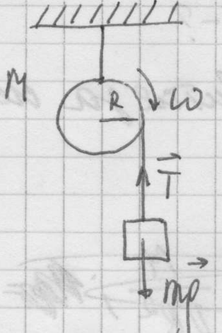
6.1



$$\begin{cases} mg - T = ma & \text{— II з-н Ньют., ур-е вращ. дв-я}\\ J\varepsilon = TR & \text{, } a = \varepsilon R \text{ (кин. связь)} \end{cases}$$

$T = m(g-a)$; $J = \frac{MR^2}{2}$ — как диск в силу невлияния распред. масс вдоль оси вращения.

$$m(g-a)R = \frac{MR^2}{2}\cdot\frac{a}{R} \Rightarrow a\left(m+\frac{M}{2}\right) = mg;$$

$$a = \frac{mg}{m+M/2}; \quad \varepsilon = \frac{mg}{R(m+M/2)} \quad (1)$$

def: $a = \frac{dv}{dt}$; $\varepsilon = \frac{d\omega}{dt}$;

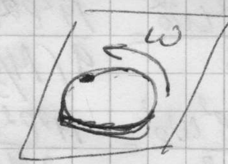
$$\frac{dv}{dt} = \frac{mg}{m+M/2} \Rightarrow v = \frac{g}{1+\frac{M}{2m}}t; \quad \omega = \frac{g}{R(1+M/2m)}t \quad (2)$$

$$W_k = \frac{J\omega^2}{2} + \frac{mv^2}{2};$$

$$W_k = \frac{MR^2}{4}\cdot\frac{g^2t^2}{R^2(1+M/2m)^2} + \frac{m}{2}\cdot\frac{g^2t^2}{(1+M/2m)^2};$$

$$W_k = \frac{1}{2}g^2t^2m\frac{\left(\frac{M}{2m}+1\right)}{\left(1+\frac{M}{2m}\right)^2} = \frac{mg^2t^2}{2\left(1+\frac{M}{2m}\right)};$$

$$W_k(t) = \frac{mg^2t^2}{2+M/m};$$

6.2. $R, \omega, \mu, N_{об} - ?$ 

$$J\frac{d\omega}{dt} = M_{тр.д.}$$

$$F_{тр} = dm \cdot \mu g; \quad dm = 2\pi r \cdot dr \, \frac{M}{\pi R^2} = 2Mrdr/R^2$$

$$M_{тр} = F_{тр} \cdot r = dm \cdot \mu g \cdot r = 2Mrdr/R^2 \cdot \mu g$$

$$M_{тр.диск} = \int_0^R M_{тр} \cdot dr = \int_0^R \frac{2Mrdr \cdot \mu g}{R^2} = \frac{2M\mu g}{3}R$$

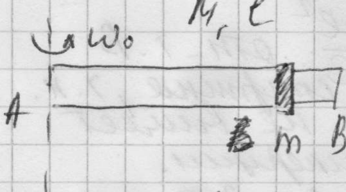
$$\frac{J\omega^2}{2} = M_{трд} \cdot \varphi, \quad т.к. \; M \cdot \varphi = F \cdot R \cdot \varphi = F\,dl = A.$$

$$\frac{J\omega^2}{2} = M_{тр.д} \cdot 2\pi N, \quad N = \frac{J\omega^2}{4\pi M_{трд}} =$$

$$= \frac{\frac{MR^2}{2} \cdot \omega^2}{4\pi \cdot \frac{2M\mu g R}{3}} = \frac{3\omega^2 R}{16\mu\pi g};$$

6.3

M, l 3-ны сохр.



$$\begin{cases} \frac{J\omega_0^2}{2} = \frac{(J+ml^2)\omega^2}{2} + \frac{mv^2}{2} \\ J\omega_0 = (J+ml^2)\omega \end{cases}$$

$$J = \frac{Ml^2}{3} \text{ (отн. т. A)}; \quad \omega = \frac{J\omega_0}{J+ml^2}$$

$$J\omega_0^2 = (J+ml^2)\, J^2\omega_0^2 / (J+ml^2)^2 + \frac{mv^2}{\cancel{2}}$$

$$J\omega_0^2 = \frac{J^2\omega_0^2}{J+ml^2} + \frac{mv^2}{\cancel{2}}$$

$$\frac{Ml^2}{3}\omega_0^2 = \left(\frac{M^2l^4}{9}\omega_0^2\right) \cdot \frac{1}{\frac{Ml^2}{3} + ml^2} + \frac{mv^2}{\cancel{2}};$$

$$\frac{Ml^2\omega_0^2}{3}=\frac{M^2l^2\omega_0^2}{9(M/3+m)}+mv^2$$

$$\frac{Ml^2\omega_0^2}{3}=\frac{M^2l^2\omega_0^2}{3M+9m}+mv^2;$$

$$Ml^2\omega_0^2\left(\underbrace{\frac{1}{3}-\frac{M}{3M+9m}}\right)=mv^2;$$

$$\frac{(3M+9m)-3M}{3(3M+9m)}=\frac{9m}{3(3M+9m)}=\frac{m}{M+3m};$$

$$Ml^2\omega_0^2\frac{\cancel{m}}{M+3m}=\cancel{m}v^2;$$

$$v^2=\frac{Ml^2\omega_0^2}{M+3m}\Rightarrow v=l\omega_0\sqrt{\frac{M}{M+3m}};$$

6.4

З-н сох. энергии:

$$\frac{mv^2}{2}=\frac{J\omega^2}{2}+\frac{mv'^2}{2}$$

$$J=\frac{Ml^2}{3}$$ отн. т. B

как для стержня, т.к. распределение масс не влияет на мом. инерции.

$mv^2=\omega^2 Ml^2/3+mv'^2$; $\omega^2=\frac{3m}{Ml^2}(v^2-v'^2)$;

З-н сохр. момента импульса:

$mv\frac{l}{2}=mv'\frac{l}{2}+\omega J$; L вр. тела $=\omega J$;

$m\frac{l}{2}(v-v')=J\omega=\frac{Ml^2}{3}\omega$; $\frac{m}{2}(v-v')=\omega\frac{Ml}{3}$

$$\frac{3m}{Ml^2}(v^2-v'^2)=\left[\frac{3\cdot\frac{m}{2}(v-v')}{Ml}\right]^2$$

$$\frac{3m}{Ml^2}(v^2-v'^2) = \frac{9m^2(v-v')^2}{4M^2l^2};$$

$$(v^2-v'^2) - \frac{3m}{4M}(v-v')^2 = 0 \quad \Big| : (v-v')$$

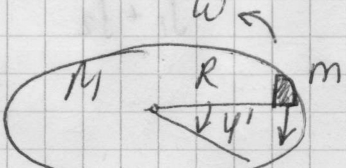
$$(v+v') - \frac{3m}{4M}(v-v') = 0;$$

$$\frac{v+v'}{v-v'} = \frac{3m}{4M}; \quad (v+v')4M = 3m(v-v')$$

$$v'(4M+3m) = v(3m-4M);$$

$$v' = \frac{3m-4M}{3m+4M}v;$$

6.5.



$J\omega + mvR = 0$ – з.с.м.и.

$v = \omega R$ – ур-е кин. св.

$$J = \frac{MR^2}{2} + mR^2 = R^2\left(\frac{M}{2}+m\right);$$

$$R^2\left(\frac{M}{2}+m\right)\omega + mvR = 0$$

$$\omega = d\varphi/dt; \quad v = \omega R = \frac{d\varphi'}{dt}R$$

$$R^2\left(\frac{M}{2}+m\right)\frac{d\varphi}{dt} + mR^2\frac{d\varphi'}{dt} = 0$$

$$\frac{d\varphi}{dt}\left(\frac{M}{2}+m\right) = -\frac{d\varphi'}{dt}m$$

$$\frac{d\varphi}{dt} = -\frac{d\varphi'}{dt}\cdot\frac{m}{\frac{M}{2}+m}; \quad \cancel{\varphi = \int -\frac{d\varphi'}{dt}}$$

$$\varphi = -\frac{2m}{M+2m}\cdot\varphi';$$

№6.6. $J_1, \omega_1; \; J_2; \omega_2$

$$\begin{cases} \dfrac{J_1\omega_1^2}{2} + \dfrac{J_2\omega_2^2}{2} = \dfrac{J\omega^2}{2} + A_{тр} \\ J_1\omega_1 + J_2\omega_2 = J\omega \\ J = J_1 + J_2 \end{cases}$$

$$\omega = \frac{J_1\omega_1 + J_2\omega_2}{J_1 + J_2};$$

$$A_{тр} = \frac{1}{2}\left(J_1\omega_1^2 + J_2\omega_2^2 - \frac{(J_1\omega_1 + J_2\omega_2)^2}{J_1 + J_2} \right) =$$

$$= \frac{1}{2}\frac{J_1 J_2}{J_1 + J_2}(\omega_1 - \omega_2)^2;$$

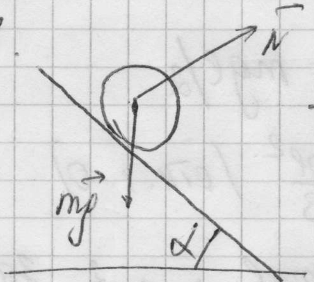
№6.7. R, ω, θ

$$\begin{cases} F_{тр}\Delta t \cdot R = J_0\omega \\ F_{тр}\Delta t = m v_x = m v \sin\alpha \\ J_0 = \dfrac{mR^2}{2}; \end{cases}$$

$$\cancel{R} = \frac{\cancel{m}R^{\cancel{2}}}{2}\cdot\omega\cdot\frac{1}{\cancel{m}v\sin\alpha}$$

$$2v\sin\alpha = \omega R; \quad v = \frac{R\omega}{2\sin\alpha};$$

6.8.



$$\begin{cases} a = \varepsilon R \\ ma = mg\sin\alpha - F_{тр} \\ J\varepsilon = F_{тр}\cdot R \end{cases}$$

$$mg\sin\alpha - \frac{J\varepsilon}{R} = m\varepsilon R \;; \quad J = \frac{mR^2}{2}$$

$$g\sin\alpha = \frac{3}{2}\varepsilon R \;, \quad \varepsilon = \frac{2g\sin\alpha}{3R} \;;$$

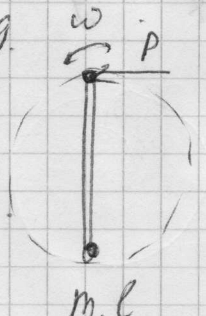
$$\varepsilon = \frac{d\omega}{dt} \;; \quad a = \frac{dv}{dt} \;; \quad \omega = \varepsilon t; \quad v = \omega R = \varepsilon R t$$

$$W_K = \frac{J\omega^2}{2} + \frac{mv^2}{2} = \frac{mR^2}{4}\cdot\frac{4g^2\sin^2\alpha}{9R^2}t^2 + \frac{mR^2}{2}\left(\frac{2g\sin\alpha\, t}{3R}\right)^2$$

$$= \frac{m}{2}\left(\frac{2}{3}g\sin\alpha\cdot\frac{t}{R}\right)^2\left(\frac{R^2}{2}\cdot\frac{1}{R^2} + 1\right) = \frac{1}{3}mg^2t^2\sin^2\alpha;$$

$$W_K(t) = \frac{1}{3}mg^2t^2\sin^2\alpha;$$

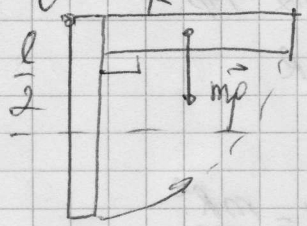
6.9.



$$J = \frac{ml^2}{12} \;; \quad p = mv; \quad v = \frac{p}{m};$$

$p.$

6.10



$$\begin{cases} \dfrac{J\omega^2}{2} = mgl/2 \\ J = \dfrac{ml^2}{3} \quad (\text{отн. т. } 0) \end{cases}$$

$$\frac{ml^2}{6}\cdot\omega^2 = \frac{mgl}{2}, \quad \omega^2 = \frac{3g}{l};$$

$$\omega = \sqrt{\frac{3g}{l}};$$

6.11. $m, r, \omega_0;\ M_0,\ J_0;\ d;$

$$\frac{1}{2}mr^2\omega_0 = (\underbrace{J_0 + \frac{1}{2}mr^2 + md^2}_{J})\omega$$

$$M_0 = J\varepsilon = J\frac{\omega}{t}$$

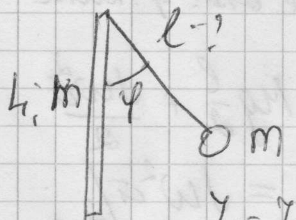
$$M_0\varphi = J\frac{\omega^2}{2} = \frac{J}{2}\,\frac{\frac{1}{4}m^2r^4\omega_0^2}{J^2} = \frac{1}{8}\,\frac{m^2r^4\omega_0^2}{J}$$

$$\varphi = 2\pi N, = \frac{m^2r^4\omega_0^2}{8JM_0};$$

$$N = \frac{m^2r^4\omega_0^2}{16\pi J M_0};$$

где $J = J_0 + \frac{1}{2}mr^2 + md^2;$

6.12.



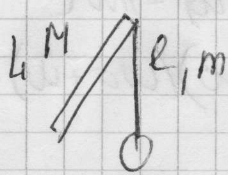
При ударе весь импульс шарика должен перейти в импульс стержня, тогда шарик остановится

$$J_1 = J_2 \qquad \frac{mL^2}{3} = ml^2$$

$$\Rightarrow l = L/\sqrt{3};$$

$$\begin{cases} \frac{J_1\omega_0^2}{2} = \frac{J_2\omega^2}{2} \\ J_1\omega_0 = J_2\omega \end{cases}$$

6.13



$\frac{M}{m}$ - ?

$$J_{ст}\omega_{ст} = -J_{ст}\omega + J\omega$$

$$\frac{J_{ст}\omega_{ст}^2}{2} = \frac{J_{ст}\omega^2}{2} + \frac{J\omega^2}{2}$$

$$J_{ст}(\omega_{ст} + \omega) = J\omega$$

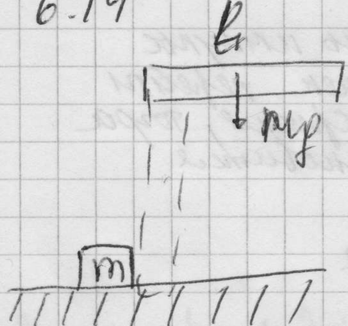
$$J_{ст}(\omega_{ст}+\omega)(\omega_{ст}-\omega) = J\omega^2$$

$$\omega_{ст} - \omega = \omega; \quad \omega_{ст} = 2\omega$$

$$2J_{ст} = -J_{ст} + J; \quad 3J_{ст} = J$$

$$\frac{3ML^2}{3} = ml^2 \Rightarrow \frac{M}{m} = \frac{L^2}{l^2};$$

6.14



$Mg\frac{l}{2}$ ке отн. ц. масс

З.С.Э. $Mgl = Mg\frac{l}{2} + \frac{J\omega_{ст}^2}{2}$

$Mg\frac{l}{2} = J\omega^2_{ст}/2;$

$J = \frac{1}{3}Ml^2$

$Mgl = \frac{1}{3}Ml^2\omega_{ст}^2; \quad \omega_{ст} = \sqrt{\frac{3g}{l}};$

З.С.М.И.

$$\begin{cases} J\omega_{ст} = J\omega + mvl \\ \frac{J\omega_{ст}^2}{2} = \frac{J\omega^2}{2} + \frac{mv^2}{2} \end{cases} \qquad \begin{cases} J(\omega_{ст} - \omega) = mvl \\ J(\omega_{ст} - \omega)(\omega_{ст} + \omega) = mv^2 \end{cases}$$

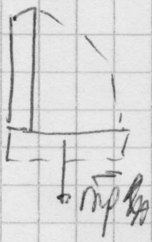
$\omega_{ст} + \omega = \frac{v}{l} \Rightarrow \omega = \frac{v}{l} - \omega_{ст}$

$mvl + \frac{Ml^2}{3}\cdot\frac{v}{l} = \frac{Ml^2}{3}(2\omega_{ст})$

$v\left(ml + \frac{Ml}{3}\right) = 2\omega_{ст}\frac{Ml^2}{3};$

$$v = \frac{\frac{2}{3}Ml\omega_{ст}}{m + M/3} = \frac{2Ml\sqrt{\frac{3g}{l}}}{M + 3m} = \frac{2M\sqrt{3gl}}{M + 3m}$$

6.15.



$$\begin{cases} mg\frac{l}{2} = J\omega \\ J = \frac{ml^2}{3} \\ v = \omega l \end{cases}$$

$$\Rightarrow v\ l\sqrt{\frac{3l}{g}};$$

2.1

$$\cancel{(M_1 - m)v_1 - mv_2 = 0}$$

$$\cancel{(M_2 - m)v_2 - mv_1 = M_2 v}$$

$$\begin{cases} M_1 v_1 - m v_1 - m v_2 = 0 \Rightarrow v_2 = \dfrac{v_1(M_1 - m)}{m} \\ M_2 v_2 - m v_2 - m v_1 = M_2 v \end{cases}$$

$$\frac{M_2}{m}(M_1 - m)v_1 - (M_1 - m)v_1 - m v_1 = M_2 v$$

$$v_1\left(\frac{M_1 M_2}{m} - M_1 - M_2\right) = M_2 v$$

$$v_1\left(\frac{M_1 M_2 - m(M_1 + M_2)}{m}\right) = M_2 v$$

$$v_1 = \frac{m M_2 v}{M_1 M_2 - m(M_1 + M_2)};$$

2.2. a_1, a_2 $\quad a_i = \frac{v_i^2}{R};$ $\quad v_1 = \sqrt{a_1 R};\ v_2 = \sqrt{a_2 R}$

$m_1, m_2;$

$$m_1 v_1 - m_2 v_2 = (m_1 + m_2)v$$

$$v = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2} \Rightarrow \frac{m_1\sqrt{a_1} - m_2\sqrt{a_2}}{m_1 + m_2} = a$$

2.3 $\quad 0 = (M+m)v - mv \Rightarrow v = \underline{\qquad}$

2.4. $ma = -\frac{dm}{dt}v$

$a\,dt = -\frac{dm}{m}v$

$a\int dt = -v\int\frac{dm}{m}$

$at = -v\ln(m) + C$, при $t=0$, $at=0=v\ln m_0+C$

$\Rightarrow at = v\ln\frac{m_0}{m}$

$\frac{at}{v} = \ln\frac{m_0}{m}$; $\frac{m_0}{m} = e^{\frac{at}{v}}$

Ответ: $m = m_0\cdot e^{-\frac{at}{v}}$;

2.5. $v = \sqrt{v_0^2 + v_p^2}$

$\Rightarrow tg\alpha = v_p / v_0$

$m\frac{dv_p}{dt} = -v\frac{dm}{dt}$; $dv_p = -v\frac{dm}{m}$;

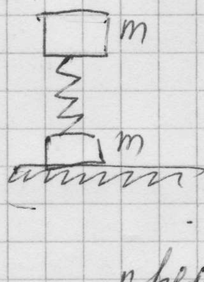
$\int dv_p = -v\int\frac{dm}{m}$; $v_p = -v\ln m + c$;

$c = v\ln m_0$ ($t=0 \Rightarrow \exists c$)

$v_p = v\ln\frac{m_0}{m}$,

$tg\alpha = \left[\frac{v}{v_0}\ln\frac{m_0}{m}\right] \Rightarrow \alpha = arctg\frac{v}{v_0}\ln\frac{m_0}{m}$

2.6



$\frac{k\Delta l^2}{2} - mg\Delta l = \frac{kx^2}{2} + mgx$

x – растяж. пружины после пережиг. нити

Усл. отрыва $kx \geq mg$

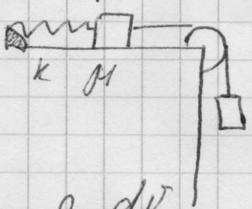
предел. случай: $x = \frac{mg}{k}$

$$\Delta l^2 - \frac{2mg}{k}\Delta l - 3\left(\frac{mg}{k}\right)^2 = 0$$

$$\Delta l = \frac{mg}{k} \pm \sqrt{\left(\frac{mg}{k}\right)^2 + 3\left(\frac{mg}{k}\right)^2} = \frac{3mg}{k}$$

Ответ: $\Delta l > 3mg/k$

2.7



$$\begin{cases} ma = mg - T \\ T - mg\mu - kx = ma \end{cases}$$

$$mg(1-\mu) - kx = 2ma$$

$$2m\frac{dv}{dt} = mg(1-\mu) - kx$$

$$2m v\,dv = mg(1-\mu)\,dx - kx\,dx$$

$$2m\int v\,dv = mg(1-\mu)\int dx - k\int x\,dx,$$

$$mv^2 = mg(1-\mu)x - \frac{kx^2}{2} + \underset{0}{\overset{\shortparallel}{C}}$$

$$v^2 = g(1-\mu)x - \frac{kx^2}{2m}$$

$$\cancel{v = \sqrt{g(1-\mu)x - kx^2/2m}}$$

$$v' = \frac{g(1-\mu) - \frac{kx}{m}}{2\sqrt{g(1-\mu)x - kx^2/2m}};$$

$$v_{max} = \sqrt{g^2 m(1-\mu)^2\frac{1}{k} - \frac{g^2 m(1-\mu)^2}{2k}} =$$

$$= g(1-\mu)\sqrt{\frac{m}{2k}};$$

2.10 З.С.И. $\begin{cases} m_1 v_0 = m_2 v_2 \cos\alpha \\ m_1 v_1 = m_2 v_2 \sin\alpha \end{cases}$

$$\frac{m_2}{m_1} = n \Rightarrow \begin{cases} \dfrac{v_0}{v_2 \cos\alpha} = n \\ \dfrac{v_1}{v_2 \sin\alpha} = n \end{cases}$$

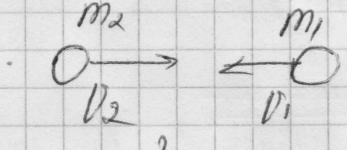
$$\sin\alpha = \frac{v_1}{n v_2} \; ; \; \cos\alpha = \frac{\sqrt{v_2^2 n^2 - v_1^2}}{n v_2}$$

$$v_0 = \sqrt{v_2^2 n^2 - v_1^2} \; ; \; v_0^2 = v_2^2 n^2 - v_1^2$$

З.С.Э $\begin{cases} \dfrac{m_1 v_1^2}{2} = \mu g m S_1 \\ \dfrac{m_2 v_2^2}{2} = \mu g m_2 S_2 \end{cases}$

$$\left.\begin{matrix} v_1^2 = 2\mu g S_1 \\ v_2 = 2\mu g S_2 \end{matrix}\right\} \Rightarrow v_0 = \sqrt{2\mu g (S_2 n^2 - S_1)}$$

2.11.



$$\frac{E_1}{E_2} = \alpha$$

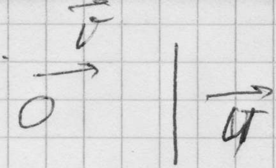
$$\frac{m_1 v_1^2}{m_2 v_2^2} = \alpha \qquad \frac{v_1}{v_2} = \sqrt{\alpha \frac{m_2}{m_1}}$$

З.С.Э: З.С.И: $\begin{cases} \dfrac{m_1 v_1^2}{2} + \dfrac{m_2 v_2^2}{2} = \dfrac{m_1 + m_2}{2} v_3^2 \\ m_2 v_2 - m_1 v_1 = (m_1 + m_2) v_3 > 0 \end{cases}$

$m_2 v_2 > m_1 v_1$, т.е. $\dfrac{m_2}{m_1} > \sqrt{\alpha \dfrac{m_2}{m_1}}$

$$\Rightarrow \frac{m_2}{m_1} > \alpha .$$

2.12.



В с.о, связанной со стенкой

$v' = v - u - u = v - 2u$

$\Delta E = \frac{mv^2}{2} - \frac{m}{2}(v-2u)^2 =$

$= \frac{m}{2}(v^2 - v^2 + 4vu - 4v^2) = \frac{m}{2}(4v(v-u)) =$

$= 2mv(v-u)$

2.13. $p_1 = mv$; $p_2 = -m(v-2u)$; $\cancel{= mv}$.

$\Delta p = mv + mv - 2mu = 2m(v-u)$

2.14

8.1. ρ, E;

$\xi = A_0 \cos(\omega t - kx)$, $v = \sqrt{\frac{E}{\rho}} = \frac{\omega}{k}$ – ск-ть распр. волны

$k \cdot \lambda = 2\pi$

$\frac{k \cdot v \cdot 2\pi}{\omega} = 2\pi \Rightarrow v = \frac{\omega}{k}$

$U_x = \frac{d\xi}{dt} = -A_0 \omega \sin(\omega t - kx)$ // ск-ть колеб. частиц произв. по времени

$\varepsilon = \frac{d\xi}{dx} = kA_0 \sin(\omega t - kx)$ // деф-я – производная по координате

$\frac{U_x}{\varepsilon} = -\frac{\omega}{k} \Rightarrow v = \sqrt{\frac{E}{\rho}} = -\frac{U_x}{\varepsilon}$;

$U_x = -\varepsilon \sqrt{\frac{E}{\rho}}$;

8.2. $\cancel{\xi = A\cos(kx)}$

$\xi = A\cos(\omega t - kx)$

$U_x = \underbrace{-\omega A}\sin(\omega t - kx)$ $\quad U_{x\,max} = \omega A$.

$v_{расп.\,в-ны} = \frac{\omega}{k}$

$\frac{U_{x\,max}}{v_{расп.\,вол.}} = \omega A \cdot \frac{k}{\omega} = A \cdot k$.

8.3. ρ; $\xi = a\cos(kx)\cos(\omega t)$

$E_{полн} = \frac{\rho\omega^2 a^2}{4}$, $E_p = E_{пол} - E_{кин}$; $E_{пол} = \frac{m\omega^2 a^2}{4}$

$U_x = -a\cos(kx)\,\omega\sin\omega t$

$E_{пол} = \frac{m\omega^2 a^2}{4}$, $E_k = \int_0^h \frac{U_x^2}{2}\rho\,dV = \int_0^h \frac{\rho}{2} a^2 \cos^2(kx)\,\omega^2 \sin^2\omega t\,dV$

$= \int_0^h \frac{\rho}{2} a^2 \cos^2(kx)\,\omega^2 \sin^2\omega t\,\cancel{dx} \cdot S \Rightarrow$

$\int_0^l \cos^2 kx\,dx = \frac{x}{2} + \frac{\sin 2kx}{4k} = l/2$

$\frac{\sin 2kx}{4k}$ → 0 – период 2 уплотн.

$$E_K = \frac{\sin^2(\omega t)\rho A^2 \omega^2 S h}{4} = \frac{\omega^2 A m}{4}\sin^2\omega t$$

$$E_n = E - E_K = \frac{m\omega^2 A}{4}(1-\sin^2\omega t) =$$

$$= \frac{m\omega^2 A}{4}\cos^2\omega t,$$

$$E_{n об} = \frac{\rho\omega^2 A}{4}\cos^2\omega t$$

8.4.

$\frac{n\lambda}{2} = L$, $n=1$ – основной тон

$n-1$, $\forall n \geq 2$ – номер обертона

$\lambda = 2L$; $\lambda = v \cdot T = v \cdot \frac{1}{\nu} = 2L$

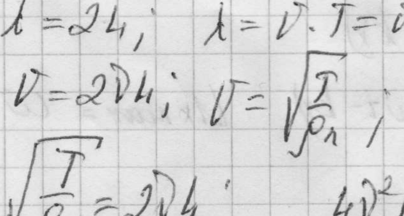
$v = 2\nu L$; $v = \sqrt{\frac{T}{\rho_л}}$, $\rho_л = \frac{m}{L} =$

$\sqrt{\frac{T}{\rho}} = 2\nu L$; $4\nu^2 L^2 = \frac{TL}{m}$

$m = \rho V = \rho \frac{\pi d^2}{4} L$;

$4\nu^2 L = \frac{T \cdot 4}{\rho\pi d^2 L}$; $T = \nu^2 L^2 d^2 \pi\rho$,

8.5



$\frac{\lambda}{2} = \frac{L}{4}$

$4 \cdot \frac{L}{8} = \lambda$, $\frac{L}{2} = \lambda$

$a = a_0 \sin\frac{\pi}{4}$, $a_0 = a\sqrt{2}$,

8.6

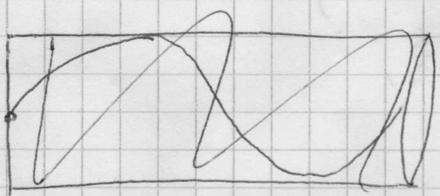
$$v = \sqrt{\frac{T}{\rho_{лин}}}$$

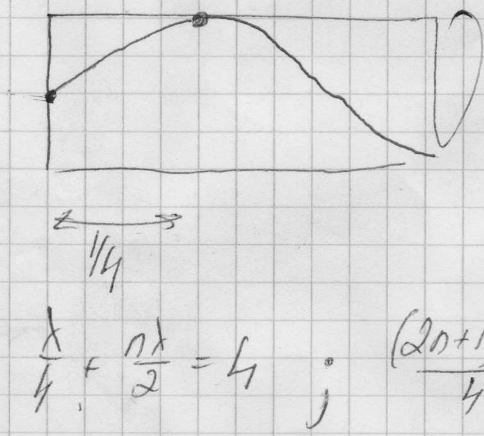
$$\frac{\lambda}{2} = l = \frac{v}{2\nu} \; ; \quad \nu = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\rho_{лин}}}$$

$$\nu_н' = \cancel{\frac{\nu n}{2l}}\sqrt{\frac{mT}{\rho_{лин}}} \quad \nu_n = \frac{v}{2l/n}\sqrt{\frac{mT}{\rho_{лин}}}$$

$$\frac{\nu_н}{\nu} = n\sqrt{m} \; ;$$

8.7.



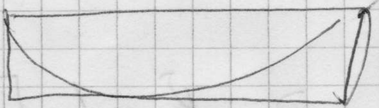


$l/4$

$$\frac{\lambda}{4} + \frac{n\lambda}{2} = l \; ; \quad \frac{(2n+1)\lambda}{4} = l$$

$$\frac{v}{2\nu} = \frac{\lambda}{2} = l \; , \quad v = 2\nu l$$

8.8. $\qquad\qquad$ l, c



морф нед-ит l

$$\frac{(2n-1)\lambda}{4} = L \qquad \frac{(2n-1)}{4}\frac{c}{\nu_n} = L$$

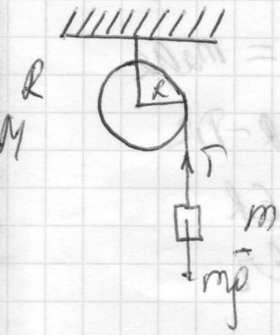
$$\nu_n = \frac{(2n-1)c}{4L} \leq \nu_0$$

$$(2n-1)c \leq 4L\nu_0 ; \qquad 2n \leq 4L\nu_0 + c$$

$$0 < n \leq 2L\nu_0 + \frac{c}{2} ;$$

8.9

5.1.



$$\begin{cases} ma = mg - T \\ J\cdot\frac{\omega}{t} = TR \\ a = \frac{v}{t} = \frac{\omega R}{t}; \end{cases}$$

$$\frac{\omega}{t} = \varepsilon = \frac{a}{R}$$

$$(l)'' = (R\varphi)''$$

$$a = R\varepsilon$$

$$mR\varepsilon = mg - T \Rightarrow T = -m(R\varepsilon - g)$$

$$J\varepsilon = Rm(g - \varepsilon R)$$

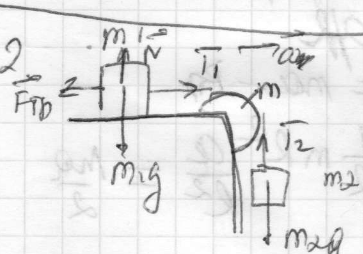
$$J\varepsilon + Rmg + \varepsilon mR^2 = 0$$

$$\varepsilon = \frac{Rmg}{J + mR^2}; \quad \varphi = \int_0^t \left(\int \varepsilon\, dt\right) dt,$$

$$\varphi = \frac{1}{2}\frac{Rmgt^2}{J + mR^2}; \quad J = \frac{1}{2} MR^2$$

$$\varphi = \frac{1}{2}\frac{Rmgt}{\frac{1}{2}MR^2 + mR^2} = \frac{1}{2}\frac{mgt}{R\left(\frac{M}{2} + m\right)} = \frac{1}{2}\frac{gt}{R\left(\frac{M}{2m} + 1\right)};$$

5-2

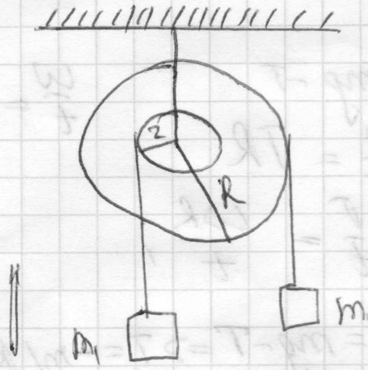


$$\begin{cases} m_2 g - T_2 = m_2 a \\ T_1 - \mu m g = m_1 a \\ \frac{1}{2} mR^2 \varepsilon = (-T_1 + T_2) R \\ a = \varepsilon R \end{cases}$$

$$m_2 g - \mu m_1 g = m_2 a + m_1 a + \frac{1}{2} mR\varepsilon; \quad a\left(m_2 + m_1 + \frac{m}{2}\right) = g(m_2 - \mu m_1)$$

$$a = g\frac{m_2 - \mu m_1}{m_2 + m_1 + m/2};$$

5.3



$$\begin{cases} m_1 g - T_1 = -m_1 a_1 \\ m_2 g - T_2 = m_2 a_2 \\ J\varepsilon = T_2 R - T_1 r \\ a_2 = \varepsilon R \\ a_1 = \varepsilon r \end{cases}$$

$T_1 = m_1(g + \varepsilon r)$

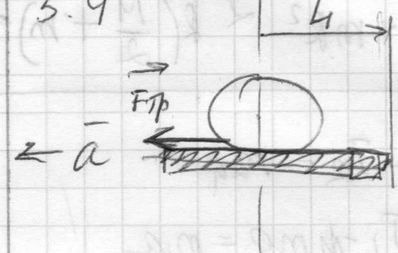
$T_2 = m_2(g - \varepsilon R)$

$J\varepsilon = m_2 R(g - \varepsilon R) - m_1 r(g + \varepsilon r)$

$J\varepsilon = m_2 R g - \varepsilon m_2 R^2 - m_1 r g - m_1 r^2 \varepsilon$

$$\varepsilon = \frac{g(m_2 R - m_1 r)}{J + m_2 R^2 + m_1 r^2};$$

5.4



$$\begin{cases} J\varepsilon = F_{тр} \cdot R \\ \varepsilon = a/R \\ ma = ma_0 - F_{тр} \end{cases}$$

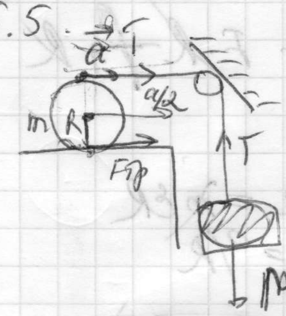
$F_{тр} = \frac{1}{2} m R^2 \frac{a}{R^2} = \frac{ma}{2}$

$ma = ma_0 - \frac{1}{2} ma$

$\frac{3}{2} a = a_0; \Rightarrow a = \frac{2}{3} a_0$

$v^2 = 2aS; \quad v = \sqrt{\frac{2 \cdot 2 l a}{3}} = 2\sqrt{\frac{la}{3}}$

5.5



$$\begin{cases} Mg - T = Ma \\ \frac{ma}{2} = F_{тр} + T \\ \varepsilon\left(\frac{mR^2}{2}\right) = R(T - F_{тр}) \end{cases}$$

$\varepsilon R = a/2$

$$F_{тр} = \frac{\varepsilon \frac{mR^2}{2} + RT}{R} = T - \frac{mR\varepsilon}{2}$$

$$\varepsilon = \frac{a}{2R}$$

$\Rightarrow \cancel{a = \frac{Mg}{\ldots}}$

$\cancel{Mg - T = Ma \Rightarrow T = M(g-a)}$

$\cancel{\frac{ma}{2} = M(g-a)}$

$\cancel{\frac{ma}{2} = F_{тр} + T = F_{тр} + M(g-a)} \Rightarrow$

$$Mg - \frac{ma}{2} = Ma - F_{тр} \Rightarrow F_{тр} = \frac{ma}{2} + M(a-g)$$

$$\varepsilon\left(\frac{mR^2}{2}\right) = R\left(T - \frac{ma}{2} - M(a-g)\right) = RT - R\frac{ma}{2} - M(a-g)R$$

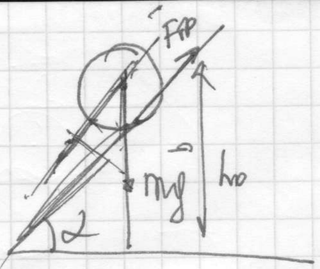
$$T = \varepsilon\frac{mR}{2} + \frac{ma}{2} + M(a-g);$$

$$Mg - \frac{a}{2R}\cdot\frac{mR}{2} - \frac{ma}{2} - M(a-g) = Ma$$

$$a\left(-\frac{m}{4} - \frac{m}{2} - 2M\right) = -2Mg$$

$$a = \frac{2Mg}{\frac{3m}{4} + 2M} = \frac{Mg}{\frac{3m}{8} + M};$$

5.7.



$$\cancel{\frac{2}{5} m R^2 \cdot \varepsilon =}$$

$$\frac{\omega^2 J_A}{2} + \frac{m v^2}{2} + mgh = const$$

$$\frac{v^2}{R^2} \cdot \frac{2 R^2 m}{5 \cdot 2} + \frac{m v^2}{2} + mgh = const$$

$$\frac{7}{10} m v^2 + mgh = const = mgh_0$$

$$x = \frac{(h_0 - h)}{\sin\alpha}$$

$$\cancel{\frac{7}{10} v^2 = 2 g x \sin\alpha}$$

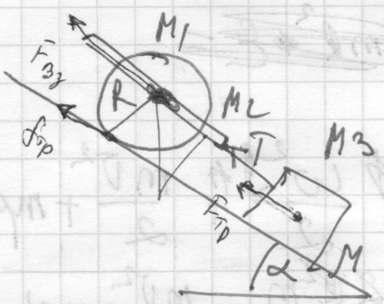
$$\frac{7}{10} v^2 = g x \sin\alpha$$

$$a = \frac{5}{7} g \sin\alpha$$

$$\frac{v^2}{2x} = a,$$

$$\Rightarrow a = \frac{5}{7} g \sin\alpha$$

5.8



$F_{Tp} = \mu M_3 g\cos\alpha$

$$\begin{cases} m_3 g\sin\alpha - T - F_{Tp} = m_3 a \\ (m_1+m_2) g\sin\alpha + T - F_{Bz} = (m_1+m_2) a \\ \cancel{\frac{3}{2}\left(\frac{mR^2}{2}+\right)\varepsilon} \quad M_1 a = M_1 g\sin\alpha - F_{Bz} - f_{Tp} \end{cases}$$

$\frac{mR^2}{2}\cancel{\varepsilon}$

$J\varepsilon = f_{Tp}R \qquad \frac{m_1R^2}{2}\cdot\frac{a}{\cancel{R}} = f_{Tp}\cancel{R}$

$a = \varepsilon R$

$(1) \Rightarrow \quad T = m_3 g\sin\alpha - m_3 a - \mu m_3 g\cos\alpha$

$(2):\ (m_1+m_2)g\sin\alpha + m_3 g\sin\alpha - m_3 a - \mu m_3 g\cos\alpha - (m_1+m_2)a = F_{Bz} =$

$= -M_1 a + M_1 g\sin\alpha - f_{Tp};$

$(m_1+m_2+m_3)g\sin\alpha - a(m_1+m_2+m_3) - \mu m_3 g\cos\alpha +$

$+ M_1(a - g\sin\alpha) = -f_{Tp} = -\frac{m_1 a}{2}$

$\cancel{a = \frac{(m_1+m_2+m_3)g\sin\alpha - \mu m_3 g\cos\alpha + M}{\frac{m_1}{2} + m_2 + m_3}}$

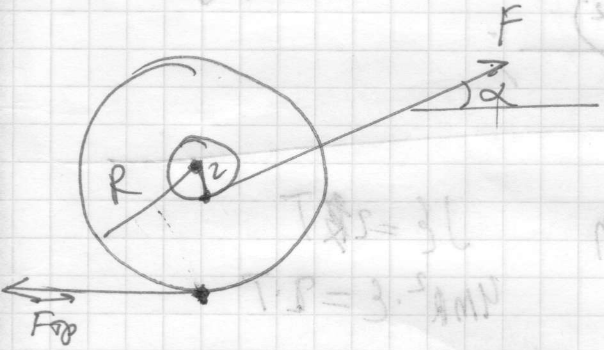
$g\sin\alpha\cdot(m_1+m_2+\cancel{m_3}) - \mu m_3 g\cos\alpha + \cancel{M_1}(\cancel{-g\sin\alpha}) = -\frac{m_1 a}{2} + a(m_1+m_2+m_3) -$

$-\cancel{M_1} a;$

$$\Rightarrow \quad a = g\,\frac{\cancel{(m_1+m_2+m_3)\sin\alpha -}}{}$$

$$a = g\,\frac{(m_1+m_2+m_3)\sin\alpha - \mu m_3\cos\alpha}{m_1+m_2+m_3}$$

5.9



$$\begin{cases} a = \varepsilon R \\ F_{тр} = F\cos\alpha - ma \\ J\varepsilon = F_{тр}\cdot R - Fr \end{cases}$$

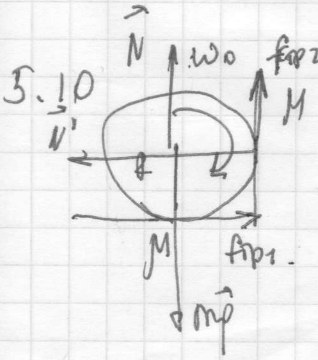
$$J\varepsilon = (F\cos\alpha - ma)R - Fr;$$

$$J\frac{a}{R} = FR\cos\alpha - maR - Fr;$$

$$a\left(\frac{J}{R} + mR\right) = F(R\cos\alpha - r);$$

$$a = \frac{F(R\cos\alpha - r)}{\frac{J}{R} + mR};$$

5.10



$$mg = f_2 + N$$

$$f_{тр1} = \mu(mg - f_2)$$

$$N' = f_{тр1}; \quad f_2 = \mu N' = \mu f_{тр1}$$

$$f_2 = \mu^2 mg - \mu^2 f_2; \quad f_2 = \frac{\mu^2 mg}{1+\mu^2};$$

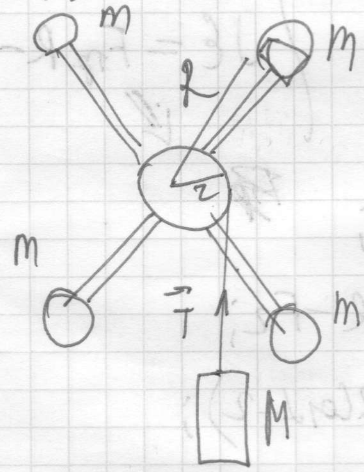
$$f_1 = \frac{\mu m g}{1+\mu^2};$$

$$\frac{mR^2}{2}\varepsilon = (f_1+f_2)R = \frac{\mu m g}{1+\mu^2}(1+\mu)$$

$$\Rightarrow \varepsilon = \frac{2\mu m g(1+\mu)}{mR(1+\mu^2)}; \quad t = \frac{\omega_0}{\varepsilon} = \frac{\omega_0(1+\mu^2)R}{2\mu g(1+\mu)};$$

$$t = \frac{\omega_0 R(1+\mu^2)}{2\mu g(1+\mu)}.$$

5.11.



$$J\varepsilon = r T$$

$$4mR^2 \cdot \varepsilon = r \cdot T$$

$$\varepsilon = \frac{rT}{4mR^2}$$

$$Ma_1 = Mg - T;$$

$$a = \varepsilon r$$

$$M\varepsilon r = Mg - T$$

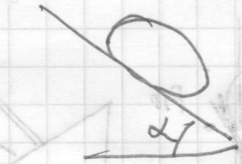
$$Mr \cdot \frac{rT}{4mR^2} = Mg - T;$$

$$T\left(\frac{Mr^2}{4mR^2}+1\right) = Mg;$$

$$T = \frac{Mg}{\frac{Mr^2}{4mR^2}+1};$$

5.12

$$\begin{cases} J\varepsilon = R f_{тр} \\ ma = mg\sin\alpha - f_{тр} \\ a = \varepsilon R \end{cases}$$



$$\cancel{m\varepsilon R = mg\sin\alpha - \frac{J\varepsilon}{R}}$$

$$ma = mg\sin\alpha - \frac{J\varepsilon}{R} = mg\sin\alpha - \frac{Ja}{R^2},$$

$$a\left(m + \frac{J}{R^2}\right) = mg\sin\alpha; \quad a = \frac{mg\sin\alpha}{m + J/R^2};$$

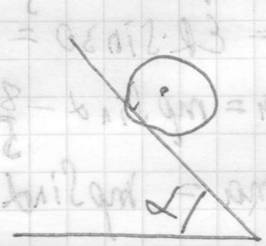
$$v = at; \quad \frac{v_{1\,шар}}{v_{2\,цил.}} = \frac{m + \frac{J_2}{R_2}}{m + \frac{J_1}{R_2}} = \frac{mR^2 + J_2}{mR_2 + J_2} =$$

$$= \frac{\frac{mR^2}{2} + mR^2}{\frac{2}{5}mR^2 + mR^2} = \frac{3/2}{14/10} = \frac{3}{14/5} = 15/14$$

5.13

$$\cancel{f_{тр} = mg\sin\alpha - ma =}$$

$= $ см. 5.12



$$a = \frac{mg\sin\alpha}{m + J/R^2} =$$

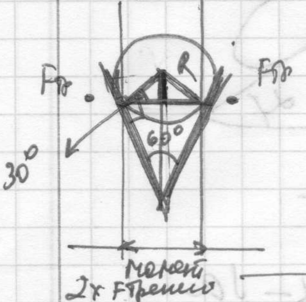
$$= \frac{mg\sin\alpha}{m + \frac{mR^2}{2R^2}} = \frac{2g\sin\alpha}{3};$$

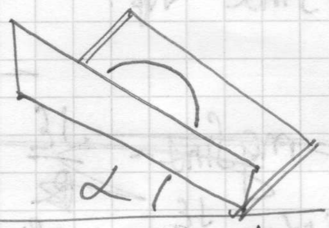
$$f_{тр} = mg\sin\alpha - ma = mg\sin\alpha - m\cdot\frac{2}{3}g\sin\alpha = \frac{1}{3}gm\sin\alpha$$

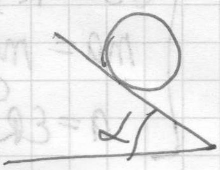
$$f_{тр.} \geq F_{тр} = \mu mg\cos\alpha; \quad \frac{1}{3}gm\sin\alpha \geq \mu mg\cos\alpha$$

$$\mu \leq \frac{\operatorname{tg}\alpha}{3}; \quad \mu_{min} = \operatorname{tg}\alpha/3$$

5.14







$$\begin{cases} ma = mg\sin\alpha - 2f_{тр} \\ J\varepsilon = 2f_{тр}R\sin 30 = \frac{R}{2}f_{тр}; \Rightarrow f_{тр} = \frac{2J\varepsilon}{R} = \cancel{\ldots} \end{cases}$$

$$f_{тр} = \frac{2\cdot 2MR^2\varepsilon}{5R} = \frac{4}{5}MR\varepsilon$$

$$\sum M = 2f_{тр}\cdot l_{\perp f_{тр}} = \frac{8}{5}MR\varepsilon$$

$$ma = mg\sin\alpha - \frac{8}{5}MR\varepsilon;$$

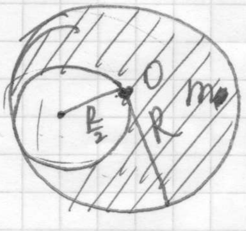
$$a = \varepsilon R\cdot\sin 30 = \frac{\varepsilon R}{2}; \quad \varepsilon = \frac{2a}{R}$$

$$ma = mg\sin\alpha - \frac{8}{5}\cdot MR\cdot\frac{2a}{R} = mg\sin\alpha - \frac{16}{5}ma$$

$$\frac{21}{5}ma = mg\sin\alpha; \quad a = \frac{5}{21}g\sin\alpha$$

~~$a = \varepsilon R\sin 30$~~

5.15.



$J_0 = \frac{m_0 R^2}{2}$ — без дырки

$J = \frac{mR^2}{4 \cdot 2} + \frac{mR^2}{4} = \frac{3mR^2}{8}$ — дырка отн. т. O

πR^2 — б.

$\frac{\pi R^2}{4}$ — м. — однород, $\Rightarrow m_0 = 4m$;

$J_0 = \frac{m_0 R^2}{2}$; $\cancel{J = \frac{3 \cdot 4 \cdot R^2}{8} = \frac{3}{2} R^2}$

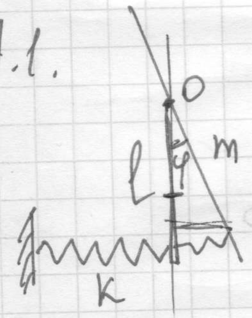
$J = \frac{3 m_0}{4 \cdot 8} \cdot R^2 = \frac{3 m_0}{32} R^2$

$J' = J_0 - J = \frac{m_0 R^2}{2} - \frac{3 m_0 R^2}{32} =$

$= \frac{16 m_0 R^2}{32} - \frac{3 m_0 R^2}{32} = \frac{13}{32} m_0 R^2$;

πR^2

7.1.



$$\frac{\omega^2 J}{2} + mg\frac{l}{2}(1-\cos\varphi) + \frac{kx^2}{2} = const$$

$$x = l\varphi;\quad \omega = \dot\varphi;\quad J = J_0 + ml^2/4;\quad J_0 = ml^2/12 \qquad J = \frac{ml^2}{3}$$

$$\Rightarrow J = \frac{ml^2}{3};\quad 1-\cos\varphi = 2\sin^2\frac{\varphi}{2} \approx 2\cdot\frac{\varphi^2}{4} = \varphi^2/2$$

$$\left[\frac{\dot\varphi^2\cdot\frac{ml^2}{3}}{2} + mg\frac{l}{2}\cdot\frac{\varphi^2}{2} + \frac{k\varphi^2 l^2}{2}\right]' = (const)'$$

$$:\dot\varphi \;\Big|\; \frac{2\dot\varphi\cdot\ddot\varphi\cdot ml^2}{6} + \frac{mgl}{4}\cdot 2\varphi\cdot\dot\varphi + \frac{k\dot\varphi\cdot 2\varphi\cdot l^2}{2} = 0$$

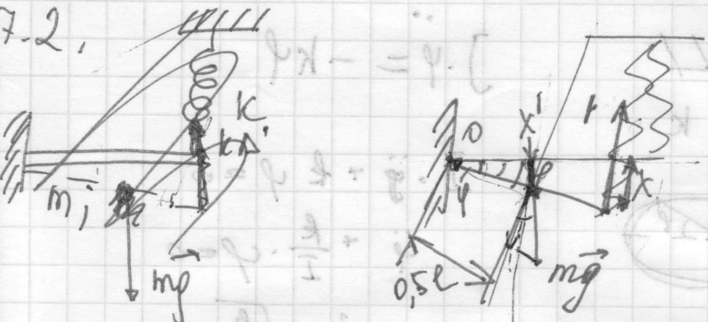
$$\ddot\varphi\,\frac{ml^2}{3} + \left[\frac{mgl}{2} + kl^2\right]\cdot\varphi = 0$$

$$\ddot\varphi + \frac{\frac{mgl}{2}+kl^2}{\frac{ml^2}{3}}\cdot\varphi = 0$$

$$\left[\frac{3g}{2l} + \frac{3k}{m}\right] = \omega^2;$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{2l}+\frac{3k}{m}};$$

7.2.



$x = l\varphi$

$$\cancel{\frac{\omega^2 J}{2} + \frac{kx^2}{2} + mgh = const}$$

$$\cancel{\frac{\omega^2 J}{2} + \frac{kl^2\varphi^2}{2} + mg\frac{l}{2}\sin\varphi = const}$$

$$\cancel{\frac{\dot\varphi^2 \cdot \frac{ml^2}{3}}{2} + \frac{kl^2\varphi^2}{2} + \frac{mgl\varphi}{2} = const}$$

$$\cancel{\frac{2\dot\varphi\cdot\ddot\varphi\, ml^2}{6} + \frac{2\dot\varphi\cdot\varphi l^2}{2} + \frac{mgl}{2} = 0}$$

$$\frac{\omega^2 J}{2} + \frac{kx^2}{2} + mgh = const; \qquad \sin\varphi = \frac{x'}{l/2} = \frac{2x'}{l}$$

$$\frac{\omega^2 J}{2} + \frac{kl^2\varphi^2}{2} + mg\frac{l\varphi}{2} = const;$$

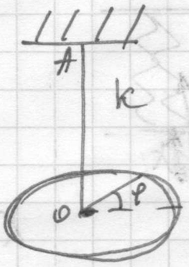
$$\frac{2\dot\varphi\ddot\varphi\, ml^2}{2\cdot 3} + \frac{2kl^2\dot\varphi\varphi}{2} + \frac{mgl\cdot\dot\varphi}{2} = 0 \qquad \sin\varphi = \frac{x'}{l/2} = \varphi$$

$$\frac{\ddot\varphi ml^2}{3} + \varphi\left[kl^2\right] + \frac{mgl}{2} = 0 \qquad 2x' = l\varphi$$

$$\frac{\ddot\varphi ml}{3} + (kl)\varphi + \frac{mg}{2} = 0; \quad \Big| : \frac{ml}{3};$$

$$\ddot\varphi + \frac{3k}{m}\varphi + \frac{3g}{2l} = 0 \Rightarrow \boxed{\omega = \sqrt{\frac{3k}{m}}}$$

7.3.



$$J \cdot \ddot{\varphi} = -k\varphi$$

$$I \cdot \ddot{\varphi} + k\varphi = 0$$

$$\ddot{\varphi} + \frac{k}{I} \cdot \varphi = 0$$

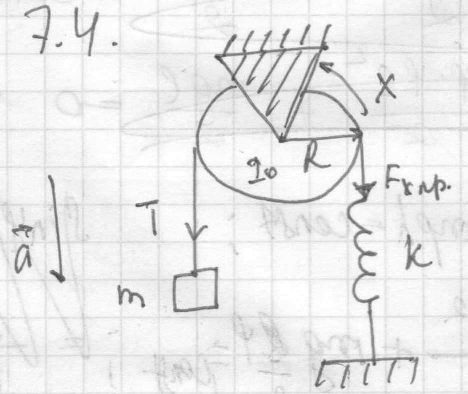
$$\omega = \sqrt{\frac{k}{I}}$$

$I = \frac{mR^2}{2}$ (диск)

$$\omega = \sqrt{\frac{2k}{mR^2}} \quad T = 2\pi\sqrt{\frac{mR^2}{2k}} = 2\pi R\sqrt{\frac{m}{2k}} \Rightarrow$$

$$T = \pi R\sqrt{\frac{2m}{k}};$$

7.4.



$$I\ddot{\varphi} = -RT + kxR$$

$$x = \varphi R; \quad \ddot{x} = \ddot{\varphi}R$$

$$a = \varepsilon R$$

$$ma = mg - T; \quad T = m(g - a)$$

$$\frac{I\ddot{\varphi}}{R} = -T + kx = m(a-g) + kx$$

$$\frac{I\ddot{x}}{R^2} = m(\ddot{x} - g) + kx;$$

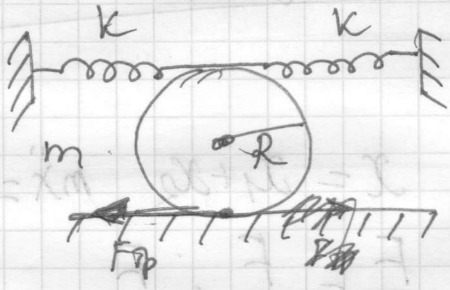
$$\ddot{x}\left(\frac{I}{R^2} - m\right) + gm - kx = 0$$

$$\ddot{x} - \frac{kx}{\frac{I}{R^2} - m} + \frac{gm}{\frac{I}{R^2} - m} = 0$$

$$\omega = \sqrt{\frac{kR^2}{I + mR^2}} = R\sqrt{\frac{k}{I + mR^2}}, \quad \omega = 2\pi\nu$$

$$\nu = \frac{R\sqrt{\frac{k}{I - mR^2}}}{2\pi};$$

7.5.



$$J\cdot\varepsilon = F_{тр}\cdot R - 2F_{упруг.}\cdot R$$

$$ma = -F_{тр} - 2F_{упруг.}$$

$$\frac{J\ddot{x}}{R} = -2F_{упр}\cdot R - R\left(\frac{m\varepsilon}{R} - 2F_{уп}\right)$$

$$a = \varepsilon R$$

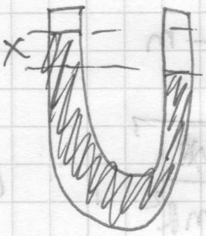
$$\frac{J\ddot{x}}{R} = -4F_{упр}R - m\cdot\varepsilon R^2 = -4\cdot kxR - m\ddot{x}R;$$

$$\ddot{x}\left(\frac{J}{R} + mR\right) + 4kRx = 0.$$

$$\ddot{x} + \frac{4kR}{\frac{J}{R} + mR}x = 0 \qquad \omega_0^2 = \frac{4kR^2}{J + mR^2} = \frac{8kR^2}{3mR^2} = \frac{8k}{3m}$$

$$T = 2\pi\sqrt{\frac{3m}{8k}} = \pi\sqrt{\frac{3m}{2k}};$$

7.6.



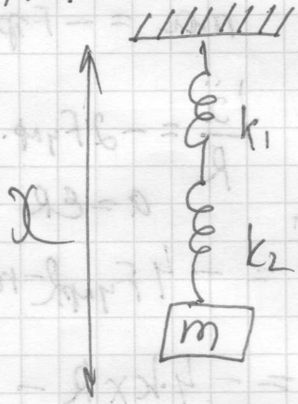
$V;\ S;$

$$m\ddot{x} = -\rho g S \cdot 2x$$

$$\omega = \frac{2\rho g S}{m} = \frac{2gS}{V};$$

$$T = 2\pi\sqrt{\frac{V}{2gS}}$$

7.7.

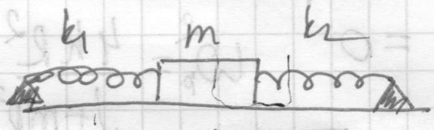


$$x = x_1 + x_2 \qquad m\ddot{x} = -kx$$

$$\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}};$$

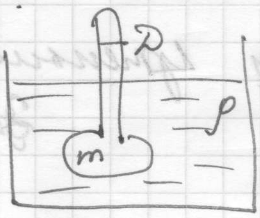
7.8.



$$m\ddot{x} = -k_1x - k_2x = -x(k_1+k_2)$$

$$T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$$

7.9.



$$m\ddot{x} = mg - \rho_ж g \cdot h_{погр}.$$

$$\cancel{h_{погр} = }$$

$$\cancel{\rho_ж h_{погр} \cdot m_{погр} = mg}$$

$$\cancel{\rho_ж \cdot g \cdot h_{погр} \cdot \frac{\pi D^2}{4} = mg}$$

$$\cancel{h_{погр} = \frac{4m}{\pi D^2 \rho_ж}};$$

$$\cancel{m\ddot{x} = mg - \rho_ж \left[g \cdot \frac{4m}{\pi D^2 \rho_ж} + x \right] Sg}$$

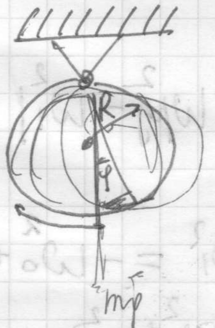
$$\cancel{\ddot{x}} +$$

$$m\ddot{x} = -\rho_ж g \frac{\pi D^2}{4} x$$

$$T = 2\pi \sqrt{\frac{4m}{\rho_ж g \pi D^2}} = \frac{4\pi}{D} \sqrt{\frac{m}{\rho_ж g \pi}} =$$

$$T = \frac{4}{D} \sqrt{\frac{\pi m}{\rho_ж g}};$$

7.10.



$$J \cdot \ddot{\varphi} = -mgR \cdot \varphi$$

$$\frac{3mR^2}{2} \cdot \ddot{\varphi} = -mgR \cdot \varphi$$

$$\ddot{\varphi} = -\frac{2g\varphi}{3R};$$

$$\omega^2 = \frac{2g}{3R};$$

$$T = 2\pi \sqrt{\frac{3R}{2g}};$$

$$T = 2\pi \sqrt{\frac{l}{g}}; \Rightarrow l = \frac{3R}{2};$$

7.11.

l; за τ в n раз уменьши ~~ся~~ энергия

$$E = E_0 \cdot e^{-2\beta\tau}$$

$$\frac{1}{n} = e^{-2\beta\tau}; \quad \ln n = 2\beta\tau$$

$$\beta = \frac{\ln n}{2\tau}; \quad T = 2\pi\sqrt{\frac{l}{g}}$$

$$Q = \frac{\pi}{\beta T} = \frac{\pi\sqrt{g}\,2\tau}{2\pi\sqrt{l}\ln n} = \frac{\tau}{\ln n}\sqrt{\frac{g}{l}};$$

7.12. ω_0; τ

$$Q = \frac{\pi}{\beta T} = \frac{\pi\tau\omega_0}{2\pi} = \frac{\tau\omega_0}{2};$$

7.13. ω_1, ω_2

$$A = \frac{f_m}{|\omega_0^2 - \omega^2|}; \quad |\omega_{рез}^2 - \omega_1^2| = |\omega_{рез}^2 - \omega_2^2|;$$

$$\omega_0^2 - \omega_1^2 = -\omega_0^2 + \omega_2^2$$

$$2\omega_0^2 = \omega_1^2 + \omega_2^2$$

$$\Rightarrow \quad \omega_{рез} = \sqrt{\frac{\omega_1^2 + \omega_2^2}{2}};$$

7.14 Q; $\frac{a_{рез}}{a} = \frac{\pi}{Q}$;

7.15. m, ω, φ; $Q - ?$

$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f_m \cos(\omega t + \varphi)$?

$x = a\cos\omega t$

$\dot{x} = -a\omega \sin\omega t$

$\ddot{x} = -a\omega^2 \cos\omega t$

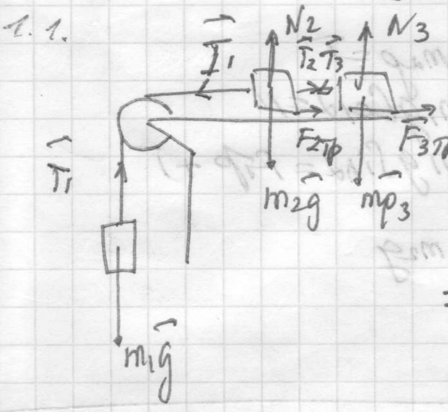
$-a\omega^2 \cos\omega t - 2\beta \cdot$

$$tg\,\varphi = \frac{2\beta\omega}{\omega_0^2 - \omega^2}; \quad \omega_0 = \sqrt{\frac{k}{m}}$$

$$\beta = \frac{tg\,\varphi(\omega_0^2 - \omega^2)}{2\omega}$$

$$Q = \frac{\pi}{\beta T} = \frac{\omega}{2\beta} = \frac{\omega^2}{tg\,\varphi(\omega_0^2 - \omega)};$$

1.1.

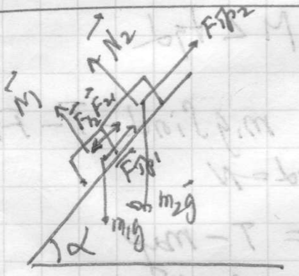


$$\begin{cases} T_2 = T_3 \\ a = a_1 = a_2 = a_3 \\ m_1 a_1 = m_1 g - T_1 \\ m_2 a_2 = T_1 - T_2 - \mu m_2 g \\ m_3 a_3 = T_3 - \mu m_3 g \end{cases}$$

$$\Rightarrow T_2 = \frac{m_3 m_1 g(1+\mu)}{m_1 + m_2 + m_3}$$

1
2
3
4
5
6
7
4

2.



$m_1, m_2, \mu_1, \mu_2, \mu_1 > \mu_2$

$$\begin{cases} \mathrm{I}: & m_1 a = m_1 g \sin\alpha + F_{21} - \mu_1 m_1 g \cos\alpha \\ \mathrm{II}: & m_2 a = m_2 g \sin\alpha - F_{12} - \mu_2 m_2 \end{cases}$$

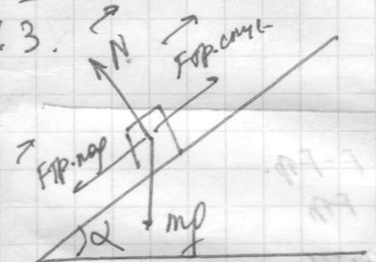
$$a = g\sin\alpha + \frac{F_{21}}{m_1} - \mu_1 g \cos\alpha$$

$$a = g\sin\alpha - \frac{F_{12}}{m_2} - \mu_2 g \cos\alpha$$

$$\frac{F_{21}}{m_1} - \mu_1 g\cos\alpha = -\frac{F_{12}}{m_2} - \mu_2 g\cos\alpha; \quad |F_{21}| = |F_{12}| = |F|$$

$$F\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = g\cos\alpha(\mu_1 - \mu_2); \quad F = \frac{g\cos\alpha(\mu_1-\mu_2)}{\frac{1}{m_1} + \frac{1}{m_2}}$$

1.3.



$$\begin{cases} a_п \cdot m = -mg\sin\alpha - \mu mg\cos\alpha \\ a_{сп} \cdot m = mg\sin\alpha - \mu mg\cos\alpha; \end{cases}$$

$$a_п = -g(\sin\alpha + \mu\cos\alpha)$$

$$a_{сп} = g(\sin\alpha - \mu\cos\alpha)$$

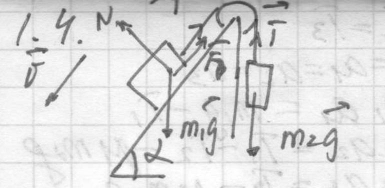
$$S_{под} = v_0 t_п - \frac{a_п t_п^2}{2}; \quad v = v_0 - a_п t_п; \; v = 0$$

$$v_0 = a_п t_п; \quad S_{под} = a_п t_п^2 - \frac{a_п t_п^2}{2} = a_п t_п^2/2;$$

$$S_{спуск} = \frac{a_с t_с^2}{2} = S_{под} = \frac{a_п t_п^2}{2}; \iff a_п t_п^2 = a_с t_с^2, \; \frac{a_п}{a_с} = \frac{t_с^2}{t_п^2} = n^2$$

$$\frac{\sin\alpha + \mu\cos\alpha}{\sin\alpha - \mu\cos\alpha} = n^2 \Rightarrow \mu = \frac{n^2-1}{n^2+1}\,\mathrm{tg}\,\alpha;$$

1.4.



$a = 0$.

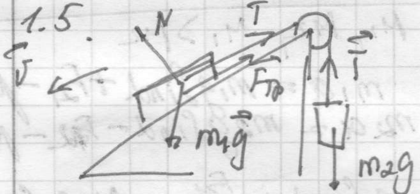
$$\begin{cases} m_2 g = T \\ m_1 g \cos\alpha = N \\ m_1 g \sin\alpha = F_{тр} + T \end{cases}$$

$m_1 g \sin\alpha = \mu m_1 g \cos\alpha + m_2 g$

$g m_1 (\sin\alpha - \mu\cos\alpha) = m_2 g$

$\Leftrightarrow \dfrac{m_1}{m_2} = \dfrac{1}{\sin\alpha - \mu\cos\alpha}$;

1.5.



$\dfrac{m_1}{m_2} = n$; $\mu < tg\alpha$

$$\begin{cases} m_1 a = m_1 g \sin\alpha - T - F_{тр}; \\ m_1 g \cos\alpha = N \\ m_2 a = T - m_2 g \end{cases}$$

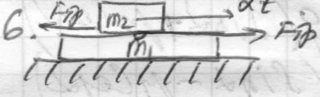
$m_1 a = m_1 g \sin\alpha - m_2 a - m_2 g - \mu m_1 g \cos\alpha$

$a(m_1 + m_2) = g(m_1 \sin\alpha - m_2 - \mu m_1 \cos\alpha)$

$a = \dfrac{m_1 \sin\alpha - m_2 - \mu m_1 \cos\alpha}{m_1 + m_2} g \;\Big|\; : m_2$;

$a = g\dfrac{n(\sin\alpha - \mu\cos\alpha) - 1}{n + 1}$;

1.6.



$$\begin{cases} m_2 a_2 = F - F_{тр}. \\ m_1 a_1 = F_{тр} \end{cases}$$

$F_{тр.max} = \mu m_2 g$, при $t < t_{кр}$ $a_1 = a_2$

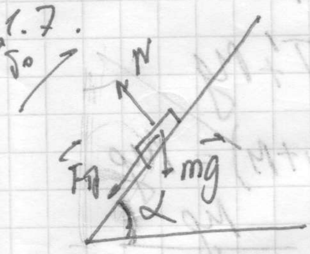
$a(m_1 + m_2) = \alpha t$; $a = \dfrac{\alpha t}{m_1 + m_2}$

при $t > t_{кр}$.

$a_1 = \dfrac{\mu m_2 g}{m_1}$; $\dfrac{\mu m_2 g}{m_1} = \dfrac{\alpha t_{кр} - \mu m_2 g}{m_2}$

$\dfrac{\mu m_2^2 g}{m_1} + \mu m_2 g = \alpha t_{кр}$; $t_{кр} = \dfrac{\mu m_2 g (m_2 + m_1)}{m_1 \alpha}$;

1.7.



$$ma = -mg\sin\alpha - \mu mg\cos\alpha$$

$$a = -g(\sin\alpha + \mu\cos\alpha)$$

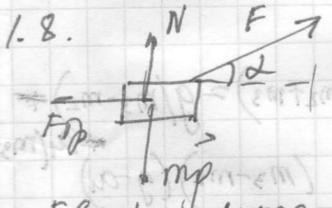
$$v_k^2 - v_x^2 = 2aS; \quad v_k = 0; \quad S = \frac{v_0^2}{2g(\sin\alpha + \mu\cos\alpha)};$$

v_0 – фиксир. величина:

$$\sin\alpha + \mu\cos\alpha = max = \sqrt{1+\mu^2}\,\sin(\underbrace{\alpha + \operatorname{arctg}\mu}_{\frac{\pi}{2}});$$

$$\alpha = \pi/2 - \operatorname{arctg}\mu;$$

1.8.



$$ma = 0; \quad \mu,$$

$$\begin{cases} F_{тр} = F\cos\alpha \\ mg = N + F\sin\alpha \end{cases}$$

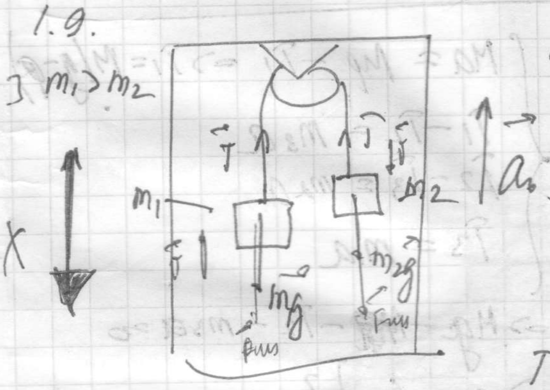
$$F\cos\alpha = \mu mg - \mu F\sin\alpha; \quad F = \frac{\mu mg}{\mu\sin\alpha + \cos\alpha} \doteq min$$

$$\mu\sin\alpha + \cos\alpha = \sqrt{1+\mu^2}\,\sin(\underbrace{\alpha + \operatorname{arcctg}\mu}_{\pi/2})$$

$$\alpha = \pi/2 - \operatorname{arcctg}\mu;$$

1.9.

$\exists\ m_1 > m_2$



$$\begin{cases} m_1 a = -T + m_1 g + m_1 a_0 \\ -m_2 a = -T + m_2 g + m_2 a_0 \end{cases}$$

$$-\frac{T}{m_1} + g + a_0 = \frac{T}{m_2} - g - a_0$$

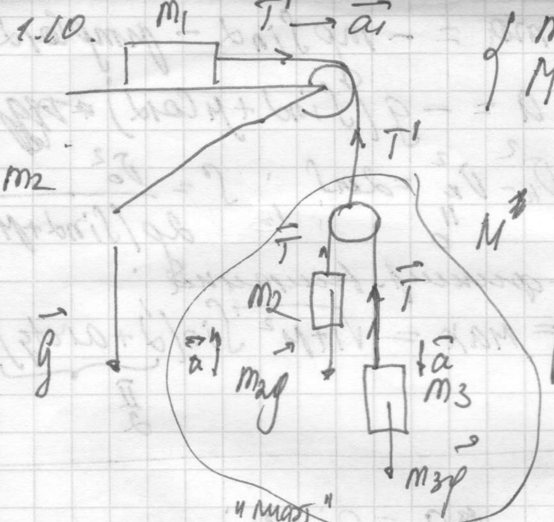
$$T\left(\frac{1}{m_2} + \frac{1}{m_1}\right) = 2g + 2a_0$$

$$T = \frac{2(g+a_0)}{\frac{1}{m_2} + \frac{1}{m_1}}$$

$$m_{бл} \cdot a_{бл} = 2T - P$$

$$2T = P = \frac{4(g+a_0)}{\frac{1}{m_2} + \frac{1}{m_1}};$$

1.10



$\exists\, m_3 > m_2$

$\vec{a}_1 \downarrow \quad \vec{g} \downarrow$

$$\begin{cases} m_1 a_1 = T' \\ M a_1 = -T' + Mg \end{cases}$$

$$a_1 (m_1 + M) = Mg$$

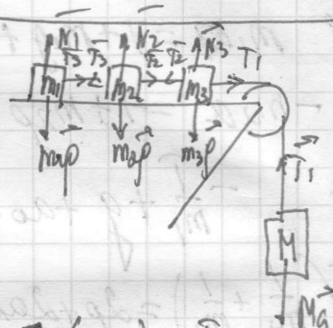
$$a_1 = \frac{Mg}{m_1 + M} = \frac{(m_2 + m_3) g}{m_1 + m_2 + m_3};$$

$F_{инерц}$

$$\begin{cases} m_3 a = m_3 g - T - m_3 a_1 \\ a m_2 = T - m_2 g + m_2 a_1 \end{cases} \Rightarrow a(m_2 + m_3) = g(m_3 - m_2) - a_1(m_3 - m_2) = (m_3 - m_2)(g - a_1)$$

$$a = \frac{(m_3 - m_2)\left(g - g\frac{(m_2 + m_3)}{m_1 + m_2 + m_3}\right)}{m_3 + m_2} = \frac{g(m_3 - m_2)\left(1 - \frac{m_3 + m_2}{m_1 + m_2 + m_3}\right)}{m_3 + m_2}$$

1.11



T_2 – ?
Тр-ней.

$$\begin{cases} Ma = Mg - T_1 \Rightarrow T_1 = M(g - a) \\ T_1 - T_2 = m_3 a \\ T_2 - T_3 = m_2 a \\ T_3 = m_1 a \end{cases}$$

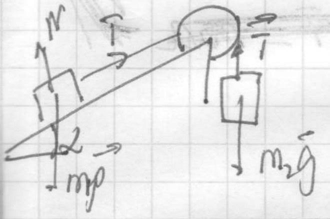
$$\begin{cases} M(g - a) - T_2 = m_3 a \Rightarrow Mg - Ma - T_2 - m_3 a = 0 \\ T_2 - m_1 a = m_2 a \Rightarrow a = \frac{T_2}{m_2 + m_1}; \end{cases}$$

$$a = \frac{Mg - T_2}{M + m_3} = \frac{T_2}{m_2 + m_1}; \quad \cancel{T_2 M - T_2 m_3 = Mg(m_2 + m_1) + T_2(m_2 + m_1)};$$

$$\cancel{T_2(M - m_3 - m_2 - m_1) = Mg(m_2 + m_1)}$$

$$T_2 = \frac{Mg(m_2+m_1)}{M+m_3+m_2+m_1}$$

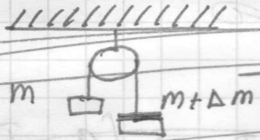
1.12



$$\begin{cases} m_2 a = m_2 g - T \\ m_1 a = T - mg\sin\alpha \end{cases}$$

$$\Rightarrow a = \frac{m_2 g - m_1 g\sin\alpha}{m_2+m_1} = \frac{g}{m_2+m_1}(m_2 - m_1\sin\alpha)$$

1.13.



$\cancel{-mg+T=+ma}$; $\cancel{T=m(g+a)}$

$\cancel{(m+\Delta m)g - T = (m+\Delta m)a}$

1.13

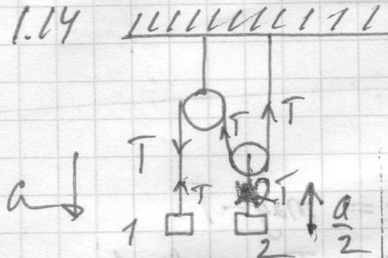
$$\begin{cases} ma = -mg+T \quad \Rightarrow T = m(g+a) \\ (m+\Delta m)g - T = (m+\Delta m)a \\ \Delta m g - F = \Delta m a \end{cases}$$

$(m+\Delta m)g - m(g+a) = (m+\Delta m)a$; $a = \frac{(m+\Delta m)g - mg}{m+\Delta m+m} = \frac{\Delta m g}{2m+\Delta m}$

$F = \Delta m(g-a) =$

$= \Delta m g\left(1 - \frac{\Delta m}{2m+\Delta m}\right) = \Delta m g\left(\frac{2m}{2m+\Delta m}\right)$;

1.14



$$\begin{cases} m_1 g - T = m_1 a \\ m_2 g - 2T = m_2 \dfrac{a}{2} \quad \cancel{\times 2}. \end{cases}$$

$$\cancel{2m_2 g - 4T = m_2 a}$$

Нитка имеет
верх. один-с уск.-е a
а у т.2 - 2 нитки

$$T + T + F = \underbrace{m \cdot a_{бл}}_{0} = 0$$

$$F = -2T$$

$$(1): a = g - T/m_1; \quad \cancel{m_2 g - 2T = \frac{m_2}{2}\left[g - \frac{T}{m_1}\right]} =$$

$$= \frac{m_2 g}{2} - \frac{m_2}{2m_1} \cdot T; \quad T\left(-2 + \frac{m_2}{2m_1}\right) = \frac{m_2 g}{2} - m_2 g$$

$$T = \frac{\frac{m_2 g}{2}}{-2 + \frac{m_2}{2m_1}} = \frac{m_2 g}{-\frac{m_2}{m_1} + 4};$$

$$(1): \quad a = g - \frac{T}{m_1}$$

$$(2): \quad a = \frac{-2m_2 g - 4T}{m_2} = -2g - \frac{4T}{m_2};$$

$$g - \frac{T}{m_1} = -2g - \frac{4T}{m_2} \quad -g = T\left(\frac{4}{m_2} - \frac{1}{m_1}\right);$$

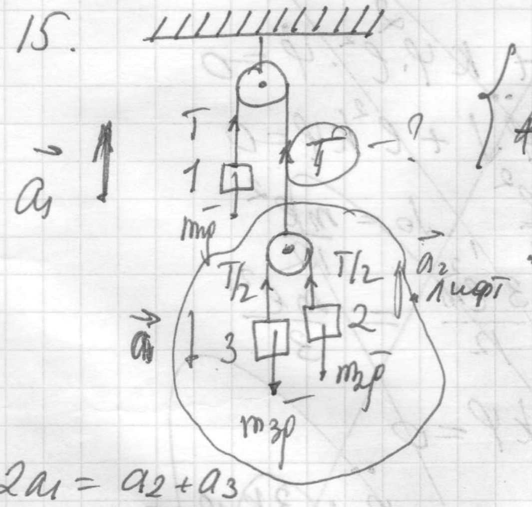
$$T = \frac{g}{\frac{4}{m_2} - \frac{1}{m_1}};$$

1.15 $\begin{cases} m_1 g - T = m_1 a \Rightarrow a = g - T/m_1; \\ m_2 g - 2T = -m_2 \dfrac{a}{2} \Rightarrow a = \dfrac{2m_2 g - 4T}{-m_2} = \dfrac{4T}{m_2} - 2g \end{cases}$

$$\frac{4T}{m_2} - 2g = g - \frac{T}{m_1}; \quad 3g = T\left(\frac{4}{m_2} + \frac{1}{m_1}\right)$$

$$T = \frac{3g}{\frac{4}{m_2} + \frac{1}{m_1}} = \frac{3g\, m_2 m_1}{m_2 + 4m_1} = \frac{3m_2 g}{4 + \frac{m_2}{m_1}};$$

1.15.



$\begin{cases} \cancel{m_1 a_1 = m_1 g - T} \\ \cancel{+ m a_1 = T + mg} \end{cases}$

$a_1 =$

$$\begin{cases} 2a_1 = a_2 + a_3 \\ T - m_1 g = a_1 m_1 \\ T/2 - m_2 g = m_2 a_2 \\ T/2 - m_3 g = -m_3 a_3 \end{cases}$$

$a_2 = \dfrac{T}{2m_2} - g$

$\cancel{g(m_3 - m_2) = m_2 a_2 + m_3 a_3 =}$
$\cancel{= m_2 (2a_1 - a_3)}$

$a_3 = g - \dfrac{T}{2m_3}$

$$2a_1 = \frac{T}{2m_2} - g + g - \frac{T}{2m_3} = \frac{T}{2}\left(\frac{1}{m_2} - \frac{1}{m_3}\right)$$

$$T = m_1(g + a_1); \Rightarrow a_1 = \frac{T}{m_1} - g;$$

$$2\left[\frac{T}{m_1} - g\right] = \frac{T}{2}\left(\frac{1}{m_2} - \frac{1}{m_3}\right); \quad T\left(\frac{2}{m_1} - \frac{1}{2m_2} + \frac{1}{2m_3}\right) = 2g$$

$$T = \frac{2g}{\frac{2}{m_1} - \frac{1}{2m_2} + \frac{1}{2m_3}}$$

$$\cancel{L = 4y - 2x = 2(2y - x) \Rightarrow}$$
$$\cancel{2y - x = L/2}$$

$\cancel{y = \frac{L}{2\sqrt{3}}}$ $\cancel{y - 2x = L\left(1 - \frac{\sqrt{3}}{2}\right)}$ ~~подставили~~

~~в 2~~

$$\cancel{0{,}75\, y'^2 = gL\left(1 - \frac{\sqrt{3}}{2}\right) = gL\frac{2-\sqrt{3}}{2}}$$

$$\cancel{y' = \sqrt{\frac{4}{3} gL \frac{2-\sqrt{3}}{2}} \Rightarrow}$$

$$\cancel{v = \sqrt{\frac{2gL(2-\sqrt{3})}{3}}}$$

~~Ответ:~~ $\cancel{v = \sqrt{\frac{2gL(2-\sqrt{3})}{3}}}$

N 3-1.

$$F_ц = m\omega^2 R \sin\theta + 2m[\vec{v}\vec{\omega}] =$$

$$= m\omega^2 R \sin\theta$$

$$\begin{cases} N\sin\theta = F_ц = m\omega^2 R\sin\theta \\ N\cos\theta = mg \end{cases}$$

$$N = m\omega^2 R \Rightarrow \cancel{m}\omega^2 R\cos\theta = \cancel{m}g$$

$$\cos\theta = \frac{g}{\omega^2 R}$$

Ответ: 1) $\theta = \arccos\frac{g}{\omega^2 R}$ – уст. пол. равнов.
2) $\theta = 0$.

№2-9.

$h = l\sin\alpha$.

$$Mgh = \frac{Mv_1^2}{2} \Rightarrow v_1 = \sqrt{2gh}$$

по ЗСИ: $Mv_1 - mv = 0 \Rightarrow$

$$v = \frac{M}{m}\sqrt{2gl\sin\alpha}$$

Ответ: $v = \frac{M}{m}\sqrt{2gl\sin\alpha}$

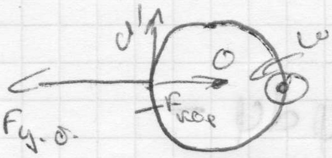
№2-10.

ЗСИ: $\begin{cases} m_1 v_0 = m_2 v_2 \cos\alpha \\ m_1 v_1 = m_2 v_2 \sin\alpha \end{cases}$

$\frac{m_2}{m_1} = n \Rightarrow \begin{cases} \frac{v_0}{v_2\cos\alpha} = n \\ \frac{v_1}{v_2\sin\alpha} = n \end{cases} \Rightarrow$

$\sin\alpha = \frac{v_1}{n v_2} \Rightarrow$

№3-2



$$\vec{F}_{уб} = -\vec{F}_{кор}$$

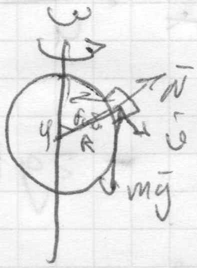
$$m\omega^2 \cdot 2R = 2m[\vec{v}'; \vec{\omega}]$$

$$\omega^2 2R = 2v'\omega$$

$$v' = \omega R \Rightarrow a' = \frac{v'^2}{R} = \omega^2 R$$

Ответ: $a' = \omega^2 R$

№3-3



$$(1)\quad m\frac{dv}{dt} = mg\sin\varphi - F_{у.б.}\cos\varphi =$$

$$= mg\sin\varphi - m\omega^2 R\sin\varphi\cos\varphi$$

$$(2)\quad \frac{mv^2}{R} = mg\cos\alpha + F_{у.б.}\sin\varphi - N$$

Условие отрыва: $N = 0$.

Найдем φ, при котором тело оторвется: из (1):

$$dv = dt(g\sin\varphi - \frac{1}{2}\omega^2 R\sin 2\varphi)$$

$$v = \frac{dl}{dt} = \frac{R d\varphi}{dt}$$

$$dt = \frac{R d\varphi}{v} \Rightarrow v\, dv = gR\cdot\sin\varphi\, d\varphi -$$

$$- \frac{\omega^2 R^2}{2} \sin 2\varphi \, d\varphi.$$

$$\int_0^{\upsilon} \upsilon \, d\upsilon = gR \int_0^{\varphi} \sin\varphi \, d\varphi -$$

$$- \frac{\omega^2 R^2}{2} \int_0^{\varphi} \sin 2\varphi \, d\varphi \text{ , т.е.}$$

$$\frac{\upsilon^2}{2} = - gR \cos\varphi + \frac{\omega^2 R^2}{4} \cos 2\varphi$$

$$\Rightarrow$$

$$\upsilon^2 = \frac{\omega^2 R^2}{2} \cos 2\varphi - 2gR\cos\varphi$$

Из 2 найдем υ^2

$$\upsilon^2 = \frac{\omega^2 R^2}{2} \cos 2\varphi - 2gR\cos\varphi =$$

$$= gR\cos\varphi - \omega^2 R^2 \sin^2\varphi$$

$$3gR\cos\varphi = \frac{\omega^2 R^2}{2} \Rightarrow$$

$$\cos\varphi = \frac{\omega^2 R}{6g} \Rightarrow$$

$$F_{у.б.} = m\omega^2 R \sin\varphi = m\omega^2 R *$$

$$\frac{\sqrt{36g^2-\omega^2 R^4}}{6g}.$$

Ответ: $F_{у.б.} = m\omega^2 R \frac{\sqrt{36g^2-\omega^2 R^4}}{6g}.$

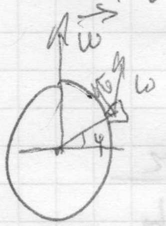
№3-4.

$$-F_{к} = 2m[\bar{v}; \bar{\omega}]$$

$$a_{к} = 2v\omega \sin\varphi.$$

$$F = \sqrt{(-F_{к})^2 + m^2g^2} = m\sqrt{v^2\omega^2\sin^2\varphi + g^2} =$$

$$= m\sqrt{v^2\left(\frac{2\pi}{T}\right)^2\sin^2\varphi + g^2}, \text{ где } T = 24 \text{ ч}.$$

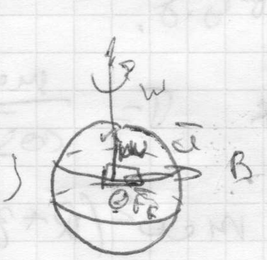


$$\bar{F} = (-\bar{F}_{к}) + m\bar{g}$$

Ответ: $F = m\sqrt{v^2\left(\frac{2\pi}{T}\right)^2\sin^2\varphi + g^2}$,

где $T = 24$ ч.

№3-5.



$$F_{к} = -2m[\bar{v}, \bar{\omega}]$$

$$|F_{к}| = |-2mv\omega| = \frac{4\pi m v}{T}$$

Ответ: $\bar{F}_{к} = -2m[\bar{v}; \bar{\omega}]$

$F_\kappa = \frac{4\pi m \upsilon}{T}$

№3-6.

$F_{тр} = \mu m g = m\omega^2 R$, где

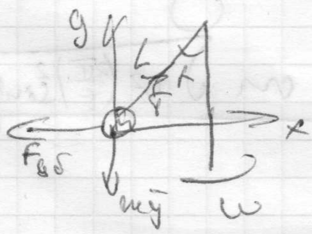
$\omega = \frac{\upsilon}{R}$

$\mu g = \frac{\upsilon^2}{R}$

$\mu = \frac{\upsilon^2}{gR}$

Ответ: $\mu = \frac{\upsilon^2}{gR}$.

№3-7.



$\sin\alpha = \frac{R+r}{R+L}$

Ox: $N + T\sin\alpha = F_{ц.б.}$

Oy: $mg = T\cos\alpha$ $\quad T = \frac{mg}{\cos\alpha}$.

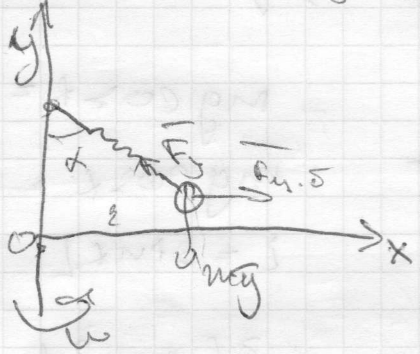
$mg\,tg\alpha = m\omega^2(R+r)$

$g\frac{(R+r)}{\sqrt{(L-r)(2R+L+r)}} = \omega^2(R+r)$

$$\omega = \sqrt{\frac{g}{(L-l)(2R+L+l)}}$$

Ответ:
$$\omega_{min} = \sqrt{\frac{g}{(L-l)(2R+L+l)}}$$

№3-8.



$Ox: F_y \sin\alpha = F_{ц.б}$

$Oy: F_y \cos\alpha = mg$

где $F_y = k\Delta L$

$F_{цб} = m\omega^2 r = m\omega^2 (L+\Delta L)\sin\alpha$

$$k\Delta L \sin\alpha = m\omega^2 (L+\Delta L)\sin\alpha.$$

$$k\Delta L = m\omega^2 L + m\omega^2 \Delta L$$

$$\Delta L (k - m\omega^2) = m\omega^2 L$$

$$\Delta L = \frac{m\omega^2 L}{k - m\omega^2}.$$

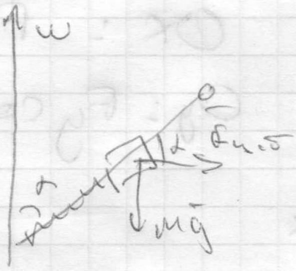
$$k\Delta L\cos\alpha = mg = k\frac{m\omega^2 L}{k-m\omega^2}\cos\alpha.$$

$$g = \frac{k\omega^2 L}{k-m\omega^2}\cos\alpha.$$

$$\cos\alpha = \frac{g(k - m\omega^2)}{k\omega^2 L}$$

Ответ: $\alpha = \arccos \frac{g(k - m\omega^2)}{k\omega^2 L}$

№3-9



$$mg\cos\alpha = F_{ц.б.}\sin\alpha$$

$$mg\cos\alpha = m\omega^2 \xi \sin\alpha$$

$$\xi = \sin\alpha L$$

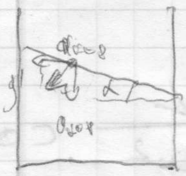
$$g\cos\alpha = \omega^2 L \sin^2\alpha.$$

$$\omega^2 = \frac{g\cos\alpha}{L\sin^2\alpha} \Rightarrow$$

$$\omega = \frac{\sqrt{g\cos\alpha}}{\sqrt{L}\sin\alpha}$$

Ответ: $\frac{1}{\sin\alpha}\sqrt{\frac{g\cos\alpha}{L}}$

№3-10.



В системе реки

$$\bar g' = \bar a_{кор} + \bar g$$

$$\bar g' = \sqrt{a_{кор}^2 + g^2}, \text{ где}$$

$$\bar a_{кор} = 2[\bar v, \bar\omega] = 2v\omega$$

$$g' = \sqrt{4v^2\omega^2 + g^2} \Rightarrow$$

$$\operatorname{tg}\varphi = \frac{a_{кор}}{g} = \frac{2v\omega}{g}.$$

Ответ: $\varphi = \operatorname{arctg}\left(\frac{2v\omega}{g}\right)$

№3-11

$$\begin{cases} m\frac{dv}{dt} = mg\sin\alpha \\ \frac{mv^2}{R} = mg\cos\alpha - N = 0. \end{cases}$$

$$v = \frac{dl}{dt} = \frac{R\,d\alpha}{dt}$$

$$dt = \frac{R\,d\alpha}{v}$$

$$\begin{cases} dv = g\sin\alpha\,dt \\ \frac{v^2}{R} = g\cos\alpha \Rightarrow \cos\alpha = \frac{v^2}{Rg} \Rightarrow \end{cases}$$

$$v^2 = 2Rg\left(1 - \frac{v^2}{Rg}\right) = 2Rg - 2v^2 \Rightarrow$$

$$v = \sqrt{\frac{2Rg}{3}}$$

Ответ: $v = \sqrt{\frac{2Rg}{3}}$

№3-12.

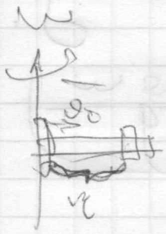
$$\bar{F} = -2m[\bar{v}; \bar{\omega}] \Rightarrow |F| = 2mv\omega$$

$$\bar{v} = v_0 + \omega\bar{z}$$

$$v = \sqrt{v_0^2 + \omega^2 v_0^2 t^2} \Rightarrow F_{кор} = 2m\omega v_0\sqrt{1+\omega^2 t^2}$$

Ответ: $\nearrow$

№3-13.



$$F_к = -2m[\bar{\omega}; \bar{v}]$$

$$|F_к| = 2mv\omega.$$

$$v = \sqrt{v_0^2 + v^2} = \sqrt{v_0^2 + (\omega r)^2}$$

$$F_{кор} = 2m\omega\sqrt{v_0^2 + (\omega r)^2}$$

Ответ: $F_{кор} = 2m\omega\sqrt{v_0^2 + (\omega r)^2}$

№3-14.

$$\bar{F}_и = (-\bar{F}_к) + \bar{F}_{ц.б.}$$

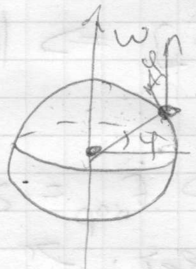
$$F_и = \sqrt{(m\omega^2 r)^2 + 4m^2(v_{отн}\omega)^2} =$$

$$= m\sqrt{4(v_{отн}\omega)^2 + (\omega^2 r)^2}$$

$$F_0 = \sqrt{g^2 + (2v_{отн}\omega)^2 + (\omega^2 r)^2}$$

Ответ: $F_0 = \sqrt{g^2 + (2v_{отн}\omega)^2 + (\omega^2 r)^2}$

№3-15.



t - время полета

$$t = \frac{l}{v}$$

Пуля отклонится на

~~восток~~ с уменьш.

$$a = 2[v\omega] = 2v\omega\sin\varphi =$$

$$= 2v\frac{2\pi}{T}\sin\varphi,\ \text{где } T = 24\text{ч}.$$

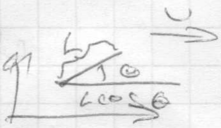
$$\Delta x = \frac{at^2}{2} = \frac{v\cdot 2\pi\sin\varphi\cdot s^2}{Tv^2} = \frac{2\pi s^2\sin\varphi}{Tv}.$$

Ответ: на восток на

$$\Delta x = \frac{2\pi s^2\sin\varphi}{Tv}.$$

~~...~~

№ 4-1.



~~lcos φ~~

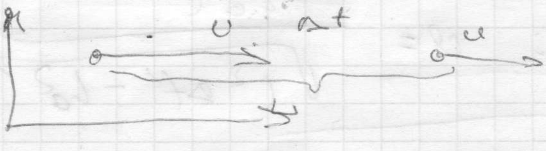
$$l\cos\theta = l_x\sqrt{1-\left(\frac{v}{c}\right)^2} \qquad l_x = \frac{l\cos\theta}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$l_\theta = \sqrt{l_y^2 + l_x^2} = \sqrt{l^2\sin^2\theta + \frac{l^2\cos^2\theta}{1-\left(\frac{v}{c}\right)^2}} =$$

$$= l\sqrt{\sin^2\theta + \frac{\cos^2\theta}{1-\left(\frac{v}{c}\right)^2}}$$

Ответ: ↑

№4-2.



$$L = v\Delta t$$

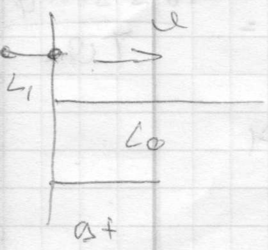
$$L = L_0\sqrt{1-\frac{9}{16}\frac{c^2}{c^2}} =$$

$$= l_0 \sqrt{\frac{16-9}{16}} = l_0 \frac{\sqrt{7}}{4}.$$

$$l_0 = \frac{4}{\sqrt{7}} L = \frac{4}{\sqrt{7}} \upsilon \Delta t.$$

ответ

№4-3



$$L_1 = l_0 \sqrt{1-\frac{\upsilon^2}{c^2}}$$

$$\frac{L_1}{\upsilon} = \Delta t.$$

$$L_1 = \upsilon \cdot \Delta t = l_0 \sqrt{1-\frac{\upsilon^2}{c^2}}$$

$$\upsilon^2 \Delta t^2 = l_0^2 - \frac{\upsilon^2}{c^2} l_0^2$$

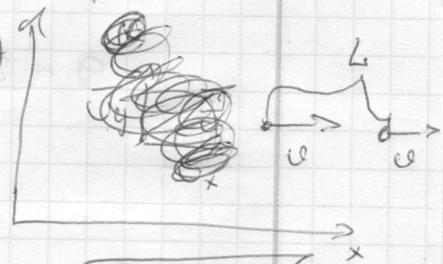
$$\upsilon^2 \Delta t^2 - \frac{\upsilon^2 l_0^2}{c^2} = l_0^2;$$

$$\upsilon^2 \left(\Delta t^2 - \frac{l_0^2}{c^2}\right) = l_0^2$$

$$\upsilon^2 = \frac{l_0^2 c^2}{(c^2 \Delta t^2 - l_0^2)} \Rightarrow \upsilon = \frac{l_0 c}{\sqrt{c^2 \Delta t^2 - l_0^2}}$$

Ответ: $\upsilon = \frac{l_0 c}{\sqrt{c^2 \Delta t^2 - l_0^2}}$

№4-4.



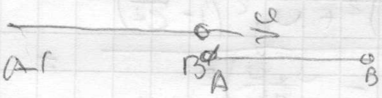
$$L^2 - c^2t^2 = L'^2$$

$$L' = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L^2 - L'^2 = c^2t^2 \Rightarrow t = \frac{\sqrt{L^2 - L'^2}}{c} =$$

$$= \frac{L}{c}\sqrt{1 - 1 + \frac{v^2}{c^2}} = \frac{L}{c}\sqrt{\frac{v^2}{c^2}} = \frac{Lv}{c^2}$$

Ответ: $t = \frac{Lv}{c^2}$



№4-8.

$$t_B = \frac{l_0}{v} \qquad t_{B'} = \frac{L'}{v} = \frac{L_0\sqrt{1-\frac{v^2}{c^2}}}{v}$$

$$\Delta t_B = t_B - t_{B'} = \frac{L_0}{v}\left(1 - \sqrt{1-\frac{v^2}{c^2}}\right) =$$

$$= \frac{L_0}{v}\left(1 - \frac{\sqrt{c^2-v^2}}{c}\right) = \frac{L_0}{vc}\left(c - \sqrt{c^2-v^2}\right)$$

Ответ: $\Delta t_B = \frac{L_0}{vc}\left(c - \sqrt{c^2-v^2}\right)$

№4-6.

$$(c^2t^2) - L^2 = inv$$

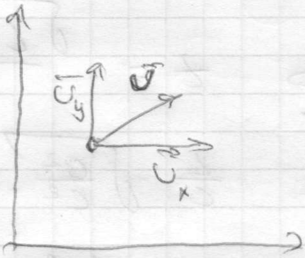
$$25 - 16 = c^2 t_2^2$$

$$t_2 = \frac{3м}{c}$$

№ 4-7.

$$9 - 25 = -l^2 \qquad l^2 = 16 \qquad l = 4 \text{ м.}$$

№ 4-8.



$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u_y' = \frac{u_y \sqrt{1 - \left(\frac{v}{c}\right)^2}}{1 - \frac{u_x v}{c^2}}$$

$$\Rightarrow u' = \sqrt{\frac{(u_x - v)^2 + u_y^2 \left(1 - \frac{v^2}{c^2}\right)^2}{\left(1 - \frac{u_x v}{g^2}\right)^2}} =$$

$$= \frac{\sqrt{u_x^2 - 2u_x v + v^2 + u_y^2 - \frac{u_y^2 v^2}{c^2}}}{1 - \frac{u_x v}{c^2}} =$$

$$= \frac{c\sqrt{c^2 u_x^2 - 2c^2 u_x v + c^2 v^2 + c^2 u_y^2 - u_y^2 v^2}}{c^2 - u_x v}$$

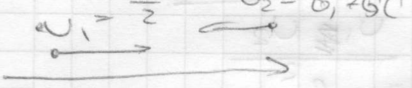
$$\boxed{\text{Ответ: } u' = c \cdot \frac{\sqrt{c^2 u_x^2 - 2c^2 u_x v + c^2 v^2 + c^2 u_y^2 - u_y^2 v^2}}{c^2 - u_x v}}$$

№4-9.

$$u_{отн} = \frac{-v_2 - v_1}{1 + \frac{v_2 v_1}{c^2}} =$$

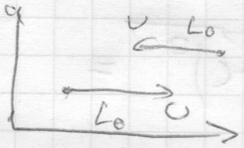
$$= \frac{(-0,5 - 0,75)c}{1 + \frac{0,5 \cdot 0,75\, c^2}{c^2}} = \frac{-1,25}{1,375} c =$$

$$= -0,49c.$$



№4-10.

$$l' = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

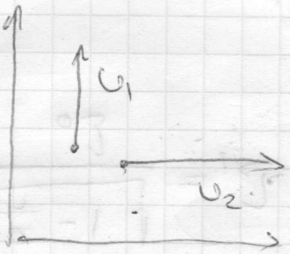


$$v' = \frac{-v - v}{1 + \frac{v^2}{c^2}} = -\frac{2v}{1 + \frac{v^2}{c}}$$

$\Rightarrow$

$$l' = l_0 \sqrt{1 - \frac{4v^2}{c(c^2 + v^2)}}$$

№4-11.



В сис. v_2:

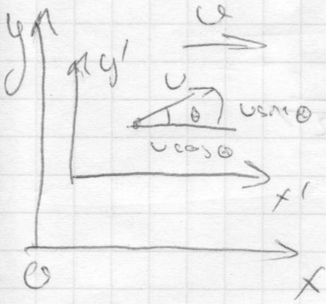
$$v'_{1x} = \frac{v_{1x} - v_2}{1 - \frac{v_{1x} v_2}{c^2}}, \text{ где}$$

$v_{1x} = 0 \Rightarrow$

$$\tau = \frac{\tau'}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{\tau_0}{\sqrt{1-\frac{u^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}}$$

Ответ: $\dfrac{\tau_0}{\sqrt{1-\frac{u^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}}$

№ 4-13.



$$u_x' = \frac{u_x - v}{1-\frac{u_x v}{c^2}}, \quad \text{где} \quad u_x = u\cos\theta$$

$$u_y' = \frac{u_y\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{u_x v}{c^2}}$$

$$u_y = u\sin\theta$$

$$\operatorname{ctg}\theta' = \frac{u_y'}{u_x'} = \frac{(u\cos\theta - v)\left(1-\frac{u\cos\theta v}{c^2}\right)}{\left(1-\frac{\cos\theta v u}{c^2}\right)\left(u\sin\theta\sqrt{1-\frac{v^2}{c^2}}\right)}$$

$$= \frac{u\cos\theta - v}{u\sin\theta\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Ответ: $\theta' = \operatorname{arcctg}\left(\dfrac{u\cos\theta - v}{u\sin\theta\sqrt{1-\frac{v^2}{c^2}}}\right)$

№4-14

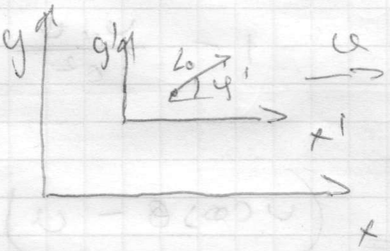
$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L = v\Delta t = \frac{24}{25}c\frac{\Delta t'}{\sqrt{1-\left(\frac{v}{c}\right)^2}} =$$

$$= \frac{24}{25}\frac{\Delta t' c^2}{\sqrt{c^2-v^2}}$$

<u>Ответ: $L = \frac{24}{25}\frac{\Delta t' c^2}{\sqrt{c^2-v^2}}$</u>

№4-15.



$$L_x = L_0\cos\varphi'\sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$L_y = L_0\sin\varphi'$$

$$\operatorname{tg}\varphi = \frac{L_y}{L_x} = \frac{\sin\varphi'}{\cos\varphi'\sqrt{1-\frac{v^2}{c^2}}}$$

<u>Ответ: $\varphi = \operatorname{arctg}\left(\frac{\operatorname{tg}\varphi'}{\sqrt{1-\frac{v^2}{c^2}}}\right)$</u>