar\dot{\vartheta} &= \hbar\omega\vartheta - f(t)\frac{x_0}{\sqrt{2}} \end{aligned}\right\}$$

ОРНС = ОРОС + ЧРНС

ОРОС: $i\hbar\dot{\vartheta} = \hbar\omega\vartheta \Rightarrow i\frac{d\vartheta}{dt} = \omega\vartheta \Rightarrow \frac{d\vartheta}{\vartheta} = -i\omega dt \Rightarrow \ln\vartheta = -i\omega t \Rightarrow \vartheta = \vartheta_0\exp(-i\omega t)$

$\vartheta_0 \equiv \vartheta_0(t)$

ЧРНС: $i\hbar(\vartheta_0(t)\exp(-i\omega t))^{\bullet} = \hbar\omega\vartheta_0(t)\exp(-i\omega t) - f(t)\frac{x_0}{\sqrt{2}} \Rightarrow$

$$i\hbar\left(\dot{\nu}_0(t)\exp(-i\omega t) - i\omega\nu_0(t)\exp(-i\omega t)\right) = \hbar\omega\nu_0\exp(-i\omega t) - f(t)\frac{x_0}{\sqrt{2}}$$

$$i\hbar\dot{\nu}_0(t) + \hbar\omega\nu_0(t) = \hbar\omega\cancel{\nu_0} - f(t)\frac{x_0}{\sqrt{2}}\exp(i\omega t)$$

$$i\hbar\dot{\nu}_0(t) = -f(t)\frac{x_0}{\sqrt{2}}\exp(i\omega t)$$

$$i\hbar\, d\nu_0(t) = -f(t)\frac{x_0}{\sqrt{2}}\exp(i\omega t)\,dt$$

$$i\hbar\, \nu_0(t) = -\frac{x_0}{\sqrt{2}}\int f(t)\exp(i\omega t)\,dt$$

$$\nu_0(t) = \frac{i x_0}{\sqrt{2}\hbar}\int f(t)\exp(i\omega t)\,dt \quad \Rightarrow \nu(t) = \nu_0(t)\exp(-i\omega t) = \frac{i x_0}{\sqrt{2}\hbar}\left[\int f(t)\exp(i\omega t)\,dt\right]\exp(-i\omega t)$$

$$i\hbar\dot{\mu} = \frac{\hbar\omega}{2} - f(t)\frac{x_0}{\sqrt{2}}\nu(t) \Rightarrow \frac{d\mu(t)}{\cancel{dt}} = \frac{-i\omega}{2}\,dt + \frac{i x_0}{\hbar\sqrt{2}} f(t)\nu(t)\,dt$$

$$\mu(t) = \frac{-i\omega t}{2} + \frac{i x_0}{\hbar\sqrt{2}}\int f(t)\nu(t)\,dt$$

(2) $H(t)$ - адиаб. измен. гамильт.

в. кажд. момент. времени $H(t)|\psi_n(t)\rangle = E_n(t)|\psi_n(t)\rangle$

Представим $\Psi(q,t)$ в виде $\Psi(q,t) = \sum_n C_n(t)\psi_n(q,t)\exp\left(-\frac{i}{\hbar}\int_0^t E_n(t')dt'\right)$

$\psi_n(q,t)$ и $E_n(t)$ - с.з. и с.ф. мгновенного гамильтониана

$\hat{H}(p,q,\lambda(t))\psi_n(q,t) = E_n(t)\psi_n(q,t)$

$i\hbar\frac{\partial}{\partial t}\Psi(q,t) = \hat{H}\Psi(q,t) \;\Big|\; *\psi_k^*(t);\ \int dq$; $\psi_n(t)$ орт $\psi_k(t)$

$$\dot{C}_k(t) = -\sum_n C_n(t)\exp\left[\frac{i}{\hbar}\int_0^t (E_k - E_n)dt'\right]\int\psi_k^*\dot{\psi}_n\,dq$$

1) $\psi_n(t)$ - веществ; $\operatorname{Im}\psi_n(t) = 0$

2) $\int\psi_n^*\dot{\psi}_n\,dq = \frac{d}{2dt}\int\psi_n^2\,dq = 0$

3) $\hat{H}(p,q,\lambda(t))\psi_n(q,t) = E_n(t)\psi_n(q,t) \;\Big|\; \frac{d}{dt};\ *(\psi_k^* = \psi_k);\ \int dq$

$$\int\psi_k\dot{\psi}_n\,dq = -\frac{1}{E_k - E_n}\int\psi_k\left(\frac{\partial\hat{H}}{\partial t}\right)\psi_n\,dq$$

$$\dot{C}_k(t) = \sum_{\substack{n\\ n\neq k}}\frac{1}{\hbar\omega_{kn}(t)}C_n(t)\left(\frac{\partial\hat{H}}{\partial t}\right)_{kn}\exp\left[i\int_0^t\omega_{kn}(t')dt'\right] \quad ; \quad \hbar\omega_{kn} = E_k - E_n$$

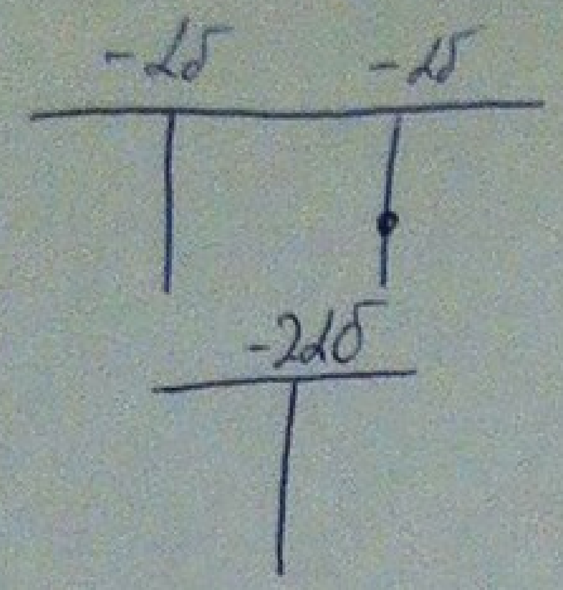
$\left(\frac{\partial H}{\partial t}\right)_{kn}$ - матричн. эл. $\frac{\partial\hat{H}}{\partial t}$

Если $\frac{\partial H}{\partial t}$ мало, то $\dot{C}_k \simeq 0$; $C_k \simeq C_{kn}^{(0)} = const = \delta_{kn}$

$$\dot{C}_{kn}^{(1)}(t) = \frac{1}{\hbar\omega_{kn}(t)}\left(\frac{\partial\hat{H}}{\partial t}\right)_{kn}\exp\left(i\int_0^t\omega_{kn}(t')dt'\right)$$

$$C_{kn}^{(1)}(t) = \frac{1}{\hbar}\int_0^t\frac{1}{\omega_{kn}(t')}\left(\frac{\partial H}{\partial t'}\right)_{kn}\exp\left(i\int_0^t\omega_{kn}(t')dt'\right)dt', \quad k\neq n$$

$$|C_{kn}^{(1)}| \sim \frac{\left|\frac{\partial\hat{H}}{\partial t}\frac{1}{\omega_{kn}}\right|}{|E_k - E_n|}$$

③ 

$U = -\alpha[\delta(x - L(t)/2) + \delta(x + L(t)/2)]$ $\underline{11.3.5 \to 8.52}$

Отдельно чётн и нечётн. решение.

$\psi_0(x)$ — в.ф. для одной ямы $U(x) = -\alpha\delta(x)$ (в правой яме)

$$\psi(t = -\infty) = \frac{1}{\sqrt{2}}(\psi^+ + \psi^-)$$

$$\psi^\pm = \frac{1}{\sqrt{2}}\left(\psi_0\left(x - \frac{L(-\infty)}{2}\right) \pm \psi_0\left(x + \frac{L(-\infty)}{2}\right)\right)$$

$L(-\infty) = \infty$ $\Rightarrow$ частица в связ. сост в поле двух ям с одинаковой энергией (уровень двукратно вырожден)

При слиянии ям остаётся единственный уровень и чётн. в.ф. для него, $\Rightarrow$ нечётн. в.ф. описывает свободн. част. Если процесс сближения ям адиабатический, то частица будет оставаться в основном состоянии. т.к. вероятность чётного состояния изначально $1/2$, то и при адиабатическом сближении вероятность остаётся $1/2$. Условие применимости: $|\dot{L}| \ll \alpha/\hbar$ при $L \sim \frac{\hbar^2}{m\alpha}$

④ $11.3.4 \to 8.51$ Плоский ротатор

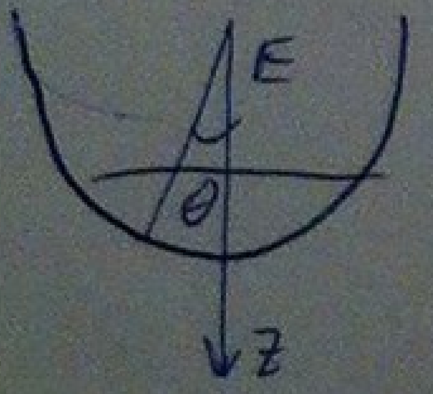
$\vec{E}(t) = E(t)\vec{n}_0$ при $t > 0$ $\quad H_0 = \frac{l_z^2}{2I}$ $\quad |\psi_m\rangle = \frac{e^{im\varphi}}{\sqrt{2\pi}}$

$E(t) = E_0[1 - \exp(-t/\tau)]$ $\quad E_m = \frac{m^2}{2I}$ $\quad H_I(t) = -dE_x(t)\cos\varphi$ поле сильное но меняется адиаб.

$P_{|\psi_m\rangle} = ?$ потенциал. $-C\cos\varphi = -C\left(1 - \frac{\varphi^2}{2}\right)$

$dE_0 t \gg \hbar^2$, но $dE_0 \tau^2 \ll \tau\hbar^3$ (сильное медленно включаемое поле)

$$\hat{H} = \frac{\hbar^2 l^2}{2I} - dE\cdot\underbrace{\cos\theta}_{1 - \frac{\theta^2}{2}} = -dE + \frac{dE}{2}\theta^2 + \frac{\hbar^2 l^2}{2I} = -dE + \frac{dE}{2}(x^2 + y^2) - \frac{\hbar^2}{2I}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$$



Наше поле заставит ротатор вращаться как гарм. осц.

$$\psi_0(\theta) \approx \left(\frac{1}{\sqrt{\pi}\,\theta_0}\right)^{1/2} \exp(-\theta^2/2\theta_0^2)$$ — основное сост. гарм. осц.

$$\theta_0 = \left(\frac{\hbar^2}{I d E_0}\right)^{1/4} \ll 1$$

$$P = \left|\int_{-\pi}^{\pi} e^{-\frac{\theta^2}{2\theta_0^2}} e^{-im\theta} d\theta\right|^2 \cdot \frac{1}{2\pi} \cdot \frac{1}{\sqrt{\pi}\,\theta_0} = \frac{1}{2\pi\sqrt{\pi}\,\theta_0}\left|\int_{-\infty}^{+\infty} e^{im\theta - \frac{\theta^2}{2\theta_0^2}} d\theta\right|^2 = \frac{1}{2\pi\sqrt{\pi}\,\theta_0} \left|2\theta_0^2 \cdot \pi\, e^{-m^2\theta_0^2/2}\right|^2 =$$

$$= \frac{\theta_0}{\sqrt{\pi}} e^{-m^2\theta_0^2}$$

12.1.4

$$u(\vec r) = -Q\delta(\vec r) - Q(\vec r - \vec a)$$

$$f(\theta) = -\frac{m}{2\pi\hbar^2}\int u(\vec r)\, e^{i\vec q\vec r}\, d\vec r =$$

$$= \frac{mQ}{2\pi\hbar^2}\int \left[\delta(\vec r) + \delta(\vec r - \vec a)\right] e^{i\vec q\vec r}\, d\vec r =$$

$$= \frac{mQ}{2\pi\hbar^2}\Big(\int \delta(x)\delta(y)\delta(z)\, e^{i(q_x\cdot x + q_y\cdot y + q_z\cdot z)}\, dx\,dy\,dz +$$

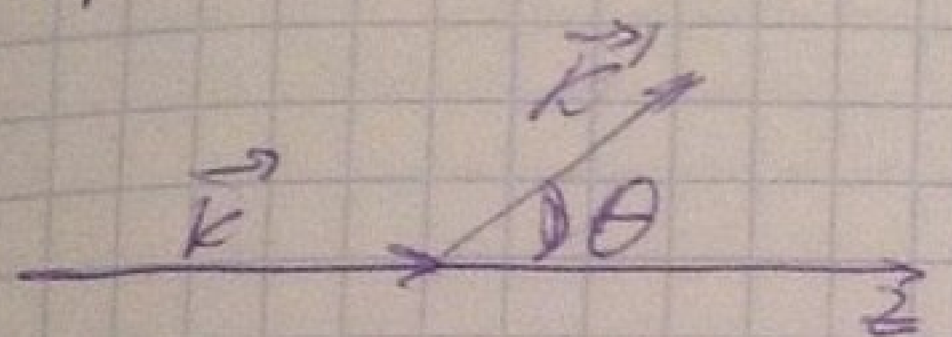
$$+ \int \delta(x)\delta(y)\delta(z-a)\, e^{i(q_x x + q_y y + q_z z)}\, dx\,dy\,dz\Big) =$$

$$= \frac{mQ}{2\pi\hbar^2}\left(1 + e^{iq_z\cdot a}\right) = \frac{mQ}{2\pi\hbar^2} e^{iq_z\frac{a}{2}}\Big(e^{-iq_z\frac{a}{2}} +$$

$$+ e^{iq_z\frac{a}{2}}\Big) = \frac{mQ}{\pi\hbar^2} e^{iq_z\frac{a}{2}} \cos\frac{q_z\cdot a}{2}$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \left(\frac{mQ}{\pi\hbar^2}\right)^2 \cos^2\frac{q_z a}{2}$$

$$q_z = k - k'\cos\theta \approx 2k\sin^2\frac{\theta}{2}$$



$$d\sigma = \left(\frac{mQ}{\pi\hbar^2}\right)^2 \cos^2\left(ka\sin^2\frac{\theta}{2}\right) 2\pi\sin\theta\, d\theta$$

$$\sigma = \int_0^\pi \ldots\, d\theta = 2\pi\left(\frac{mQ}{\pi\hbar^2}\right)^2 \int_0^\pi \cos^2\left(ka\sin^2\frac{\theta}{2}\right) 2\,\cdot$$

$$\cdot \sin\frac{\theta}{2}\cdot 2\, d\left(\sin\frac{\theta}{2}\right) = 8\pi\cdot\left(\frac{mQ}{\pi\hbar^2}\right)^2 \cdot$$

$$\cdot \int_0^\pi \cos^2\left(ka\sin^2\frac{\theta}{2}\right)\cdot\frac{1}{2}\, d\left(\sin^2\frac{\theta}{2}\right) =$$

$$= \left[\xi = ka\sin^2\frac{\theta}{2}\right] = \frac{4\pi}{ka}\left(\frac{mQ}{\pi\hbar^2}\right)^2 \cdot$$

$$\cdot \int_0^{ka} \cos^2\xi\, d\xi = \ldots \int_0^{ka} \frac{1 + \cos 2\xi}{2}\, d\xi =$$

$$= \ldots \left(\frac{\xi}{2} + \frac{\sin 2\xi}{4}\right)\Big|_0^{ka} = \frac{\pi}{ka}\left(\frac{mQ}{\pi\hbar^2}\right)^2 \cdot$$

$$\cdot (2ka + \sin 2ka) = 2\pi\cdot\left(\frac{mQ}{\pi\hbar^2}\right)^2 (1 + \mathrm{sinc}\, 2ka)$$

Дз на 3.12.14

① $V=Q\delta(\bar{x})$

$f(\theta)=?$

① → 12.1.1 → 13.4a

$$f(\theta)=-\frac{1}{4\pi}\frac{2m}{\hbar^2}\int e^{i\bar{q}\bar{x}}V(\bar{x})d\bar{x}=-\frac{2m}{\hbar^2}\int_0^\infty r^2dr\,\frac{\sin qr}{qr}V(r)=$$

$$=\frac{-2m}{\hbar^2}\int_0^\infty r^2dr\cdot\operatorname{sinc}(qr)V(r)=$$

$$=\frac{-2m}{\hbar^2}\int_0^\infty r^2dr\cdot\operatorname{sinc}(qr)\cdot Q\delta(r)=\frac{-2mQ}{\hbar^2}\cdot 0^2\cdot\operatorname{sinc}''(0)=0\,?$$

$V=Q\delta(r-R)$

$$f(\theta)=\frac{-2m}{\hbar^2}\int_0^\infty r^2dr\cdot\operatorname{sinc}(qr)\,Q\delta(r-R)=\frac{-2mQ}{\hbar^2}R^2\cdot\operatorname{sinc}(qR)=\frac{-2mQ}{\hbar^2}R^2\frac{\sin qR}{qR}$$

$$\frac{d\sigma}{d\Omega}=|f(\theta)|^2\Rightarrow\sigma=\int|f|^2d\Omega=2\pi\int_0^\pi|f|^2\sin\theta d\theta=\frac{\pi\hbar^2}{2mE}\int_0^{8mE/\hbar^2}f^2(q)dq^2=$$

$$=\frac{4\pi mQ^2R^2}{\hbar^2E}\int_0^{\sqrt{8mER^2/\hbar^2}}\frac{\sin^2x}{x}dx$$

$q=2k\sin\theta/2\Rightarrow q^2=4k^2\sin^2\theta/2=$
$=4k^2\frac{(1-\cos\theta)}{2}=2k^2(1-\cos\theta)$
$dq^2=2kd(1-\cos\theta)=2k^2d(-\cos\theta)$
$2\pi\int\sin\theta d\theta=2\pi\int d(1-\cos\theta)=\frac{2\pi}{2k^2}\int dq^2$
$\begin{cases}x=1\\ \hbar,x=\pm3\end{cases}\Rightarrow k=p/\hbar$
$k^2=p^2/\hbar^2\Rightarrow k^2=\frac{2mE}{\hbar^2}\Rightarrow\frac{\pi\hbar^2}{2mE}\int|f|^2dq^2$
$E=p^2/2m$

12.1.4

② $V=Q\delta(\bar{x})-Q\delta(\bar{x}-\bar{a})$

$f(\theta)=0$ $\quad f(\theta)=-\frac{2mQ}{\hbar^2}\frac{a^2\sin qa}{qa}$

$$\sigma_{tot}=\sigma_1+\sigma_2=0+\frac{4\pi mQ^2a^2}{\hbar^2E}\int_0^{\sqrt{8mEa^2/\hbar^2}}\frac{\sin^2x}{x}dx$$

---

③ а) $V(r)=ge^{-r/a}$

б) $V(r)=g/ch(r/a)$

12.15, 13.4

$\boxed{\text{из табл }\int_0^\infty xe^{-ax}\sin bx\,dx=\frac{2ab}{(a^2+b^2)^2}}$

а) $$f(\theta)=-\frac{2m}{\hbar^2}\int_0^\infty r^2dr\cdot\frac{\sin(qr)}{qr}V(r)=$$

$$=\frac{-2m}{\hbar^2}\int_0^\infty r^2\frac{\sin qr}{qr}\cdot ge^{-r/a}dr=-\frac{2mg}{\hbar^2q}\int_0^\infty r\sin qr\cdot e^{-r/a}dr=$$

$$=\frac{-2mg}{q\hbar^2}\cdot\frac{2\frac{1}{a}q}{(\frac{1}{a^2}+q^2)^2}=\frac{-4mg}{\hbar^2}\frac{1/a}{1/a^4(1+a^2q^2)^2}=$$

$$=-\frac{4mga^3}{\hbar^2(1+a^2q^2)^2}$$

б) $$f(\theta)=-\frac{2m}{\hbar^2}\int_0^\infty r^2dr\frac{\sin qr}{qr}\cdot\frac{g}{ch(r/a)}=-\frac{2m}{\hbar^2}\frac{g}{q}\int_0^\infty\frac{r\sin qr\,dr}{ch(r/a)}=-\frac{2mg}{\hbar^2q}\left(\frac{\pi a}{2}\right)^2\frac{sh\left(\frac{q\pi a}{2}\right)}{\left(ch\frac{q\pi a}{2}\right)^2}$$

$$\text{из табл }\int_0^\infty x\frac{\sin\alpha x}{ch\beta x}dx=\frac{\pi^2}{4\beta^2}\cdot\frac{sh\frac{\alpha\pi}{\beta}}{ch^2\frac{\alpha\pi}{\beta}}$$

$$\int_0^\infty\frac{\sin\alpha x}{ch\beta x}x^{2m+1}dx=(-1)^{m+1}\frac{\pi}{2\beta}\cdot\frac{\partial^{2m+1}}{\partial\alpha^{2m+1}}\left(1/ch\frac{\alpha\pi}{2\beta}\right)$$

при $m=0$ $$\int=(-1)\frac{\pi}{2\beta}\frac{\partial}{\partial\alpha}\left(\frac{1}{ch\frac{\alpha\pi}{2\beta}}\right)=$$

$$=\left(\frac{\pi}{2\beta}\right)^2\cdot\frac{sh\frac{\alpha\pi}{2\beta}}{\left(ch\frac{\alpha\pi}{2\beta}\right)^2}$$

а) $$\frac{d\sigma}{d\Omega}=|f(q)|^2=\frac{16m^2g^2a^6}{\hbar^4(1+a^2q^2)^4}$$

$$\frac{\sigma}{\pi\hbar^2/2mE}=\int|f(q)|^2dq^2=\frac{16m^2g^2a^6}{\hbar^4}\int_0^{8mE/\hbar^2}(1+a^2q^2)^{-4}dq^2=$$

$$=\frac{m^2g^2a^6}{\hbar^4a^2}\int_0^{t_0}\frac{1}{(1+t)^4}d(t+1)=\frac{8m^2g^2a^4}{\hbar^2}\left(-\frac{1}{3(1+t)^3}\right)\Bigg|_0^{t_0}$$

$$=\frac{16m^2g^2a^4}{3\hbar^4}\left(1-\frac{1}{(1+8mEa^2/\hbar^2)^3}\right)=$$

$$=\sigma\Big/\frac{\pi\hbar^2}{2mE}$$

б) $\frac{d\sigma}{d\Omega} = |f(q)|^2 = \frac{4m^2g^2}{\hbar^4 q^2}\left(\frac{\pi a}{2}\right)^4 \frac{Sh^2(q\pi a/2)}{Ch^4(q\pi a/2)} \Rightarrow \frac{4m^2g^2}{\hbar^4 q^2}\left(\frac{\pi a}{2}\right)^4 \frac{Ch^2(q\pi a/2)-1}{Ch^4(q\pi a/2)}$

$Ch^2 - Sh^2 = 1 \Rightarrow Sh^2 = Ch^2 - 1$

$$\sigma = \frac{\pi\hbar^2}{2mE}\int_0^{8mE/\hbar^2} |f(q)|^2 dq^2 = \frac{\pi\hbar^2}{2mE}\frac{4m^2g^2}{\hbar^4 q^2}\frac{\pi^4 a^4}{16}\left(\int \frac{dq^2}{Ch^2(q\pi a/2)} - \int\frac{dq^2}{Ch^4(q\pi a/2)}\right) =$$

$$= \frac{\pi^5 mg^2a^4}{4E\hbar^2q^2}\left(\int_0^{q_0}\frac{q\,dq}{Ch^2(q\beta)} - \int_0^{q_0}\frac{q\,dq}{Ch^4(q\beta)}\right) = \frac{\pi^5 mg^2a^4}{4E\hbar^2q^2}\left(\frac{\beta q_0\, th(\beta q_0) - \ln(Ch(\beta q_0))}{\beta^2} + \right.$$

$$\left. + \frac{1}{6\beta^2}\left(-1 - 4\ln(ch(\beta q_0)) + \frac{1}{Ch^2(\beta q_0)} + 2\beta q_0\left(2 + \frac{1}{Ch^2(\beta q_0)}\right)th(\beta q_0)\right)\right) =$$

$$= \frac{\pi^3 mg^2a^2}{E\hbar^2q^4}\left(-''- + \frac{1}{6} -''- \right)$$

---

④ → 12.1.3 → 13.42

$V(r) = A/r^2$

$\int_0^x \frac{\sin t}{t}dt = Si(x)$

$\lim_{x\to\infty} Si(x) = \frac{\pi}{2}$

$$f(\theta) = -\frac{2m}{\hbar^2}\int_0^\infty r^2 \cdot \frac{\sin qr}{qr}\cdot\frac{A}{r^2}\,dr = -\frac{2mA}{q\hbar^2}\int_0^\infty \frac{\sin qr}{qr}\,d(qr) =$$

$$= -\frac{2mA}{q\hbar^2}\cdot\frac{\pi}{2} = -\frac{\pi mA}{q\hbar^2}$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{\pi^2m^2A^2}{q^2\hbar^4}$$

$$\sigma = \frac{\pi\hbar^2}{2mE}\int_0^{8mE/\hbar^2}|f(q)|^2dq^2 = \frac{\pi\hbar}{2mE}\cdot\frac{\pi^2m^2A^2}{\hbar^4}\int_0^{q_0^2}\frac{1}{q^2}dq^2 = \frac{\pi^3mA^2}{2E\hbar^3}\left(\ln q_0^2 - \underset{-\infty}{\ln 0}\right)$$

$= \infty$

$q^2 \to 2k^2(1-\cos\theta)$

$E \to \hbar^2k^2/2m$

---

⑤ $V(r) \sim \frac{1}{r^n}$; $V(r) = \frac{\alpha}{r^n}$

12.1.2. $\sigma_{tot} = ?$

$\frac{d\sigma}{d\Omega} = ?$

Метод размерности

$\alpha = [E\cdot r^n]$

$m = [M]$

$\hbar = [E\cdot T]$

$V = [L\cdot T^{-1}]$

$E = [M\cdot L^2\cdot T^{-2}]$

$L^2 = V^y\cdot m^z\cdot \hbar^p\cdot \underbrace{(E\cdot L^n)^x}_{\alpha^x}$

$$L^2 = M^x L^{2x+nx} T^{-2x} L^y T^{-y} \cdot M^z M^p L^{2p} T^{-p}$$

$$\begin{cases} 2 = (2+n)x + y + z + 2p \\ 0 = x + z + p \\ 0 = -2x - y - p \end{cases}$$

$x = 2$ т.к. Борновское приближение

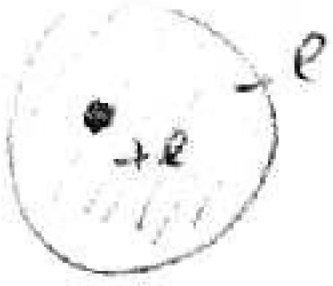
$p = -4 - y$

$$\begin{cases} 0 = 2 + z - 4 - y = z - y - 2 \\ 2 = 4 + 2n + y + z + 2p \end{cases} \Rightarrow z = 2n - 4$$

$$\sigma \sim \alpha^2 V^{2(n-3)} m^{2(n-2)} \hbar^{2(1-n)}$$

⑥ 12.1.6. → 13.48

атом H осн. сост.



Эл. потенциал $\varphi(r) = \underbrace{\frac{Ze}{r}}_{\text{от ядра}} - \underbrace{e\int\frac{\rho(r'')dv''}{|\bar r''-\bar r|}}_{\text{от электронов}}$

$$U(r) = e_1\varphi(r) = \frac{Zee_1}{r} - ee_1\int\frac{\rho(r'')dv''}{|\bar r''-\bar r|}$$

$$A(\theta) = -\frac{1}{4\pi}\frac{2m}{\hbar^2}\int e^{i\bar q\bar r'}U(\bar r')dv' \Rightarrow$$

$$\Rightarrow A(\theta) = -\frac{2m}{\hbar^2}\frac{Zee_1}{4\pi}\overbrace{\int\frac{e^{i\bar q\bar r'}}{r'}dv'}^{I_1} + \frac{2m}{\hbar^2}\frac{ee_1}{4\pi}\overbrace{\int e^{i\bar q\bar r'}dv'\int\frac{\rho(\bar r'')dv''}{|\bar r''-\bar r'|}}^{I_2}$$

$\varphi(r'') = \int\frac{e^{i\bar q\bar r'}}{|\bar r''-\bar r'|}dv'$ → потенциал в $\bar r''$ с плотностью $\rho(\bar r') = e^{i\bar q\bar r'}$

$$\left.\begin{aligned}&\nabla^2\varphi(\bar r') = -4\pi\rho(\bar r') = -4\pi e^{i\bar q\bar r'}\\ &\nabla^2\varphi(\bar r') = \nabla^2 e\rho(r') = -|\bar q|^2 e\rho(r') = -|\bar q|^2\varphi(r')\end{aligned}\right\} \Rightarrow \varphi(r') = \frac{4\pi e^{i\bar q\bar r'}}{|\bar q|^2}$$

$$I_1 = \int\frac{e^{i\bar q\bar r'}}{r'}dv' = \int\frac{e^{i\bar q\bar r'}}{|\bar r'|}dv' = \frac{4\pi}{|\bar q|^2}$$

$$I_2 = \int e^{i\bar q\bar r'}dv'\int\frac{\rho(\bar r'')dv''}{|\bar r''-\bar r'|} = \int dv''\rho(r'')\int\frac{e^{i\bar q\bar r'}}{|\bar r''-\bar r'|}dv' = \int dv''\rho(r'')\frac{4\pi e^{i\bar q\bar r''}}{|\bar q|^2} = \frac{4\pi}{|\bar q|^2}\int dv\,\rho(\bar r)e^{i\bar q\bar r} =$$

$$= \left\{\begin{aligned}&dv = r^2dr\sin\theta d\theta d\varphi\\ &\bar q\bar r = qr\cos\theta\end{aligned}\right\} = \frac{4\pi}{|\bar q|^2}\int_0^\infty\rho(r)r^2dr\int_0^{2\pi}d\varphi\int_0^\pi e^{ikr\cos\theta}\sin\theta d\theta = \frac{(4\pi)^2}{|\bar q|^2}\int_0^\infty\rho(r)r^2\frac{\sin qr}{qr}dr$$

$$A(\theta) = -\frac{2m}{\hbar^2}\frac{ee_1}{4\pi}\frac{4\pi}{|\bar q|^2}\left(Z - 4\pi\int_0^\infty\frac{\sin qr}{qr}\rho(r)r^2dr\right)$$

$$q^2 = 4k^2\sin^2\frac{\theta}{2}$$

$$A(\theta) = -\frac{2m}{\hbar^2}\frac{ee_1}{4k^2\sin^2\theta/2}(Z - F(\theta)),\ \text{где}\ F(\theta) = 4\pi\int_0^\infty\frac{\sin qr}{qr}\rho(r)r^2dr$$

$$Z=1,\ \rho = \frac{1}{\pi a_0^3}e^{-2r/a_0}$$

$$F(\theta) = \int dv\, e^{i\bar q\bar r}\rho(\bar r) = \frac{1}{\pi a_0^3}\underbrace{\int_0^{2\pi}d\varphi}_{2\pi}\int dr\, e^{iqr\cos\theta - 2r/a}\cdot r^2\sin\theta d\theta = +\frac{2}{a_0^3}\int_0^\infty r^2e^{-2r/a_0}dr\int_{-1}^{+1}e^{iqr\cos\theta}d(\cos\theta) =$$

по Силаеву

$$= +\frac{2}{a_0^3}\int r^2e^{-2r/a_0}dr\cdot\frac{1}{iqr}e^{iqr\cos\theta}\Big|_{-1}^{1} = -\frac{2}{a_0^3}\int_0^\infty r^2e^{-2r/a_0}dr\frac{1}{iqr}\left(e^{iqr} - e^{-iqr}\right) =$$

$$= +\frac{2i}{qa_0^3}\left(\int re^{iqr-2r/a}dr - \int re^{-iqr-2r/a}dr\right) = \frac{2i}{qa_0^3}\left(\frac{1}{(2/a-iq)^2}\left(-e^{-r(2/a-iq)}\cdot((2/a-iq)r+1)\right) -\right.$$

$$\left.\left. - \frac{1}{(2/a+iq)^2}\left(-e^{-r(2/a+iq)}((2/a+iq)r+1)\right)\right)\right|_0^\infty = \frac{2i}{qa_0^3}\left(\frac{1}{(2/a-iq)^2} - \frac{1}{(2/a+iq)^2}\right) = \frac{2i}{qa_0^3}\frac{\left(4/a^2 + 4iq/a - q^2 - 4/a^2 + 4iq/a + q^2\right)}{(4/a^2+q^2)^2}$$

$$= \frac{16}{(4+a_0^2q^2)^2} \Rightarrow A(\theta) = -\frac{2m}{\hbar^2}\frac{e^2}{4k^2\sin^2\theta/2}\left(1 - \frac{16}{(4+a_0^2q^2)^2}\right) = \frac{2me^2}{\hbar^2q^2}\left(1 - \frac{16}{(4+a_0^2q^2)^2}\right)$$

⑦ 12.2.7 → 13.13/4 При выполнении каких условий сечение рассеяния на совокупности большого кол-ва центров будет равняться сумме сечений рассеяния на отдельных центрах?

Для рассеяния на двух центрах:

$$f(\theta) = -\frac{m}{2\pi\hbar^2}\int e^{-i\vec q\vec r}\left[U_0(\vec r) + U_0(\vec r - \vec a)\right] dV = f_0(q)(1 + e^{-i\vec q\vec a}) \quad \left\{ \begin{array}{l} \text{замена} \\ \vec r' \to \vec r + a \end{array} \right\}$$

т.к. $q^2 = 2k^2(1-\cos\theta)$ => при $ka \ll 1$ и $qa \ll 1$, при этом $e^{-i\vec q\vec a} \simeq 1$

$$f^Б_{2ц} = 2f^Б_0 \Rightarrow \frac{d\sigma}{d\Omega} = |f(\theta)|^2 \Rightarrow \frac{d\sigma_{2ц}}{d\Omega} = 4\frac{d\sigma_0}{d\Omega}$$

$$\sigma_{2ц} = 2\int |f^Б_0(q)|^2 (1 + \cos qa)\, d\Omega$$

В случае $kR \gtrsim 1$ и $a \gg R$ имеем $ka \gg 1$ => qa сильно меняется при изменении θ => $\cos qa$ сильно осциллирует и даёт вклад меньше 1. $= \sigma_{2ц} \simeq 2\sigma_0$

Для многих центров

$$f^Б_N(q) = f_0(q)\sum_n e^{-i\vec q\vec a_n} = f_0(q)\, G_N(q)$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega}|G_N(q)|^2$$

---

⑧ 12.3.1 → 13.50 Неупругое рассеяние на ¹H с возбуждением до 2S.

$\overrightarrow{k}\; e^-$ $\quad \underset{1S}{\odot} \Rightarrow \underset{2S}{\odot} \overset{e^-}{\nearrow} \vec k'$ $\qquad |\vec k'| \neq |\vec k| \qquad f(\theta) = ?$

$$|\psi_i\rangle = \frac{e^{i\vec k\vec x}}{\sqrt{L^3}}\psi_{100}(\vec y) \qquad |\psi_f\rangle = \frac{e^{i\vec k'\vec x}}{\sqrt{L^3}}\psi_{200}(\vec y)$$

$$\psi_{100} = \frac{2}{\sqrt{a^3}}\exp(-r/a_0)\cdot\frac{1}{\sqrt{4\pi}}$$

$$\psi_{200} = \frac{1}{\sqrt{4\pi}}\cdot\frac{1}{\sqrt{2a_0^3}}\left(1 - \frac{1}{2}\,r/a_0\right)\exp(-r/2a_0)$$

$$E_i = \frac{\hbar^2k^2}{2m} + E_{1S} = \frac{\hbar^2k^2}{2m} - Ry \qquad E_i = E_f$$

$$E_f = \frac{\hbar^2k'^2}{2m} + E_{2S} = \frac{\hbar^2k'^2}{2m} - \frac{1}{4}Ry$$

$$\delta(E_f - E_i) = \delta\left(\frac{\hbar^2}{2m}(k^2 - k'^2) + \frac{3}{4}Ry\right) = \frac{2m}{\hbar^2}\delta\left(k^2 - k'^2 + \frac{3}{4}Ry\frac{2m}{\hbar^2}\right)$$

$$\int dx\,\delta(f(x)) = \frac{1}{\left|\frac{\partial f}{\partial x}(x_0)\right|}$$

$$\to k'^2 = k^2 - \frac{3}{4}Ry\frac{2m}{\hbar^2}$$

рассеяние неупругое

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\sum_f |\langle\psi_f|H_s|\psi_i\rangle|^2\delta(E_f - E_i)$$

$$\sum_f = \sum_{n_x n_y n_y} \Rightarrow \int d\vec k = \sum_{n_x n_y n_z}\left(\frac{2\pi}{L}\right)^3 \Rightarrow \sum_{n_x n_y n_z} = \left(\frac{L}{2\pi}\right)^3\int d^3k$$

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\int\left(\frac{L}{2\pi}\right)^3 d\Omega_{k'}\cdot k'^2dk'\left|\int d^3r\,\frac{2e^{-i\vec k'\vec r}}{\sqrt{4\pi a_0^3}}\frac{\exp(-r/a_0)}{\sqrt{L^3}}\cdot V(r)\cdot\frac{1}{\sqrt{8\pi a_0^3}}(1 - r/2a_0)\frac{e^{i\vec k\vec r}\exp(-r/2a_0)}{\sqrt{L^3}}\right|^2\frac{2m}{\hbar^2}\delta\left(k'^2 - k^2 + \frac{3}{4}Ry\frac{2m}{\hbar^2}\right)$$

$$= \frac{12\pi}{L^6\hbar}\int\left(\frac{L}{2\pi}\right)^3 d\Omega_{k'}\left(k^2 - \frac{3}{4}Ry\frac{2m}{\hbar^2}\right)\left|\int d^3r\,\frac{2}{\pi a_0^3 4\sqrt2}\exp(-r/a_0)V(r)(1 - r/2a_0)\exp(-r/2a_0)\right|^2\frac{2m}{\hbar^2}\frac{1}{2k} =$$

$$=\frac{2\pi}{\hbar}\left(\frac{L}{2\pi}\right)^3\frac{2m}{\hbar^2}\frac{1}{2k}\left(k^2-\frac{3}{4}Ry\frac{2m}{\hbar^2}\right)\frac{4}{\pi^2 a_0^6\,32}\frac{1}{L^6}\left|\int d^3r\,e^{-ik'r}e^{-r/a_0}V(r)e^{ikr}e^{-r/2a_0}(1-r/2a_0)\right|^2\int d\Omega_{k'}$$

$E=\frac{\hbar^2k^2}{2m}$

при $L\to\infty$; $\frac{1}{\tau}=0$

$\sigma_{tot}=\frac{1/\tau}{j_0}$; где $j_0=\frac{\hbar k}{m}\frac{1}{L^3}$

$$\sigma_{tot}=\frac{1}{\tau}\frac{mL^3}{\hbar k}=\frac{2\pi}{\hbar}\left(\frac{L}{2\pi}\right)^3\frac{2m}{\hbar^2}\frac{1}{2k}\frac{1}{L^6}\left(k^2-\frac{3}{4}Ry\frac{2m}{\hbar^2}\right)\frac{4}{\pi^2a_0^6\cdot 32}\left|\dots\right|^2\frac{mL^3}{\hbar k}=$$

$$=\frac{1}{(2\pi)^2}\frac{m^2}{\hbar^4}\left(1-\frac{3}{4}Ry\frac{2m}{\hbar^2k^2}\right)\frac{1}{\pi^2a_0^6\,8}\left|\dots\right|^2=\frac{(2m)^2}{(4\pi)^2\hbar^4}(\dots)\frac{1}{\dots}\left|\dots\right|^2\int d\Omega_{k'}=$$

$$=\left(1-\frac{3}{4}Ry\frac{2m}{\hbar^2k^2}\right)\frac{1}{\pi^2a_0^6 8}\int d\Omega_{k'}\left|-\frac{1}{4\pi}\int d^3r\,e^{i(\vec k-\vec k')\vec r}\cdot\frac{2m}{\hbar^2}V(x)\right|^2$$

$$\frac{d\sigma}{d\Omega}=\left(1-\frac{3}{4}Ry\frac{2m}{\hbar^2k^2}\right)\frac{1}{\pi^2a_0^6 8}\Big|\underbrace{-\frac{1}{4\pi}\int_0^\infty d^3r\,e^{i(\vec k-\vec k')\vec r}\cdot\frac{2m}{\hbar^2}\exp(-r/a_0-r/2a_0)(1-r/2a_0)}\Big|^2$$

$$I=-\frac{1}{4\pi}\frac{2m}{\hbar^2}\int d^3r\,e^{i\vec q\vec r}\exp(-\tfrac{3}{2}r/a_0)(1-r/2a_0)\overset{I}{=}-\frac{2m}{\hbar^2}\int_0^\infty r^2dr\frac{\sin qr}{qr}\exp(-\tfrac{3}{2}r/a_0)(1-r/2a_0)=$$

$$=-\frac{2m}{\hbar^2}\Big[\underbrace{\frac{1}{q}\int_0^\infty r\sin qr\exp(-\tfrac{3}{2}r/a_0)}_{\frac{1}{q}\frac{2\cdot 3/2a_0\cdot q}{((3/2a_0)^2+q^2)^2}=\frac{48a^3}{(9+4a^2q^2)^2}}-\underbrace{\frac{1}{2a_0q}\int_0^\infty r^2\sin qr\exp(-\tfrac{3}{2a_0}r)}_{\frac{16a^3(27-4a^2q^2)}{(9+4a^2q^2)^3}}\Big]=$$

$$=-\frac{2m}{\hbar^2}\frac{a^3}{(9+4a^2q^2)^3}\left(48(9+4a^2q^2)-16(27-4a^2q^2)\right)=-\frac{2m}{\hbar^2}\frac{a^3\cdot16}{(9+4a^2q^2)^3}\left(27+12a^2q^2-27+4a^2q^2\right)=$$

$$=-\frac{2m}{\hbar^2}\frac{a^3\cdot16^2a^2q^2}{(9+4a^2q^2)^3}=f(\theta)$$

$$\frac{d\sigma}{d\Omega}=\left(1-\tfrac{3}{4}Ry/E\right)\frac{1}{8\pi^2a_0^6}\left|-\frac{2m}{\hbar^2}\frac{a^3 16a^2q^2}{(9+4a^2q^2)^2}\right|^2=\left(1-\frac{3Ry}{4E}\right)\frac{1}{8\pi^2a_0^6}\frac{4m^2}{\hbar^4}\frac{a^6\cdot256\cdot a^4q^4}{(9+4a^2q^2)^4}=$$

$$=\left(1-\frac{3Ry}{4E}\right)\frac{m^2\cdot2^7a^4q^4}{\pi^2\hbar^4(9+4a^2q^2)^4}$$

$t_0=8mEa^2 4/\hbar^2=$
$=32mEa^2/\hbar^2$

$$\sigma=\frac{\pi\hbar^2}{2mE}\int\dots dq^2=\left(1-\frac{3Ry}{4E}\right)\frac{m^22^7a^4}{\pi^2\hbar^4}\frac{\pi\hbar^2}{2mE}\int_0^{8mE/\hbar^2}\frac{q^4dq^2}{(9+4a^2q^2)^4}=$$

$$=\left(1-\frac{3Ry}{4E}\right)\frac{m2^6a^4}{\pi\hbar^2E\,4a^2}\int_0^{t_0}\frac{t^2dt}{(9+t)^4}=\left(1-\frac{3Ry}{4E}\right)\frac{m\cdot2^4a^2}{\pi\hbar^2E}\left(-\frac{t^2+9t+27}{(t+9)^3}\right)\Big|_0^{t_0}=$$

$$=\left(1-\frac{3Ry}{4E}\right)\frac{m\cdot2^4a^2}{\pi\hbar^2E}\left(\frac{27}{9^3}-\frac{t_0^2+9t_0+27}{(t_0+9)^3}\right)$$

2'

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l + \frac{6\cos\theta}{k^2}\sum_{l=0}^{\infty}(l+1)\sin\delta_l\sin\delta_{l+1}\cos(\delta_{l+1}-\delta_l) +$$

$$+\frac{5}{k^2}\frac{3\cos^2\theta-1}{2}\sum_{l=0}^{\infty}\left\{\frac{l(l+1)(2l+1)}{(2l-1)(2l+3)}\sin^2\delta_l + \frac{3(l+1)(l+3)}{2l+3}\sin\delta_l\sin\delta_{l+2}\cos(\delta_{l+2}-\delta_l)\right\} + \dots$$

$$\frac{d\sigma}{d\Omega}(\theta) = \alpha + \beta\cos\theta, \quad \beta \ll \alpha \qquad , \delta_0, \delta_1 = ?$$

$$\Rightarrow \alpha = \frac{1}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l = \frac{1}{k^2}(\sin^2\delta_0 + 3\sin^2\delta_1 + 5\sin^2\delta_2 + \dots$$

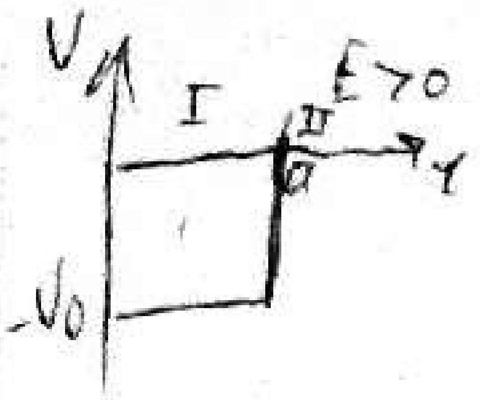
$$\beta = \frac{6}{k^2}\sum_{l=0}^{\infty}(l+1)\sin\delta_l\sin\delta_{l+1}\cos(\delta_{l+1}-\delta_l) = \frac{6}{k^2}\left(\sin\delta_0\sin\delta_1\cos(\delta_1-\delta_0) + 2\sin\delta_1\sin\delta_2\cos(\delta_2-\delta_1) + \dots\right)$$

Медлен. част, сферич. симм. потенц. $\Rightarrow l=0$

$$\alpha = \frac{1}{k^2}\sin^2\delta_0 = \frac{1}{2k^2}(1-\cos 2\delta_0) \Rightarrow \cos 2\delta_0 = 1-2k^2\alpha \Rightarrow \delta_0 = \frac{1}{2}\arccos(1-2k^2\alpha)$$

$$\beta = \frac{6}{k^2}\sin\delta_0\sin\delta_1\cos(\delta_1-\delta_0)$$

$$ V = \begin{cases} -V_0, & r \le a \\ 0, & r > a \end{cases} $$



$$ R_\ell'' + \frac{2}{r} R_\ell' + \left[k^2 - \frac{2m}{\hbar^2} V(r) - \frac{\ell(\ell+1)}{r^2}\right] R_\ell = 0 $$

$$ R_\ell = \frac{w}{\sqrt{r}} $$

$$ w'' + \frac{1}{r} w' + \left(k^2 - \frac{(\ell + 1/2)^2}{r^2}\right) w = 0 $$

$$ \sqrt{\frac{2m}{\hbar^2}(E - V(r))} \qquad \sqrt{\frac{2m}{\hbar^2} E} $$

$$ \text{II}\quad w_{II} = A J_{\ell+1/2}(kr) + B N_{\ell+1/2}(kr) $$

$$ \text{I:}\quad w_I = C J_{\ell+1/2}(\varkappa r) \quad ; \quad \varkappa = \sqrt{\frac{2m}{\hbar^2}(E + V_0)} $$

$$ \begin{cases} w_I(a) = w_{II}(a) \Rightarrow C J_{\ell+1/2}(\varkappa a) = A J_{\ell+1/2}(ka) + B N_{\ell+1/2}(ka) \\ w_I'(a) = w_{II}'(a) \Rightarrow C \varkappa J_{\ell+1/2}'(\varkappa a) = A k J_{\ell+1/2}' + B k N_{\ell+1/2}'(ka) \end{cases} $$

$$ \frac{1}{w_I} \frac{\partial w_I}{\partial r}\Big|_{r=a} = f_\ell $$

$$ f_\ell = \varkappa \frac{J_{\ell+1/2}'(\varkappa a)}{J_{\ell+1/2}(\varkappa a)} $$

$$ R_\ell = \frac{w}{\sqrt{r}} \Rightarrow w = R_\ell \sqrt{r} $$

$$ C\varkappa J_{\ell+1/2}'(\varkappa a) = k J_{\ell+1/2}'(ka) \frac{\left(C J_{\ell+1/2}(\varkappa a) - B N_{\ell+1/2}(ka)\right)}{J_{\ell+1/2}(ka)} + B k N_{\ell+1/2}'(ka) $$

$$ C\varkappa J_{\ell+1/2}'(\varkappa a) J_{\ell+1/2}(ka) = k J_{\ell+1/2}'(ka) \cdot C J_{\ell+1/2}(\varkappa a) + B\left(k N_{\ell+1/2}' (ka) J_{\ell+1/2}(ka) + J'(ka) N(ka)\right) $$

$$ B = C \frac{\frac{\varkappa}{k} J'(\varkappa a) J(ka) - J'(ka) J(\varkappa a)}{N'(ka) J(ka) - J'(ka) N(ka)} $$

$$ A = C \frac{\frac{\varkappa}{k} J'(\varkappa a) N(ka) - J(\varkappa a) N'(ka)}{J'(ka) N(ka) - J(ka) N'(ka)} = -C \frac{\frac{\varkappa}{k} J'(\varkappa a) N(ka) - J(\varkappa a) N'(ka)}{N'(ka) J(ka) - J'(ka) N(ka)} = $$

$$ = -B \frac{\frac{\varkappa}{k} J'(\varkappa a) N(ka) - J(\varkappa a) N'(ka)}{\frac{\varkappa}{k} J'(\varkappa a) J(ka) - J'(ka) J(\varkappa a)} $$

$$ J_\nu(x)\big|_{x\to\infty} = \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi\nu}{2} - \frac{\pi}{4}\right) $$

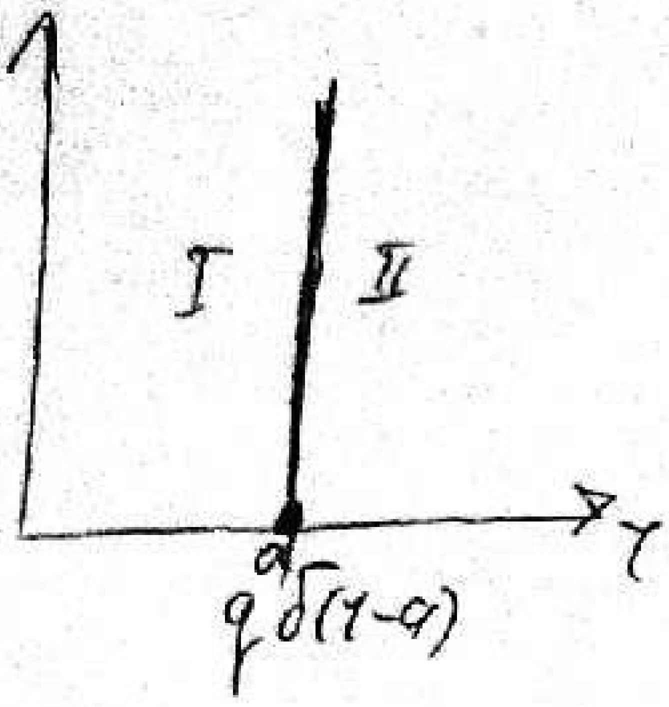
$$ N_\nu(x)\big|_{x\to\infty} = \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi\nu}{2} - \frac{\pi}{4}\right) $$

$$ R = \frac{1}{\sqrt{r}} w\big|_{r\to\infty} = \frac{1}{\sqrt{r}}\left[A\sqrt{\frac{2}{\pi k r}} \cos\left(kr - \frac{\pi(\ell+1/2)}{2} - \frac{\pi}{4}\right) + B\sqrt{\frac{2}{\pi k r}} \sin\left(kr - \frac{\pi(\ell+1/2)}{2} - \frac{\pi}{4}\right)\right] $$

$$ \frac{C}{r}\cos\left(kr - \frac{\pi(\ell+1)}{2} + \delta_\ell\right) = \frac{C}{r}\cos\left(kr - \frac{\pi(\ell+1)}{2}\right)\cos\delta_\ell - \frac{C}{r}\sin\left(kr - \frac{\pi(\ell+1)}{2}\right)\sin\delta_\ell $$

$$ \operatorname{tg}\delta_\ell = \frac{-B}{A} = \frac{\frac{\varkappa}{k} J'(\varkappa a) J(ka) - J'(ka) J(\varkappa a)}{\frac{\varkappa}{k} J'(\varkappa a) N(ka) - J(\varkappa a) N'(ka)} $$

в)



$$W_I = C\cdot J_{l+1/2}(ka)$$

$$W_{II} = A J_{l+1/2}(ka) + B N_{l+1/2}(ka)$$

1) $W_I(a) = W_{II}(a)$: $C J_{l+1/2}(ka) = A J_{l+1/2}(ka) + B N_{l+1/2}(ka)$

2) $\cancel{W_I'(a-0) = W_{II}'(a+0) + \frac{2mq}{\hbar^2} W(a)}$

$$W'(a+0) - W'(a-0) = \frac{2mq}{\hbar^2} W(a)$$

$$Ak J'_{l+1/2}(ka) + Bk N'_{l+1/2}(ka) - Ck J'_{l+1/2}(ka) = \frac{2mq}{\hbar^2} C J_{l+1/2}(ka)$$

1) $A = \frac{C J(ka) - B N(ka)}{J(ka)}$; $kJ'\frac{CJ - BN}{J} + BkN' - CkJ' = \frac{2mq}{\hbar^2} C J$

$$\cancel{kJ'C} - BN\frac{J'}{J}k + BkN' - \cancel{CkJ'} = \frac{2mq}{\hbar^2} C J$$

$$Bk(NJ' - N'J) = \frac{2mq}{\hbar^2} C J^2$$

$$B = \frac{2mq}{k\hbar^2} C \frac{J^2}{NJ' - N'J}$$

2) $B = \frac{CJ - AJ}{N}$

$$AkJ' + \frac{CJ - AJ}{N} N'k - CkJ' = \frac{2mq}{\hbar^2} C J$$

$$Ak(J'N - JN') = \frac{2mq}{\hbar^2} C (JN - kJN' + kJ'N)$$

$$A = \frac{2mq}{\hbar^2} C \frac{\left(\frac{JN}{k} - JN' + J'N\right)}{J'N - JN'} = -\frac{2mq}{\hbar^2} C \frac{\left(\frac{1}{k}JN - JN' + J'N\right)}{J'N - JN'} = -B\frac{\left(\frac{1}{k}JN - JN' + J'N\right)}{\frac{1}{k}J^2}$$

$J_\nu(x)|_{x\to\infty} = \sqrt{\frac{2}{\pi x}}\cos\left(x - \frac{\pi\nu}{2} - \frac{\pi}{4}\right)$; $N_\nu(x)|_{x\to\infty} = \sqrt{\frac{2}{\pi x}}\sin\left(x - \frac{\pi\nu}{2} - \frac{\pi}{4}\right)$

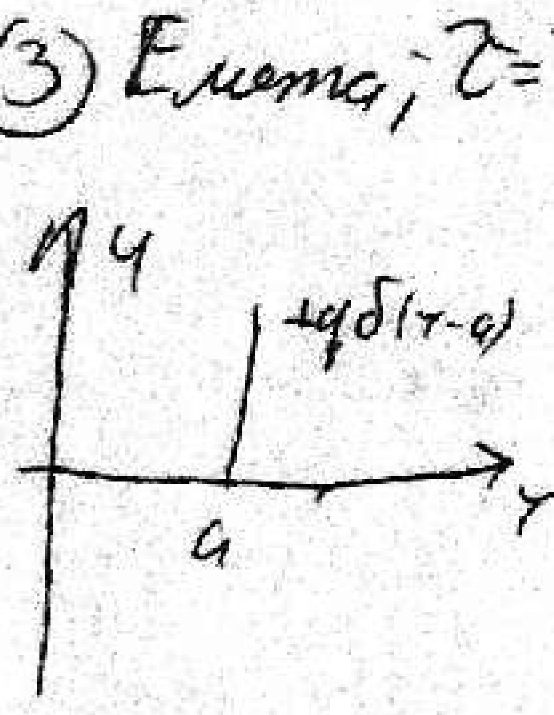
$$R = \sqrt{r}\, W|_{r\to\infty} = \sqrt{r}\left[A\sqrt{\frac{2}{\pi k r}}\cos\left(kr - \frac{\pi(l+1)}{2}\right) + B\sqrt{\frac{2}{\pi k r}}\sin\left(kr - \frac{\pi(l+1)}{2}\right)\right]$$

$$\frac{C}{r}\cos\left(kr - \frac{\pi(l+1)}{2} + \delta_l\right) = \frac{C}{r}\cos\left(kr - \frac{\pi(l+1)}{2}\right)\cos\delta_l - \frac{C}{r}\sin\left(kr - \frac{\pi(l+1)}{2}\right)\sin\delta_l$$

$$\mathrm{tg}\,\delta_l = \frac{-B}{A} = \frac{\frac{1}{k}J^2}{\frac{1}{k}JN - JN' + J'N} = \frac{\frac{1}{k}J^2_{l+1/2}(ka)}{\frac{1}{k}J_{l+1/2}(ka)N_{l+1/2}(ka) - J_{l+1/2}(ka)N'_{l+1/2}(ka) + J'_{l+1/2}(ka)N_{l+1/2}(ka)}$$

+ http://nuclphys.sinp.msu.ru/qti/qti_03.htm

③ $E_{мета}$; $\tau = ?$



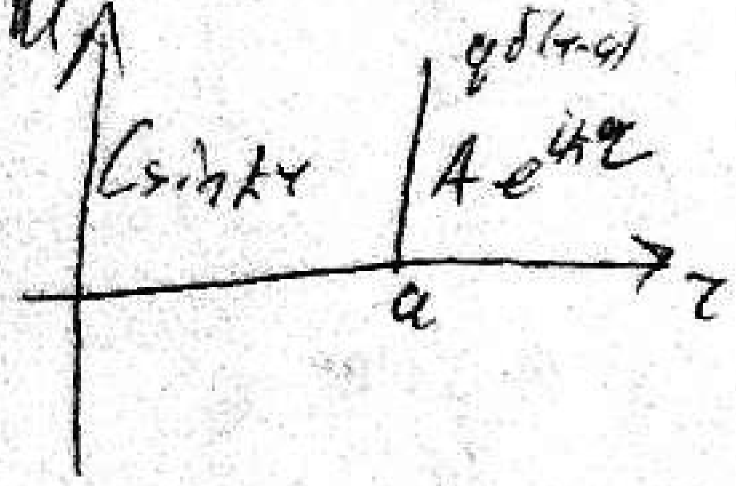
$$E = E_0 - iE_1, \quad E_1 \ll E_0$$

$$e^{-\frac{i}{\hbar}Et} = e^{-\frac{i}{\hbar}E_0 t} e^{-\frac{i}{\hbar}E_1 t}$$

$$e^{-\frac{2}{\hbar}E_1 t} = P_{E_1} = e^{-t/\tau} \Rightarrow \tau = \frac{\hbar}{2E_1}$$

$$\psi(r \to \infty) = Ae^{ikr}$$

$$E_0 - iE_1 = \frac{\hbar^2}{2m}k^2 = \frac{\hbar^2}{2m}(k_0 - ik_1)^2 = \frac{\hbar^2}{2m}(\underbrace{k_0^2 - k_1^2}_{E_0} - i \underbrace{2k_0 k_1}_{E_1})$$



$$\begin{cases} C\sin ka = Ae^{ika} \\ ikAe^{ika} - Ck\cos ka = \frac{2mq}{\hbar^2} C\sin ka \end{cases}$$

$$ik \cdot C\sin ka - Ck\cos ka = \frac{2mq}{\hbar^2} C\sin ka$$

$$-ke^{-ika} = \frac{2mq}{\hbar^2} \frac{e^{ika} - \exp(-ika)}{2i}$$

$$-k\frac{\hbar^2}{mq} i = e^{2ika} - 1 = \underline{e^{2ika} = 1 - k \cdot \varepsilon}, \quad \varepsilon = \frac{\hbar^2}{mq}$$

0: $e^{2ik^{(0)}a} = 1 \Rightarrow k^{(0)} = \frac{\pi n}{a} \rightarrow$

1: $\underbrace{e^{2ik^{(0)}a}}_{1} e^{2ik^{(1)}a} = 1 - i\varepsilon k^{(0)}$.

$$1 + 2ik^{(1)}a = 1 - i\varepsilon k^{(0)} \Rightarrow k^{(1)} = -\frac{\varepsilon k^{(0)}}{2a} = -\frac{\varepsilon}{2a}\frac{\pi n}{a}$$

2: $\underbrace{e^{2ik^{(0)}a}}_{1} e^{2ik^{(1)}a} e^{2ik^{(2)}a} = 1 - i\varepsilon k^{(0)} - i\varepsilon k^{(1)}$

$$\left(1 + 2ik^{(1)}a + \tfrac{1}{2}(2ik^{(1)}a)^2\right)\left(1 + 2ik^{(2)}a\right) = 1 - i\varepsilon k^{(0)} - i\varepsilon k^{(1)}$$

$$1 + 2ik^{(1)}a + \tfrac{1}{2}(2ik^{(1)}a)^2 + 2ik^{(2)}a + \cancelto{0}{2ik^{(1)}a\, 2ik^{(2)}a} + \cancelto{0}{\tfrac{1}{2}(2ik^{(1)}a)^2\, 2ik^{(2)}a} = 1 - i\varepsilon k^{(0)} - i\varepsilon k^{(1)}$$

$$\underbrace{1 + 2ik^{(1)}a}_{1 - i\varepsilon k^{(0)}} - 2(k^{(1)})^2 a^2 + 2ik^{(2)}a = 1 - i\varepsilon k^{(0)} - i\varepsilon k^{(1)}$$

$$-2(k^{(1)})^2 a^2 + 2ik^{(2)}a = -i\varepsilon k^{(1)}$$

$$k^{(2)} = -\frac{\varepsilon}{2a}k^{(1)} - ia(k^{(1)})^2 = -\frac{\varepsilon}{2a}\left(-\frac{\varepsilon}{2a}\frac{\pi n}{a}\right) - ia\left(-\frac{\varepsilon}{2a}\frac{\pi n}{a}\right)^2 = \frac{\varepsilon^2 \pi n}{4a^3} - i\frac{\varepsilon^2 \pi^2 n^2}{4a^3}$$

$$k = k^{(0)} + k^{(1)} + k^{(2)} = \frac{\pi n}{a} - \frac{\varepsilon}{2a^2}\pi n + \frac{\varepsilon^2 \pi n}{4a^3} - i\frac{\varepsilon^2 \pi^2 n^2}{4a^3} = k_0 - ik_1$$

$$k_0 = \operatorname{Re} k = \frac{\pi n}{a} - \frac{\varepsilon \pi n}{2a^2} + \frac{\varepsilon^2 \pi n}{4a^3}$$

$$k_1 = \operatorname{Im} k = +\frac{\varepsilon^2 \pi^2 n^2}{4a^3}$$

$$E_1 = 2k_0 k_1 = 2\left(\frac{\pi n}{a} - \frac{\varepsilon \pi n}{2a^2} + \frac{\varepsilon^2 \pi n}{4a^3}\right)\left(+\frac{\varepsilon^2 \pi^2 n^2}{4a^3}\right) = \left(+\frac{\varepsilon^2 \pi^3 n^3}{2a^4} - \frac{\varepsilon^3 \pi^3 n^3}{4a^5} + \frac{\varepsilon^4 \pi^3 n^3}{8a^6}\right)$$

$$= \frac{\varepsilon^2 \pi^3 n^3}{2a^4}\left(1 - \frac{\varepsilon}{2a} + \frac{\varepsilon^2}{4a^2}\right) = \frac{\varepsilon^2 \pi^3 n^3}{2a^4}\left(\frac{4a^2 - 2a\varepsilon + \varepsilon^2}{4a^2}\right)$$

$$\tau = \frac{\hbar}{2E_1}$$

④ 12.3.2. → 13.54

$$V(r)=\begin{cases} -iU_0, & r<a, \quad U_0>0 \\ 0, & r>a \end{cases}, \quad |V_0| \ll \hbar^2/mR^2$$

S волна

$\sigma_{упр}, \sigma_{неупр}$ - ?

$$f \approx f_0 = \frac{1}{2ik}\left(e^{2i\delta_0}-1\right) \approx \frac{1}{2ik}\left(1+2i\delta_0-1\right) = \frac{\delta_0(k)}{k}$$

Борн. прибл. $(qr \le 2ka \ll 1)$

$$f \approx -\frac{m}{2\pi\hbar^2}\int U(r)\,d\vec{r} = -\frac{m}{2\pi\hbar^2}\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta \int_0^a r^2 dr \cdot (-iU_0) =$$

$$= -\frac{m}{2\pi\hbar^2}\cdot 2\pi\cdot 2\cdot\frac{a^3}{3}(-iU_0) = \frac{-2ma^3}{3\hbar^2}(-iU_0)$$

$$\sigma_{упр} = \int |f|^2 d\Omega = 4\pi\cdot|f|^2 = \frac{4m^2a^6}{9\hbar^4}\cdot 4\pi\cdot U_0^2 = \frac{16\pi m^2 a^6}{9\hbar^4}\cdot U_0^2$$

$$\begin{cases} f \approx \dfrac{\delta_0(k)}{k} \\ f \approx -\dfrac{2ma^3}{3\hbar^2}(0-iU_0) \end{cases}$$

$$\delta_0(k) = \frac{-2mka^3}{3\hbar^2}(-iU_0)$$

$$Im\,\delta_0(k) = \frac{2mka^3U_0}{3\hbar^2}$$

$\delta_0(k) = Re\,\delta_0 + i\,Im\,\delta_0$

$$f = \frac{1}{2ik}\left(e^{2i\delta_0}-1\right) = \frac{1}{2ik}\left(e^{2i\cdot i\,Im\,\delta_0}-1\right) = \frac{1}{2ik}\left(e^{-2Im\,\delta_0}-1\right)$$

$$\sigma_{неупр} = 4\pi|f|^2 = \frac{4\pi}{4i^2k^2}\left(e^{-2Im\,\delta_0}-1\right)^2 \approx \frac{\pi}{i^2k}\left(e^{-4Im\,\delta_0}-1\right) = \frac{\pi}{k^2}\left(1-e^{-4Im\,\delta_0}\right) \approx \frac{\pi}{k^2}\left(1-\left(1-4Im\,\delta_0\right)\right) =$$

$$= \frac{4\pi\, Im\,\delta_0}{k^2} = \frac{8\pi m a^3 U_0}{3k\hbar^2}$$

---

⑤ Для быстрых частиц $kR \gg 1$ $\quad \sigma(E) = 4\pi\int_0^\infty \left\{1-\cos\left[\frac{m}{k\hbar^2}\int_{-\infty}^{+\infty} U\left(\sqrt{\rho^2+z^2}\right)dz\right]\right\}\rho\, d\rho$

12.3.3.

13.39 $\quad$ Оптическая теорема $\sigma = \frac{4\pi}{k} Im\, f(k,\theta=0) = \frac{2\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(1-\cos 2\delta_\ell)$

При $kR \gg 1$ существ роль играет большое число слаг. с $\ell \sim kR$

δ_ℓ из квазиклассич. выр.

$$\delta_\ell \approx \int_{r_0}^{\infty}\left[\frac{1}{\hbar}\sqrt{2m(E-U)-\hbar^2\frac{(\ell+1/2)^2}{r^2}}-k\right]dr + \frac{\pi}{2}(\ell+1/2) - kr_0$$

При $|U| \ll E$ получаем $\delta_\ell \approx -\frac{m}{k\hbar^2}\int_{r_0=\frac{\ell+1/2}{k}}^{\infty}\frac{rU(r)dr}{\sqrt{r^2-\frac{(\ell+1/2)^2}{k^2}}} = -\frac{m}{2k\hbar^2}\int_{-\infty}^{+\infty}U\left(\sqrt{\rho^2+r_0^2}\right)dz$

При замене ℓ на $(\ell+1)$

$$|\Delta\delta_\ell| \sim \left|\frac{\partial}{\partial\ell}\delta_\ell\right| \sim \frac{m}{k\hbar^2}|U(r_0)|\frac{1}{k^2R} \sim \frac{m|U(r_0)|}{\hbar^2k^2} \ll 1 \Rightarrow \frac{2\pi}{k^2}\sum_{0}^{\infty}(2\ell+1)(1-\cos 2\delta_\ell) = 4\pi\int_0^\infty \rho(1-\cos 2\delta_\ell)d\rho, \quad \rho = \frac{\ell+1/2}{k}$$

При $|U(r_0)| \gtrsim E$: $|\delta_\ell| \gg 1$ и $|\Delta\delta_\ell| > 1 \Rightarrow \cos 2\delta_\ell$ мало

$$\sigma(E) = 4\pi\int_0^\infty \rho(1-\cos 2\delta_\ell)d\rho = 4\pi\int_0^\infty \rho\, d\rho\left[1-\cos\left(\frac{m}{k\hbar^2}\int_{-\infty}^{\infty}U\left(\sqrt{\rho^2+z^2}\right)dz\right)\right]$$

⑥ 12.2.6. → Гольд. 13.

$$|\psi_i\rangle = \begin{pmatrix}1\\0\end{pmatrix}_n \begin{pmatrix}0\\1\end{pmatrix}_p = \cancel{\frac{1}{\sqrt2}\left[\frac{1}{\sqrt2}\right.}$$

$\uparrow^{\vec n}$ $\; n\uparrow \quad \downarrow p$

$$= \frac{1}{\sqrt2}\left[\frac{1}{\sqrt2}\underbrace{\left\{\begin{pmatrix}1\\0\end{pmatrix}_n\begin{pmatrix}0\\1\end{pmatrix}_p + \begin{pmatrix}0\\1\end{pmatrix}_n\begin{pmatrix}1\\0\end{pmatrix}_p\right\}}_{\text{трипл.}} + \frac{1}{\sqrt2}\underbrace{\left\{\begin{pmatrix}1\\0\end{pmatrix}_n\begin{pmatrix}0\\1\end{pmatrix}_p - \begin{pmatrix}0\\1\end{pmatrix}_n\begin{pmatrix}1\\0\end{pmatrix}_p\right\}}_{\text{синг.}}\right]$$

$$|\psi_f\rangle = \frac{e^{ikr}}{r}\frac{1}{\sqrt2}\left[f_3\frac{1}{\sqrt2}\left\{\begin{pmatrix}1\\0\end{pmatrix}_n\begin{pmatrix}0\\1\end{pmatrix}_p + \begin{pmatrix}0\\1\end{pmatrix}_n\begin{pmatrix}1\\0\end{pmatrix}_p\right\} + f_1\frac{1}{\sqrt2}\left\{\begin{pmatrix}1\\0\end{pmatrix}_n\begin{pmatrix}0\\1\end{pmatrix}_p - \begin{pmatrix}0\\1\end{pmatrix}_n\begin{pmatrix}1\\0\end{pmatrix}_p\right\}\right] =$$

← переориентация

$$= \frac{e^{ikr}}{r}\left[\underbrace{\frac{f_3+f_1}{2}}_{\text{ампл.}}\begin{pmatrix}1\\0\end{pmatrix}_n\begin{pmatrix}0\\1\end{pmatrix}_p + \underbrace{\frac{f_3-f_1}{2}}_{\text{ампл.}}\begin{pmatrix}0\\1\end{pmatrix}_n\begin{pmatrix}1\\0\end{pmatrix}_p\right]$$

Нормировка: $A^2\left(\frac{f_3+f_1}{2}\right)^2 + A^2\left(\frac{f_3-f_1}{2}\right)^2 = 1 \Rightarrow 1/A^2 = \frac{1}{4}(f_1^2+f_3^2+2f_1f_3+f_1^2+f_3^2-2f_1f_3)$

$1/A^2 = \frac{1}{2}(f_1^2+f_3^2) \Rightarrow A^2 = \frac{2}{(f_1^2+f_3^2)}$

Вероятность смены спина $P = A^2\frac{(f_3-f_1)^2}{4} = \frac{1}{2}\frac{(f_3-f_1)^2}{(f_1^2+f_3^2)}$

Дз на 17.12.19.

① → 13.1.1

$I = 10^8 \text{ Вт} \cdot \text{см}^{-2} = 10^{12} \text{ Вт}/\text{м}^2$

$\omega = 1{,}77 \cdot 10^{15}$ Гц

$\langle n \rangle = ?$

$$P = \frac{E}{t}; \quad I = \frac{P}{S}; \quad E = N\hbar\omega$$

$$P = \frac{N\hbar\omega}{t} \Rightarrow I = \frac{N\hbar\omega}{St} = \frac{N\hbar\omega c}{\underbrace{Sct}_{V}} = \underbrace{\frac{N}{V}}_{n}\hbar\omega c = n\hbar\omega c$$

$$n = \frac{I}{\hbar\omega c}$$

$$n = \frac{10^{12} \text{ Дж}/(\text{с} \cdot \text{м}^2)}{1{,}055 \cdot 10^{-34} \text{ Дж} \cdot \text{с} \cdot 1{,}77 \cdot 10^{15} \frac{1}{\text{с}} \cdot 3 \cdot 10^8 \frac{\text{м}}{\text{с}}} =$$

$$= 10^{23} \cdot 0{,}1786 \approx 1{,}7 \cdot 10^{22} \approx 2 \cdot 10^{22} \frac{1}{\text{м}^3}$$

---

② а) $\langle \alpha | \hat{x} | \alpha \rangle$ $\quad a|\alpha\rangle = \alpha|\alpha\rangle$

б) $\langle \alpha | \hat{p} | \alpha \rangle$ $\quad \langle\alpha| a^+ = \langle\alpha|\alpha^*$

в) D_x $\quad x = \frac{x_0}{\sqrt{2}}(a + a^+)$

г) D_p $\quad p = \frac{p_0}{i\sqrt{2}}(a - a^+)$

$$\langle \alpha | \hat{x} | \alpha \rangle = \langle \alpha | \frac{x_0}{\sqrt{2}}(a + a^+) | \alpha \rangle = \frac{x_0}{\sqrt{2}}\left(\langle\alpha|a|\alpha\rangle + \langle\alpha|a^+|\alpha\rangle\right) = \frac{x_0}{\sqrt{2}}(\alpha + \alpha^*)$$

$$\langle \alpha | \hat{p} | \alpha \rangle = \langle \alpha | \frac{p_0}{i\sqrt{2}}(a - a^+) | \alpha \rangle = \frac{p_0}{i\sqrt{2}}(\alpha - \alpha^*)$$

$$\langle \alpha | \hat{x}^2 | \alpha \rangle = \langle \alpha | \frac{x_0^2}{2}(a^2 + \underbrace{aa^+}_{1 + a^+a} + a^+a + a^{+2}) | \alpha \rangle = \frac{x_0^2}{2}\left(\alpha^2 + 1 + 2\alpha^*\alpha + (\alpha^*)^2\right) = \frac{x_0^2}{2}\left(1 + (\alpha + \alpha^*)^2\right)$$

$$= \frac{x_0^2}{2}\left(1 + \alpha^2 + (\alpha^*)^2 + 2|\alpha|^2\right)$$

$$\langle \alpha | \hat{p}^2 | \alpha \rangle = \langle \alpha | \frac{p_0^2}{i^2 2}(a^2 - \underbrace{aa^+ - a^+a}_{-(1 + aa^+)} + a^{+2}) | \alpha \rangle = \frac{p_0^2}{i^2 2}\left(\alpha^2 - 1 - 2\alpha^*\alpha + (\alpha^*)^2\right) = \frac{p_0^2}{i^2 2}\left(-1 + (\alpha - \alpha^*)^2\right) =$$

$$= \frac{p_0^2}{2}\left(1 - (\alpha - \alpha^*)^2\right)$$

$$D_x = \frac{x_0^2}{2}\left(1 + (\alpha + \alpha^*)^2\right) - \frac{x_0^2}{2}(\alpha + \alpha^*)^2 = \frac{x_0^2}{2}$$

$$D_p = \frac{p_0^2}{2}\left(1 - (\alpha - \alpha^*)^2\right) - \frac{p_0^2}{-2}(\alpha - \alpha^*)^2 = \frac{p_0^2}{2}$$

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③ 13.1.4 Предположим $a^+|\beta\rangle = \beta|\beta\rangle \rightarrow \sum_n c_n\sqrt{n+1}\,|n+1\rangle = \sum_n \beta\,|n\rangle c_n \Rightarrow \{n \to n+1\} \Rightarrow$

$\langle\beta| a = \langle\beta|\beta^*$

разложение $\quad |\beta\rangle = \sum_n c_n |n\rangle$

$$\Rightarrow \sum_n c_n\sqrt{n+1}\,|n+1\rangle = \sum_n \beta\,|n+1\rangle c_{n+1}$$

$$c_{n+1} = \frac{c_n\sqrt{n+1}}{\beta} \Rightarrow c_n = \frac{c_0\sqrt{n!}}{\beta^n}$$

Нормировка $\langle\beta|\beta\rangle = 1$

$$\langle\beta|\beta\rangle = \langle n|\sum_n c_n^* \cdot \sum_n c_n|n\rangle = \langle n|\sum_n \frac{c_0\sqrt{n!}}{(\beta^*)^n}\sum \frac{c_0\sqrt{n!}}{\beta^n}|n\rangle \Rightarrow \sum_{n=0}^{\infty}\frac{n!}{|\beta|^{2n}} = 1/c_0^2$$

Ряд расходится при любых конечных β $\quad$ ч.т.д.

④ $H = \hbar\omega(a^+a + 1/2) + i\hbar(ga^2 - g^*(a^+)^2)$

13.8

а) Представление Гайзенберга $\quad \dfrac{d}{dt}\hat{A}_r(t) = \dfrac{\partial}{\partial t}A_r(t) + \dfrac{i}{\hbar}[H, A_r(t)]$

$$\frac{d}{dt}\hat{a}_r(t) = 0 + \frac{i}{\hbar}[H, \hat{a}_r(t)]$$

$$[H, \hat{a}_r(t)] = [\hbar\omega(a_r^+a_r + 1/2) + i\hbar(g\underset{0}{a_r^2} - g^*(a_r^+)^2), a_r] = \hbar\omega[a_r^+a_r + 1/2, a_r] + i\hbar[ga_r^2, a_r] - i\hbar g^*[a_r^{+2}, a_r] =$$

$$= \hbar\omega[a_r^+a_r, a_r] + \hbar\omega[\overset{0}{1/2}, a_r] + i\hbar g[\overset{0}{a_r^2}, a_r] - i\hbar g^*[a_r^{+2}, a_r] = \hbar\omega a_r^+\cancel{[a_r, a_r]} + \hbar\omega\underset{=-1}{[a_r^+, a_r]}a_r - i\hbar g^* a_r^+\underset{=-1}{[a_r^+, a_r]} -$$

$$- i\hbar g^*\underset{=-1}{[a_r^+, a_r]}a_r^+ = -\hbar\omega a_r + 2i\hbar g^* a_r^+$$

$$\frac{d}{dt}\hat{a}_r(t) = \frac{i}{\hbar}(-\hbar\omega\hat{a}_r(t) + 2i\hbar g^* a_r^+(t)) = -i\omega\hat{a}_r(t) - 2g^* a_r^+(t)$$

$$\frac{d}{dt}\hat{a}_r^+(t) = 0 + \frac{i}{\hbar}[H, \hat{a}_r^+(t)]$$

$$[H, a_r^+(t)] = [\hbar\omega(a_r^+a_r + 1/2) + i\hbar(ga_r^2 - g^*(a_r^+)^2), a_r^+] = \hbar\omega\cancel{[a_r^+, a_r^+]}a_r + \hbar\omega a_r^+[a_r, a_r^+] + \hbar\omega\cancel{[1/2, a_r^+]} + i\hbar g\, a_r[a_r, a_r^+] +$$

$$+ i\hbar g[a_r, a_r^+]a - i\hbar g^*\cancel{[(a_r^+)^2, a_r^+]} = \hbar\omega a_r^+ + 2i\hbar g a_r$$

$$\frac{d}{dt}\hat{a}_r^+(t) = \frac{i}{\hbar}(\hbar\omega\hat{a}_r^+(t) + 2i\hbar g\hat{a}_r(t)) = i\omega\hat{a}_r^+(t) - 2g\hat{a}_r(t)$$

$$\left.\begin{aligned} \frac{d\hat{a}_r(t)}{dt} &= -i\omega a_r(t) - 2g^* a_r^+(t) \quad (1) \\ \frac{d\hat{a}_r^+(t)}{dt} &= i\omega a_r^+(t) - 2g\hat{a}_r(t) \quad (2) \end{aligned}\right\} \Rightarrow \hat{a}_r(t) = \frac{i\omega a_r^+(t)}{2g} - \frac{d\hat{a}_r^+(t)}{dt}\frac{1}{2g}$$

$$\frac{d}{dt}\left(\frac{i\omega a_r^+(t)}{2g} - \frac{d\hat{a}_r^+(t)}{dt}\frac{1}{2g}\right) = -i\omega\left(\frac{i\omega a_r^+(t)}{2g} - \frac{d}{dt}a_r^+(t)\frac{1}{2g}\right) - 2g^* a_r^+(t)$$

$$i\omega\dot{a}_r^+ - \ddot{a}_r^+ = \omega^2 a_r^+ + i\omega\dot{a}_r^+ - 4|g|^2 a_r^+(t)$$

$$\ddot{a}_r^+ + a_r^+(\omega^2 - 4|g|^2) = 0 \Rightarrow \ddot{a}_r^+ + a_r^+\bar{\omega}^2 = 0$$

$$a_r^+(t) = C_1\sin\bar{\omega}t + C_2\cos\bar{\omega}t \quad ; \quad a_r^+(0) = C_2$$

$$(2)\ C_1\bar{\omega}\cos\bar{\omega}t - C_2\bar{\omega}\sin\bar{\omega}t = C_1\bar{\omega}\cos\bar{\omega}t - \bar{\omega}\sin\bar{\omega}t \cdot a_r^+(0) = \frac{da_r^+(t)}{dt}$$

$$\left.\frac{da_r^+(t)}{dt}\right|_{t=0} = C_1\bar{\omega} = i\bar{\omega}\cdot a_r^+(0) - 2g\,a_r(0) \Rightarrow C_1 = \frac{\omega i}{\bar{\omega}}a_r^+(0) - \frac{2g}{\bar{\omega}}a_r(0)$$

$$a_r^+(t) = \frac{\omega i\, a_r^+(0)}{\bar{\omega}}\sin\bar{\omega}t + a_r^+(0)\cos\bar{\omega}t - \frac{2g}{\bar{\omega}}a_r(0)\sin\bar{\omega}t =$$

$$= a_r^+(0)\cos\bar{\omega}t + \left(a_r^+(0)\frac{i\omega}{\bar{\omega}} - \frac{2g}{\bar{\omega}}a_r(0)\right)\sin\bar{\omega}t$$

$(1),(2) \Rightarrow \hat{a}_r^+(t) = \left(-i\,\omega a_r(t) - \frac{da_r(t)}{dt}\right)\frac{1}{2g^*}$

$$\frac{d}{dt}\left(-i\omega a_r(t) - \frac{da_r(t)}{dt}\right)\frac{1}{2g^*} = i\frac{\omega}{2g^*}\left(-i\omega a_r(t) - \frac{da_r(t)}{dt}\right) - 2g a_r(t)$$

$$-i\omega\dot{a}_r(t) - \ddot{a}_r(t) = \omega^2 a_r(t) - i\omega\dot{a}_r(t) - 4|g|^2 a_r(t)$$

$$\ddot{a}_r(t) + a_r(t)(\omega^2 - 4|g|^2) = 0 \Rightarrow \ddot{a}_r(t) + a_r(t)\bar{\omega}^2 = 0$$

$$a_r(t) = C_1\sin\bar{\omega}t + C_2\cos\bar{\omega}t;\ a_r(0) = C_2$$

$$C_1\bar{\omega}\cos\bar{\omega}t - C_2\bar{\omega}\sin\bar{\omega}t = \frac{da_r(t)}{dt}$$

$$\left.\frac{da_r(t)}{dt}\right|_{t=0} = C_1\bar{\omega} = -i\omega a_r(0) - 2g^* a_r^+(t) \Rightarrow C_1 = -\frac{i\omega a_r(0)}{\bar{\omega}} - \frac{2g^* a_r^+(0)}{\bar{\omega}}$$

$$\begin{cases} a_r(t) = \left(-\frac{i\omega a_r(0)}{\bar{\omega}} - \frac{2g^* a_r^+(0)}{\bar{\omega}}\right)\sin\bar{\omega}t + a_r(0)\cos\bar{\omega}t \\ a_r^+(t) = \left(\frac{i\omega a_r^+(0)}{\bar{\omega}} - \frac{2g a_r(0)}{\bar{\omega}}\right)\sin\bar{\omega}t + a_r^+(0)\cos\bar{\omega}t \end{cases}$$

$$\hat{X} = \frac{x_0}{\sqrt{2}}(a+a^+) = \frac{x_0}{\sqrt{2}}\left[(a_r(0)+a_r^+(0))\cos\bar{\omega}t + (a_r^+(0)-a_r(0))\frac{i\omega}{\bar{\omega}}\sin\bar{\omega}t - \frac{2}{\bar{\omega}}(g^* a_r^+(0) + g a_r(0))\sin\bar{\omega}t\right]$$

$$\hat{P} = \frac{p}{i\sqrt{2}}(a-a^+) = \frac{p}{i\sqrt{2}}\left[(a_r(0)-a_r^+(0))\cos\bar{\omega}t - (a_r^+(0)+a_r(0))\frac{i\omega}{\bar{\omega}}\sin\bar{\omega}t - \frac{2}{\bar{\omega}}(g^* a_r^+(0) - g a_r(0))\sin\bar{\omega}t\right]$$

$$\langle\alpha|\hat{X}|\alpha\rangle = \frac{x_0}{\sqrt{2}}\left[(\alpha+\alpha^*)\cos\bar{\omega}t + (\alpha^*-\alpha)\frac{i\omega}{\bar{\omega}}\sin\bar{\omega}t - \frac{2}{\bar{\omega}}(g^*\alpha^* + g\alpha)\sin\bar{\omega}t\right]$$

$$\langle\alpha|\hat{P}|\alpha\rangle = \frac{p}{i\sqrt{2}}\left[(\alpha-\alpha^*)\cos\bar{\omega}t - (\alpha^*+\alpha)\frac{i\omega}{\bar{\omega}}\sin\bar{\omega}t - \frac{2}{\bar{\omega}}(g^*\alpha^* - g\alpha)\sin\bar{\omega}t\right]$$

$$\langle\alpha|\hat{X}^2|\alpha\rangle = \frac{x_0}{2}\Big[(a_r(0)+a_r^+(0))^2\cos^2\bar{\omega}t - (a_r^+(0)-a_r(0))^2\frac{\omega^2}{\bar{\omega}^2}\sin^2\bar{\omega}t + \frac{4}{\bar{\omega}^2}(g^* a_r^+(0) + g a_r(0))^2\sin^2\bar{\omega}t +$$
$$+ 2(a_r(0)+a_r^+(0))(a_r^+(0)-a_r(0))\frac{i\omega}{\bar{\omega}}\cos\bar{\omega}t\sin\bar{\omega}t + \{(a_r(0)+a_r^+(0))(g^* a_r^+(0) + g a_r(0)) + (g^* a_r^+(0) + g a_r(0))(a_r(0)+a_r^+(0))\}\cdot$$
$$\cdot\left(-\frac{2}{\bar{\omega}}\right)\sin\bar{\omega}t\cos\bar{\omega}t + \{(a_r^+(0)-a_r(0))(g^* a_r^+(0) + g a_r(0)) + (g^* a_r^+(0) + g a_r(0))(a_r^+(0)-a_r(0))\}\frac{i\omega\cdot(-2)}{\bar{\omega}^2}\sin^2\bar{\omega}t\Big] =$$

$$= \frac{x_0}{2}\Big[(a_r^2(0) + 1 + 2a^+a + a^{+2})\cos^2\bar{\omega}t - (a^{+2} + a^2 - a^+a - a a^+)\frac{\omega^2}{\bar{\omega}^2}\sin^2\bar{\omega}t + \frac{4}{\bar{\omega}^2}(g^{*2}a^{+2} + g^2a^2 + |g|^2a^+a + |g|^2 + |g|^2a^+a)\sin^2\bar{\omega}t +$$
$$+ 2(a^{+2} - a^2)\frac{i\omega}{\bar{\omega}}\cos\bar{\omega}t\sin\bar{\omega}t + \{g^* + g + 2ga^2 + 2g^*a^{+2} + 2ga^+a + 2g^*a^+a\}\left(-\frac{2}{\bar{\omega}}\right)\sin\bar{\omega}t\cos\bar{\omega}t +$$
$$+ \{2g^*a^{+2} - 2ga^2 + 2ga^+a - 2g^*a^+a + g - g^*\}\frac{i\omega(-2)}{\bar{\omega}^2}\sin^2\bar{\omega}t\Big] =$$

$$= \frac{x_0}{2}\Big[(\alpha^2 + 1 + 2\alpha^*\alpha + \alpha^{*2})\cos^2\bar{\omega}t - (\alpha^{*2} + \alpha^2 - 1 - 2\alpha\alpha^*)\frac{\omega^2}{\bar{\omega}^2}\sin^2\bar{\omega}t + \frac{4}{\bar{\omega}^2}(g^{*2}\alpha^{*2} + g^2\alpha^2 + 2|g|^2|\alpha|^2 + |g|^2)\sin^2\bar{\omega}t +$$
$$+ 2(\alpha^{*2} - \alpha^2)\frac{i\omega}{\bar{\omega}}\cos\bar{\omega}t\sin\bar{\omega}t + \{g^* + g + 2g\alpha^2 + 2g^*\alpha^{*2} + 2g|\alpha|^2 + 2g^*|\alpha|^2\}\left(-\frac{2}{\bar{\omega}}\right)\sin\bar{\omega}t\cos\bar{\omega}t +$$
$$+ \{2g^*\alpha^{*2} - 2g\alpha^2 + 2g|\alpha|^2 - 2g^*|\alpha|^2 + g - g^*\}\frac{i\omega(-2)}{\bar{\omega}^2}\sin^2\bar{\omega}t\Big] = x_0$$

$$= \frac{x_0}{2}\Big[\{(\alpha+\alpha^*)^2 + 1\}\cos^2\bar{\omega}t - \{(\alpha^*-\alpha)^2 - 1\}\frac{\omega^2}{\bar{\omega}^2}\sin^2\bar{\omega}t + \frac{4}{\bar{\omega}^2}\{(\alpha^*g^* + \alpha g)^2 + |g|^2\}\sin^2\bar{\omega}t +$$
$$+ 2(\alpha^{*2} - \alpha^2)\frac{i\omega}{\bar{\omega}}\cos\bar{\omega}t\sin\bar{\omega}t + \{g^* + g + 2(\alpha+\alpha^*)(g\alpha + g^*\alpha^*)\}\left(-\frac{2}{\bar{\omega}}\right)\sin\bar{\omega}t\cos\bar{\omega}t +$$
$$+ \{g - g^* + 2(\alpha-\alpha^*)(g^*\alpha^* + g\alpha)\}\frac{i\omega(-2)}{\bar{\omega}^2}\sin^2\bar{\omega}t$$

$$\langle\alpha|\hat{x}|\alpha\rangle^2 = \frac{x_0}{2}\Big[(\alpha+\alpha^*)^2\cos^2\bar\omega t + (\alpha-\alpha^*)^2\frac{i\omega^2}{\bar\omega^2}\sin\bar\omega t + \frac{4}{\bar\omega^2}(g^*\alpha^*+g\alpha)^2\sin^2\bar\omega t + 2(\alpha^{*2}-\alpha^2)\cos\bar\omega t\sin\bar\omega t\cdot\frac{i\omega}{\bar\omega} +$$

$$+ 2(\alpha+\alpha^*)(g^*\alpha^*+g\alpha)\left(\frac{-2}{\bar\omega}\right)\cos\bar\omega t\sin\bar\omega t + 2(\alpha^*-\alpha)(g^*\alpha^*+g\alpha)\frac{i\omega(-2)}{\bar\omega^2}\sin^2\bar\omega t\Big]$$

$$\sigma = \langle\alpha|\hat{x}^2|\alpha\rangle - \langle\alpha|\hat{x}|\alpha\rangle^2 = \frac{x_0^2}{2}\Big[\cos^2\bar\omega t + \frac{\omega^2}{\bar\omega^2}\sin^2\bar\omega t + \frac{4}{\bar\omega^2}|g|^2\sin^2\bar\omega t + (g^*+g)\left(-\frac{2}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t +$$

$$+(g-g^*)\left(\frac{-2i\omega}{\bar\omega^2}\right)\sin^2\bar\omega t\Big] = \frac{x_0^2}{2}\Big[\cos^2\bar\omega t + \frac{\omega^2+4|g|^2}{\omega^2-4|g|^2}\sin^2\bar\omega t + \cancel{\ldots} - \frac{2}{\bar\omega}\sin\bar\omega t\Big(g^*\big(\cos\bar\omega t - \frac{i\omega}{\bar\omega}\sin\bar\omega t\big) +$$

$$+ g\big(\cos\bar\omega t + \frac{i\omega}{\bar\omega}\sin\bar\omega t\big)\Big)$$ хрень какая-то. Силаев сказал что D_x и D_p постоянны, хотя у Елютина написано, что нет зависимости от времени

$$\langle\alpha|\hat{p}^2|\alpha\rangle = -\frac{p_0^2}{2}\Big[(a^2+a^{+2}-2a^+a-1)\cos^2\bar\omega t - (a^2+a^{+2}+2a^+a+1)\frac{\omega^2}{\bar\omega^2}\sin^2\bar\omega t + \frac{4}{\bar\omega^2}(g^{*2}a^{+2}+g^2a^2-2|g|^2a^+a-|g|^2)\sin^2\bar\omega t +$$

$$+2(a-a^+)(a^++a)\left(-\frac{i\omega}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t + \{(a-a^+)(g^*a^+-ga)+(g^*a^+-ga)(a-a^+)\}\left(\frac{-2}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t +$$

$$+\{(a^++a)(g^*a^+-ga)+(g^*a^+-ga)(a^++a)\}\frac{2i\omega}{\bar\omega^2}\sin^2\bar\omega t\Big] =$$

$$= \frac{-p_0^2}{2}\Big[(a^2+a^{+2}-2a^+a-1)\cos^2\bar\omega t - (a^2+a^{+2}+2a^+a+1)\frac{\omega^2}{\bar\omega^2}\sin^2\bar\omega t + \frac{4}{\bar\omega^2}(g^{*2}a^{+2}+g^2a^2-2|g|^2a^+a-|g|^2)\sin^2\bar\omega t +$$

$$+2(a^2-a^{+2})\left(-\frac{i\omega}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t + \{g^*+g+2g^*a^+a+2ga^+a-2ga^2-2g^*a^{+2}\}\left(-\frac{2}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t +$$

$$+\{g^*-g+2g^*a^{+2}-2ga^2-2ga^+a+2g^*a^+a\}\frac{2i\omega}{\bar\omega^2}\sin^2\bar\omega t\Big] =$$

$$= -\frac{p_0^2}{2}\Big[(\alpha^2+\alpha^{*2}-2\alpha^*\alpha-1)\cos^2\bar\omega t - (\alpha^2+\alpha^{*2}+2\alpha^*\alpha+1)\frac{\omega^2}{\bar\omega^2}\sin^2\bar\omega t + \frac{4}{\bar\omega^2}(g^{*2}\alpha^{*2}+\alpha^2g^2-2|g|^2\alpha^*\alpha-|g|^2)\sin^2\bar\omega t +$$

$$+2(\alpha^2-\alpha^{*2})\left(-\frac{i\omega}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t + \{g^*+g+2g^*\alpha^*\alpha+2g\alpha^*\alpha-2g\alpha^2-2g^*\alpha^{*2}\}\left(-\frac{2}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t +$$

$$+\{g^*-g+2g^*\alpha^{*2}-2g\alpha^2-2g\alpha^*\alpha-2g^*\alpha^*\alpha\}\frac{2i\omega}{\bar\omega^2}\sin^2\bar\omega t\Big] =$$

$$= -\frac{p_0^2}{2}\Big[\{(\alpha-\alpha^*)^2-1\}\cos^2\bar\omega t - \{(\alpha+\alpha^*)^2+1\}\frac{\omega^2}{\bar\omega^2}\sin^2\bar\omega t + \frac{4}{\bar\omega^2}(\{g^*\alpha^*-g\alpha\}^2-|g|^2)\sin^2\bar\omega t +$$

$$+2(\alpha^2-\alpha^{*2})\left(-\frac{i\omega}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t + \{2(\alpha-\alpha^*)(g^*\alpha^*-g\alpha)+g^*+g\}\left(-\frac{2}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t +$$

$$+\{2(\alpha^*+\alpha)(g^*\alpha^*-g\alpha)+g^*-g\}\frac{2i\omega}{\bar\omega^2}\sin^2\bar\omega t\Big]$$

$$\langle\alpha|\hat{p}|\alpha\rangle^2 = -\frac{p_0^2}{2}\Big[(\alpha-\alpha^*)^2\cos^2\bar\omega t - (\alpha+\alpha^*)^2\frac{\omega^2}{\bar\omega^2}\sin^2\bar\omega t + \frac{4}{\bar\omega^2}(g^*\alpha^*-g\alpha)^2\sin^2\bar\omega t +$$

$$+2(\alpha^2-\alpha^{*2})\left(-\frac{i\omega}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t + 2(\alpha-\alpha^*)(g^*\alpha^*-g\alpha)\left(-\frac{2}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t +$$

$$+2(\alpha^*+\alpha)(g^*\alpha^*-g\alpha)\frac{2i\omega}{\bar\omega^2}\sin^2\bar\omega t\Big]$$

$$\sigma = \langle\alpha|\hat{p}^2|\alpha\rangle - \langle\alpha|\hat{p}|\alpha\rangle^2 = -\frac{p_0^2}{2}\Big[-\cos^2\bar\omega t - \frac{\omega^2}{\bar\omega^2}\sin^2\bar\omega t - \frac{4}{\bar\omega^2}|g|^2\sin^2\bar\omega t +$$

$$+(g^*+g)\left(-\frac{2}{\bar\omega}\right)\sin\bar\omega t\cos\bar\omega t + (g^*-g)\frac{2i\omega}{\bar\omega^2}\sin^2\bar\omega t\Big] = \frac{p_0^2}{2}\Big[\cos^2\bar\omega t + \frac{\omega^2}{\bar\omega^2}\sin^2\bar\omega t + \frac{4|g|^2}{\bar\omega^2}\sin^2\bar\omega t +$$

$$+(g^*+g)\frac{2}{\bar\omega}\sin\bar\omega t\cos\bar\omega t - (g^*-g)\frac{2i\omega}{\bar\omega^2}\sin^2\bar\omega t\Big] = \Big[\cos^2\bar\omega t + \frac{\omega^2+4|g|^2}{\omega^2-4|g|^2}\sin^2\bar\omega t +$$

$$+\frac{2\sin\bar\omega t}{\bar\omega}\Big(g^*\big(\cos - \frac{i\omega}{\bar\omega}\sin\bar\omega t\big) + g\big(\cos\bar\omega t + \frac{i\omega}{\bar\omega}\sin\bar\omega t\big)\Big)\Big]$$ та же хрень.

13.1.7

(5) $H=\hbar\omega(a^+a+1/2)-f(t)\frac{x_0}{\sqrt{2}}(a+a^+)$ Списано у Дворянова

$|\psi(0)\rangle=|\alpha\rangle$

$e^{\mu(t)}e^{a^+\nu(t)}|0\rangle$

$|\psi(t)\rangle=?$

$P_{nn'}=?$

Чемютина: $\hat{H}=\underbrace{\hbar\omega a^+a}_{H_0}+\underbrace{\hbar f(t)(\hat{a}^++\hat{a})}_{V}$

$i\hbar\frac{\partial\psi}{\partial t}=(\hat{H}_0+\hat{V})\psi$

$\psi(t)=\exp\left(\frac{-iH_0t}{\hbar}\right)\psi_0(t)=e^{-i\omega a^+at}\psi_0(t)$

$i\hbar\frac{\partial\psi}{\partial t}=H_0\psi+i\hbar\frac{\partial\psi_0}{\partial t}e^{-i\omega a^+at}=H_0\psi+Ve^{-i\omega a^+at}\psi_0$

представление взаимодействия

$i\hbar\frac{\partial\psi_0}{\partial t}=\hat{W}\psi_0$, где $\hat{W}=e^{i\omega a^+at}Ve^{-i\omega a^+at}$

Найдем: $e^{i\omega a^+at}ae^{-i\omega a^+at}=e^{i\omega a^+at}a\sum\frac{(-i\omega a^+at)^n}{n!}=e^{i\omega a^+at}e^{-i\omega a^+at}a=$

$=e^{i\omega t}a$

$\{a(a^+a)^n=(aa^+aa^+aa^+a\ldots)a=(aa^+)^na\}$

$[\hat{a}^+a]=a^+a-aa^+=1$

$\hat{D}(\alpha)=\exp(\alpha a^+-\alpha^*a)$ оператор сдвига

$\frac{\partial\psi_0}{\partial t}=-if(t)[e^{i\omega t}\hat{a}^++e^{-i\omega t}\hat{a}]\psi_0(t)$

$\psi_0(t)=D(\beta)\psi_0(0)\Rightarrow\beta=-i\int_0^t f(\tau)e^{i\omega\tau}d\tau$, т.е. $|\alpha\rangle\to|\alpha+\beta\rangle$

(6) $|\alpha_{k_0p_0}\rangle=e^{-\frac{|\alpha|^2}{2}}e^{\alpha a^+_{k_0p_0}}|0\rangle$

13.1.6

$\langle\bar{E}\rangle=?$

D_{Ex}

D_{Ey}

D_{Ez}

~~$\langle\alpha_{k_0p_0}|E_z(x,t)|\alpha_{k_0p}\rangle=\langle\alpha_{k_0p_0}|\sum_{k,p}\sqrt{\frac{2\pi\hbar c^2}{\omega_kL^3}}(\bar{e}_{kp})_z(a_{kp}e^{ik\bar{x}-i\omega_kt}+a^+_{kp}e^{-ik\bar{x}+i\omega_kt})|\alpha_{k_0p_0}\rangle=$~~

~~$=\{k\to k_0, p\to p_0\}=\langle\alpha|(|\alpha|e^{i\varphi}e^{ik_0\bar{x}-i\omega_kt}+|\alpha|e^{-i\varphi}e^{-ik\bar{x}+i\omega_kt})|\alpha_{k_0p_0}\rangle$~~

~~$=2|\alpha|\cos(k\bar{x}-\omega_kt+\varphi)\sqrt{\frac{2\pi\hbar c^2}{\omega_kL^3}}(\bar{e}_{k_0p_0})_z$~~

$\bar{A}(x,t)=\sum_{\bar{k},p}\sqrt{\frac{2\pi\hbar c^2}{\omega_kL^3}}(\bar{e}_{kp})_z(\hat{a}_{kp}e^{i\bar{k}\bar{x}-i\omega_kt}+\hat{a}^+_{kp}e^{(i\bar{k}\bar{x}-i\omega_kt)})$

$E=-\frac{1}{c}\frac{\partial\bar{A}}{\partial t}=\sum_{\bar{k},p}\sqrt{\frac{2\pi\hbar c^2}{\omega_kL^3}}(\bar{e}_{kp})_z(\hat{a}_{kp}e^{i\bar{k}\bar{x}-i\omega_kt}\cdot\frac{i\omega_k}{c}-a^+_{kp}e^{(i\bar{k}\bar{x}-i\omega_kt)}\cdot\frac{i\omega_k}{c})$

$\langle\alpha_{k_0p_0}|E_z(\bar{x},t)|\alpha_{k_0p}\rangle=\langle\alpha_{k_0p}|\sum_{k,p}\sqrt{\frac{2\pi\hbar c^2}{\omega_kL^3}}(\bar{e}_{kp})_z(\hat{a}_{kp}e^{i\bar{k}\bar{x}-i\omega_kt}-a^+_{kp}e^{-(i\bar{k}\bar{x}-i\omega_kt)})\cdot\frac{i\omega_k}{c}|\alpha_{k_0p}\rangle=\{k\to k_0, p\to p_0\}=$

$=\sqrt{\frac{2\pi\hbar c^2}{\omega_{k_0}L^3}}(\bar{e}_{k_0p})_z\cdot\frac{i\omega_k}{c}\left(\frac{|\alpha|e^{i\varphi}e^{i\bar{k}\bar{x}-i\omega_kt}-|\alpha|e^{-i\varphi}e^{-(i\bar{k}\bar{x}-i\omega_kt)}}{2i}\right)2i=-\sqrt{\frac{2\pi\hbar c^2}{\omega_{k_0}L^3}}|\alpha|\frac{2\omega_k}{c}(\bar{e}_{k_0p})\sin(\bar{k}\bar{x}-\omega t+\varphi)$

$\langle\alpha_{k_0p_0}|E_z(\bar{x},t)|\alpha_{k_0p}\rangle^2=\frac{2\pi\hbar c^2|\alpha|^24\omega^2_{k_0}}{\omega_kL^3c^2}(\bar{e}_{k_0p})^2_z\sin^2(\bar{k}\bar{x}-\omega t+\varphi)=\frac{2\pi\hbar c^2}{\omega_{k_0}L^3}\frac{\omega^2_{k_0}}{c^2}(\bar{e}_{k_0p})^2_z4|\alpha|^2\sin^2\psi$

$$E^2 = \frac{2\pi\hbar c^2}{\omega_k L^3}(\bar e_{\bar k p})_z^2\left(a_{\bar k p}^2 e^{2(i\bar k\bar x - i\omega t)} - 2a^+a - 1 + a_{\bar k p}^{2+} e^{-2(i\bar k\bar x - i\omega t)}\right)\frac{i^2\omega_k^2}{c^2}$$

$$\langle\alpha|E^2|\alpha\rangle = \frac{2\pi\hbar c^2}{\omega_k L^3}\frac{i^2\omega_k^2}{c^2}(\bar e_{\bar k p})_z^2\left(|\alpha|^2 e^{2i\varphi} e^{2(i\bar k\bar x - i\omega t)} - 2|\alpha|^2 e^{i\varphi}e^{-i\varphi} - 1 + |\alpha|^2 e^{-2i\varphi} e^{-2(i\bar k\bar x - i\omega t)}\right) =$$

$$= \frac{2\pi\hbar c^2}{\omega_k L^3}\frac{i^2\omega_k^2}{c^2}(\bar e_{\bar k p})_z^2\left(2|\alpha|^2\cos 2\psi - 2|\alpha|^2 - 1\right) = \frac{2\pi\hbar c^2}{\omega_k L^3}\frac{\omega_k^2}{c^2}(\bar e_{\bar k p})_z^2\left(1 + 2|\alpha|^2(1-\cos 2\psi)\right) =$$

$$= \frac{2\pi\hbar c^2}{\omega_k L^3}\frac{\omega_k^2}{c^2}(\bar e_{\bar k p})_z^2\left(1 + 4|\alpha|^2\sin^2\psi\right)$$

$$D_{E_z} = \langle\alpha|E^2|\alpha\rangle - \langle\alpha|E|\alpha\rangle^2 = \frac{2\pi\hbar c^2}{\omega_k L^3}\frac{\omega_k^2}{c^2}(\bar e_{\bar k p})_z^2 = \frac{2\pi\hbar\omega_k}{L^3}(\bar e_{\bar k p})_z^2$$

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(7) 13.1.9

$\tau = 100$ фмс

$P_{2\varphi} \le \frac{1}{100} P_{1\varphi}$

$\lambda = 550$ нм

$W = ?$

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha a^+}|0\rangle$$

$$\left.\begin{aligned}|\alpha\rangle &= \sum_n C_n|n\rangle\\ C_n &= C_0\frac{\alpha^n}{\sqrt{n!}}\\ C_0 &= \exp(-|\alpha|^2/2)\end{aligned}\right\}\quad |\alpha\rangle = \sum_n e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

$$P_n = |\langle n|\alpha\rangle|^2 = \frac{|\alpha|^{2n}}{n!}\exp(-|\alpha|^2)$$

$$P_{n=1} = |\alpha|^2\exp(-|\alpha|^2)$$

$$P_{n=2} = \frac{|\alpha|^4}{2}\exp(-|\alpha|^2)$$

$$P_{n=2} \le 0{,}01 P_{n=1} \Rightarrow \frac{|\alpha|^4}{2}\exp(-|\alpha|^2) \le \frac{1}{100}|\alpha|^2\exp(-|\alpha|^2)$$

$$|\alpha|^2 \le \frac{1}{50}$$

$$W = \frac{\mathcal{E}}{\tau};\ \langle W\rangle = \frac{\langle\mathcal{E}\rangle}{\tau} = \frac{1}{\tau}\langle\underbrace{\frac{E^2}{4\pi}}_{w}\rangle\cdot\underbrace{L^3}_{V} = \frac{L^3}{4\pi\tau}\langle E^2\rangle$$

$$\langle E^2\rangle = \frac{2\pi\hbar c^2}{\omega_k L^3}\frac{\omega_k^2}{c^2}(\bar e_{\bar k p})_z^2\left(1 + 4|\alpha|^2\sin^2(\bar k\bar x - \omega t + \varphi)\right)$$

$$\langle\sin^2\rangle = \frac{1}{2}$$

$$\langle W\rangle = \frac{L^3}{4\pi\tau}\cdot\frac{2\pi\hbar\omega}{L^3}(1+2|\alpha|^2) = \frac{\hbar\omega}{2\tau}(1+2|\alpha|^2) = \frac{\hbar c\cdot 2\pi}{2\tau\lambda}(1+2|\alpha|^2) = \frac{hc}{2\tau\lambda}(1+2|\alpha|^2)$$

$$\langle W\rangle = \frac{6{,}626\cdot 10^{-34}\,\text{Дж}\cdot\text{с}\cdot 3\cdot 10^8\,\frac{\text{м}}{\text{с}}}{2\cdot\underbrace{100\cdot 10^{-15}}_{10^{-13}}\text{с}\cdot 550\cdot 10^{-9}\,\text{м}}(1 + 2/50) = 10^{-4}\cdot 18{,}1\cdot 10^{-3}\cdot 1{,}04 = 18{,}8\cdot 10^{-7} \simeq 2\cdot 10^{-6}\,\frac{\text{Дж}}{\text{с}}$$

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(8) 13.2.3 Бесспин. водород на $2p \to 1s$ Переход E1 $\quad x = \frac{x_0}{\sqrt 2}(a + a^+)$

$\tau = ?$

$\frac{dI}{d\Omega} = ?$

$$\frac{1}{\tau} = \frac{e^2\omega^3}{2\pi\hbar c^3}\sum_p\int d\Omega_k\left|\langle\Psi_2|\bar x|\Psi_1\rangle\bar e_{\bar k p}\right|^2$$

$$\Psi_1 = \Psi_{2p}(r)|0\rangle$$

$$\Psi_{2p}(r) = \Psi_{210} = \sqrt{\frac{3}{4\pi}}\cos\theta\frac{r}{2\sqrt{6a^5}}e^{-r/2a} = \sqrt{\frac{1}{4\pi\cdot 8a^5}}\cos\theta\, r\, e^{-r/2a}$$

$$\langle\Psi_2 = \langle 0|\Psi_{1s}$$

$$\Psi_{1s} = \Psi_{100} = \sqrt{\frac{1}{\pi a^3}}e^{-r/a}$$

$$\langle\Psi_2|\bar x|\Psi_1\rangle = \int d^3r\,\Psi_{100}^*\,\bar r\,\Psi_{210} = \frac{1}{\pi a^4\cdot 4\sqrt 2}\int e^{-r/a}\cdot\overbrace{(r\cos\theta)}^{\bar z}\cdot\cos\theta\, r\, e^{-r/2a}\cdot r^2 dr\cdot\sin\theta d\theta d\varphi =$$

$$= \frac{1}{4\pi\sqrt{2}a^4}\int\limits_0^\infty r^4 e^{-r\cdot 3/2a_0}\int\limits_{-1}^{1}\cos^2\theta\, d(\cos\theta)\cdot\int d\varphi = \frac{1}{4\pi\sqrt{2}a^4}\,\frac{2^8}{3^4}a^5\cdot\frac{2}{3}\cdot 2\pi = \frac{2^7\sqrt{2}}{3^5}a$$

две поляр. фотона

$$\frac{1}{\tau} = 2\frac{e^2\omega^3}{2\pi\hbar c^3}\underbrace{\int d\Omega_k}_{\frac{4\pi}{3}}\cdot\left(\frac{2^7\sqrt{2}a}{3^5}\right)^2 = \frac{e^2\omega^3}{\pi\hbar c^3}\cdot\frac{4\pi}{3}\left(\frac{2^7\sqrt{2}a}{3^5}\right)^2 = \frac{e^2\omega^3}{\hbar c^3}a^2\cdot\frac{4}{3}\left(\frac{2}{3}\right)^{10}\cdot 32$$

$$\omega = \frac{1}{\hbar}\left(E_{2p}-E_{1s}\right) = \frac{3}{8}\frac{me^4}{\hbar^3} = \frac{3}{4\hbar}Ry$$

$$\frac{1}{\tau} = \frac{e^2}{\hbar c^3}\cdot\frac{4}{3}\left(\frac{2}{3}\right)^{10}\cdot 32\cdot\left(\frac{3}{2}\right)^3\frac{1}{2^6}\frac{m^3e^{12}}{\hbar^6}\cdot\underbrace{\frac{\hbar^2}{me^2}}_{a}\cdot a = \left(\frac{2}{3}\right)^8\left(\frac{e^2}{\hbar c}\right)^4\frac{c}{a}$$

$$\tau \approx 1{,}6\cdot 10^{-9}\,c$$

$$\frac{dI}{d\Omega} = 2\cdot\frac{e^2\omega^4}{2\pi c^3}\sum_p |\langle\psi_2|\bar{x}|\psi_1\rangle\,\bar{e}_{kp}|^2 = \frac{\hbar\omega}{\pi 4/3}\,\frac{1}{\tau}$$

$$\frac{dI}{d\Omega} = \frac{3/4\,\frac{Ry}{\hbar}\cdot\hbar}{4\pi/3\cdot 1{,}6\cdot 10^{-9}\,c} = \frac{Ry\cdot\frac{9}{16\pi}}{1{,}6\cdot 10^{-9}} = \frac{13{,}6\,\text{эВ}\cdot\frac{9}{16\pi}}{1{,}6\cdot 10^{-9}\,c} = 1{,}52\cdot 10^{9}\,\frac{\text{эВ}}{c} = 2{,}4\cdot 10^{-10}\,\frac{\text{Дж}}{c}$$

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9 M1 $S=1/2$ в $\mathcal{H}$

13.2.4

14.7

$$H_I = -2\mu_0 S_z H_z$$

$——|\downarrow\rangle\ +\mu_0 H_z$ $\qquad |\psi_1\rangle = |\downarrow\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

$——|\uparrow\rangle\ -\mu_0 H_z$ $\qquad |\psi_2\rangle = |\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

$\tau = ?$

$\frac{dI}{d\Omega} = ?$

$$\frac{1}{\tau} = \frac{e^2\omega}{2\pi\hbar c^3}\sum_p\int d\Omega_k\left|[\bar{e}_{kp}\times\bar{k}]\,\langle\psi_2|\frac{\hbar}{2m}(\underset{0}{\underset{\|}{\bar{l}}}+2\bar{s})|\psi_1\rangle\right|^2$$

$$\hbar\omega = E_2 - E_1 = 2\mu_0 H_0\ ;\ \bar{s} = \tfrac{\hbar}{2}\bar{\sigma}$$

$$|k| = \frac{\omega}{c}$$

$$\frac{1}{\tau} = \frac{e^2\omega}{2\pi\hbar c^3}\sum_p\underbrace{\int d\Omega_k}_{\frac{4\pi}{3}}\left|[\bar{e}_{kp}\times\bar{k}]\,\langle\psi_2|\frac{\hbar}{2m}\bar{\sigma}|\psi_1\rangle\right|^2$$

две поляризац.

$$\sum_{p=1}^{2}\left|[\bar{e}_{kp}\times\bar{k}]\,\langle{\textstyle{1\atop 0}}|\frac{\hbar}{2m}\bar{\sigma}|{\textstyle{0\atop 1}}\rangle\right|^2 = 2\left||k|\cdot\frac{\hbar}{2m}\cdot\bar{\sigma}_{12}\right|^2 = 2\cdot|k|^2\cdot\frac{\hbar^2}{4m^2}\cdot|\bar{\sigma}_{12}|^2 = \frac{2\omega^2}{c^2}\frac{\hbar^2}{4m^2}|\bar{\sigma}_{12}|^2$$

$$\sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix};\ \sigma_y = \begin{pmatrix}0&-i\\i&0\end{pmatrix},\ \sigma_z = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$(\sigma_{12})_x = 1\ ;(\sigma_{12})_y = -i\quad(\sigma_{12})_z = 0 \qquad \Rightarrow |\sigma_{12}|^2 = 2$$

$$\frac{1}{\tau} = \frac{e^2\omega}{2\pi\hbar c^3}\cdot\frac{4\pi}{3}\cdot\frac{2\omega^2}{c^2}\frac{\hbar^2}{4m^2}2 = \frac{8}{3}\frac{\omega^3}{\hbar c^3}\frac{e^2\hbar^2}{4m^2c^2} = \frac{8\omega^3\mu_0^2}{3\hbar c^3}\ ;\ \omega = \frac{2\mu_0 H_0}{\hbar}$$

$$\frac{e^2\hbar^2}{4m^2c^2} = \mu_0^2$$

$$\frac{1}{\tau} = \frac{64\,\mu_0^5\mathcal{H}^3}{3\,\hbar^4c^3}$$

$$\frac{dI}{d\Omega} = \frac{\hbar\omega}{4\pi/3}\frac{1}{\tau} = \frac{64\mu_0^5\mathcal{H}^3}{3\hbar^4c^3}\,\frac{2\mu_0 H_0}{4\pi/3} = \frac{32\,\mu_0^6 H_0^4}{\hbar^4c^3}$$

(10) $2S_{1/2} \to 2P_{1/2}$ E1

13.2.1.
14.4.

$\tau = ?$

$2S_{1/2}$ —— ΔE_{λ} —— $2p_{1/2}$, —— $2p_{3/2}$

$\Delta E_\lambda = 7\cdot 10^{-18}$ эрг $= 7\cdot 10^{-25}$ Дж $\Rightarrow \omega = \frac{\Delta E}{\hbar}$

$$\psi_1 = \psi_{200} = \sqrt{\frac{1}{4\pi}}\,\frac{r}{2\sqrt{6a^5}}\,e^{-r/2a}$$

$$\psi_2 = \psi_{210} = \sqrt{\frac{3}{4\pi}}\cos\theta\,\frac{r}{2\sqrt{6a^5}}\,e^{-r/2a}$$

$$\frac{1}{\tau} = \frac{e^2\omega^3}{2\pi\hbar c^3}\sum_p \int d\Omega_k \left|\langle\psi_2|\bar{x}|\psi_1\rangle \bar{e}_{\bar{k}p}\right|^2$$

$$\langle\psi_2|\bar{x}|\psi_1\rangle = \int d^3r\,\psi_{210}^*\,\bar{r}\,\psi_{200} = \frac{\sqrt{3}}{4\pi\, 24a^5}\int e^{-r/a} r^3\cos^2\theta\cdot r^2dr\, d(-\cos\theta)\, d\varphi =$$

$$= \frac{\sqrt{3}}{4\pi\cdot 24a^5}\int_0^\infty e^{-r/a}r^5dr\int_{-1}^{1}\cos^2\theta\, d(\cos\theta)\cdot\int_0^{2\pi}d\varphi = \frac{\sqrt{3}}{4\pi\cdot 24a^5}\cdot 120a^6\cdot\frac{2}{3}\cdot 2\pi = \frac{\sqrt{3}\,a}{3\cdot 5} = \frac{a}{5\sqrt{3}}$$

поляриз.

$$\frac{1}{\tau} = \frac{e^2\cdot\omega^3}{2\pi\hbar c^3}\cdot 2\cdot\frac{4\pi}{3}\cdot\frac{a^2}{25\cdot 3} = \frac{4e^2\omega^3a^2}{9\cdot 25\hbar c^3} = \frac{4e^2a^2\Delta E^3}{9\cdot 25\hbar c^3\hbar^3} = \frac{4}{9\cdot 25}\left(\frac{e^2}{\hbar c}\right)\frac{a^2\Delta E^3}{\hbar^3c^2} = \frac{4}{9\cdot 25}\frac{e^2}{\hbar c}\frac{a^2}{c^2}\left(\frac{\Delta E}{\hbar}\right)^3$$

$$\frac{1}{\tau} = \frac{4}{9\cdot 25}\,\frac{1}{137}\left(\frac{0{,}529\cdot 10^{-8}}{3\cdot 10^{10}}\right)^2\cdot\left(\frac{7\cdot 10^{-18}}{1{,}0546\cdot 10^{-27}}\right)^3 \approx 1{,}8\cdot 10^{-12}$$