m, e ; $\vec{H} = H_0\vec{e_z}$ и поле тяжести $\vec{g} = -g\vec{e_z}$ на пов. $az = x^2+y^2$ L -? з-н движ. -?

1

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$ $\quad x^2+y^2=\rho^2=az \Rightarrow z = \frac{\rho^2}{a}$

$$L = T-U = \frac{m}{2}(\dot\rho^2+\rho^2\dot\varphi^2+\dot z^2)+\frac{e}{c}(\vec{A}\vec{v}) - mgz\,, \quad (\vec{A}\vec{v}) = \frac{1}{2}\rho^2\dot\varphi H_0$$

$$L = \frac{m}{2}(\dot\rho^2+\rho^2\dot\varphi^2+\dot z^2)+\frac{e}{2c}\rho^2\dot\varphi H_0 - \frac{mg\rho^2}{a}\,, \quad \dot z = \frac{2\dot\rho\rho}{a}$$

$$L = \frac{m}{2}\left(\dot\rho^2+\rho^2\dot\varphi^2+\left(\frac{2\dot\rho\rho}{a}\right)^2\right)+\frac{e}{2c}\rho^2\dot\varphi H_0 - \frac{mg\rho^2}{a}$$

1) $\frac{\partial L}{\partial\varphi}=0 \quad \frac{\partial L}{\partial\dot\varphi} = m\rho^2\dot\varphi^2+\frac{H_0 e}{2c}\rho^2 = p_\varphi = const$

2) $\frac{\partial L}{\partial t}=0 \quad E = \frac{\partial L}{\partial\dot q}\dot q - L = \frac{m}{2}\left(\dot\rho^2+\rho^2\dot\varphi^2+\left(\frac{2\dot\rho\rho}{a}\right)^2\right)+mg\frac{\rho^2}{a}$

$$\dot\varphi = \frac{p_\varphi - \frac{eH_0}{2c}\rho^2}{m\rho^2}$$

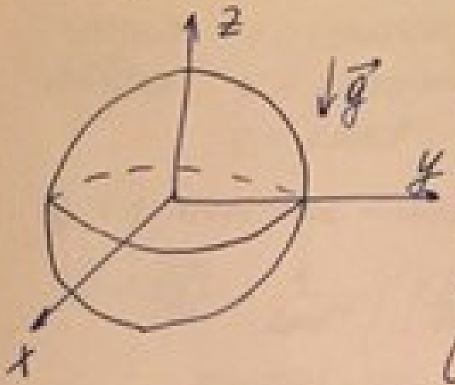
$$E = \frac{m}{2}\left(\dot\rho^2\left(1+\frac{4\rho^2}{a^2}\right)+\rho^2\left(\frac{p_\varphi-\frac{eH_0}{2c}\rho^2}{m\rho^2}\right)^2\right)+mg\frac{\rho^2}{a}$$

$$\dot\rho = \pm\sqrt{\frac{\frac{2}{m}\left(E-\frac{mg\rho^2}{a}\right)-\rho^2\left(\frac{p_\varphi-\frac{eH_0}{2c}\rho^2}{m\rho^2}\right)^2}{1+\frac{4\rho^2}{a^2}}}$$

$$\frac{\dot\varphi}{\dot\rho} = \frac{d\varphi}{d\rho} = \frac{\frac{p_\varphi-\frac{eH_0}{2c}\rho^2}{m\rho^2}}{\pm\sqrt{\;}} \Rightarrow \varphi-\varphi_0 = \pm\int\dots d\rho.$$

m движ. по R в поле силы тяжести. L в цил. коорд. — ? з-н движ. — ?

2



$$\begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \\ z = z \end{cases} \quad \rho = R$$

$$\begin{cases} x = \sqrt{R^2 - z^2}\cos\varphi \\ y = \sqrt{R^2 - z^2}\sin\varphi \\ z = z \end{cases}$$

$$L = \frac{m}{2}\left[(R^2 - z^2)\dot\varphi^2 + \frac{R^2\dot z^2}{R^2 - z^2}\right] - mgz$$

интегр. движения $\frac{\partial L}{\partial \dot\varphi} = m(R^2 - z^2)\dot\varphi = P_\varphi$

$$E = \underbrace{\dot\varphi\frac{\partial L}{\partial \dot\varphi} + \dot z\frac{\partial L}{\partial \dot z}}_{\sum_{i=1}^{2}\dot q_i \frac{\partial L}{\partial \dot q_i}} - L = \frac{m}{2}\left[(R^2 - z^2)\dot\varphi^2 + \frac{R^2\dot z^2}{R^2 - z^2}\right] = E_0 = const$$

з-н движение $$\int_{-R}^{R}\frac{dz}{\sqrt{\frac{2g}{R^2}\left[\left(\frac{E_0}{mg} - z\right)(R^2 - z^2) - \frac{P_\varphi^2}{2m^2g}\right]}} = t + t_0 .$$

Част. по сфере R в поле силы тяж. 3-и физ. методы

сферич. коорд. $x=\rho\sin\theta\cos\varphi$ $\quad$ $\Gamma.\mathcal{Я}-?$

$y=\rho\sin\theta\sin\varphi$ $\quad \rho=R$ $\quad L=\frac{m}{2}(R^2\sin^2\theta\dot\varphi^2+R^2\dot\theta^2)-mgR\cos\theta$

$z=\rho\cos\theta$

3

Ур. Гамильтона - Якоби: $0=\frac{\partial F}{\partial t}+H\left(\frac{\partial F}{\partial q},q,t\right)$

$$0=\frac{\partial F}{\partial t}+\frac{1}{2mR^2}\left(\left(\frac{\partial F}{\partial\theta}\right)^2+\frac{\left(\frac{\partial F}{\partial\varphi}\right)^2}{\sin^2\theta}\right)+mgR\cos\theta$$

$$F=T(t)+\Theta(\theta)+\Psi(\varphi)$$

$$0=T'+\frac{1}{2mR^2}\left(\Theta'^2+\frac{\Psi'^2}{\sin^2\theta}\right)+mgR\cos\theta$$

1) $-T'=\ldots=C_1=E \Rightarrow T=-Et$

2) $E=\frac{1}{2mR^2}\left(\Theta'^2+\frac{\Psi'^2}{\sin^2\theta}\right)+mgR\cos\theta$

$E\cdot 2mR^2\sin^2\theta=\Theta'^2\sin^2\theta+\Psi'^2+2m^2gR^3\cos\theta\sin^2\theta$

$E\,2mR^2\sin^2\theta-2mgR^3\sin^2\theta\cos\theta-\Theta'^2\sin^2\theta=\Psi'^2$

$\Psi'=\pm\sqrt{}=C_2 \quad \Psi(\varphi)=C_2\varphi$

$\Theta'^2=-\frac{C_2^2}{\sin^2\theta}+2mR^2E-2m^2gR^3\cos\theta$

$\Theta'=\pm\sqrt{-\frac{C_2^2}{\sin^2\theta}+2mR^2E+2m^2gR^3\cos\theta}$

$F=-Et+C_2\varphi\pm\int d\theta\sqrt{}$

$\frac{\partial F}{\partial\varphi}=p_\varphi=C_2 \quad \frac{\partial F}{\partial\theta}=p_\theta$

$E=\frac{1}{2mR^2}\left(\frac{p_\varphi^2}{\sin^2\theta}-p_\theta^2\right)+mgR\cos\theta$

3) $\frac{\partial F}{\partial C_1}=\alpha_1=-t\pm\int\limits^{\theta} d\theta\,\frac{2mR^2}{2\sqrt{}}$

$t-t_0=\pm\int\limits_{\theta_0}\frac{d\theta\, mR^2}{2\sqrt{}}$

$\frac{\partial F}{\partial C_2}=\varphi\pm\int d\theta\,\frac{(-C_2)}{\sin^2\theta\cdot 2\sqrt{}}$

$\varphi-\varphi_0=\pm\int\limits_{\theta_0}^{\theta}\ldots$

$= \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) - a\cos\varphi$ интегр. движ. — ?

$P = q + e^{-q} + \ln p, \quad Q = pe^{q}$ — явл. канонич. найти производ. ф-цию

4

Критерий каноничности:
$$\begin{cases} \{P_i(p,q,t), Q_i(p,q,t)\}_{pq} = c\delta_{ij} \\ \{P_i(p,q,t), P_j(p,q,t)\}_{pq} = 0 \\ \{Q_i(p,q,t), Q_j(p,q,t)\}_{pq} = 0 \end{cases}$$

$$\{P_i, P_j\} = \{q + e^{-q} + \ln p, q + e^{-q} + \ln p\} = \overbrace{\{q, q\}}^{0} + \overbrace{\{q, e^{-q}\}}^{0} + \{q, \ln p\} +$$
$$+ \underbrace{\{e^{-q}, q\}}_{0} + \underbrace{\{e^{-q}, e^{-q}\}}_{0} + \{e^{-q}, \ln p\} + \{\ln p, q\} + \{\ln p, e^{-q}\} + \underbrace{\{\ln p, \ln p\}}_{0} = 0$$

Очевидно, $\{Q_i, Q_j\} = 0$ из-за некоммутативности скобок Пуассона.

$$\{P, Q\}_{pq} = \frac{\partial P}{\partial p}\frac{\partial Q}{\partial q} - \frac{\partial P}{\partial q}\frac{\partial Q}{\partial p} = \frac{1}{p}pe^{q} - (1 - e^{-q})e^{q} = e^{q} - e^{q} + 1 = 1 \checkmark$$

$$\det \frac{\partial Q_i}{\partial p_j} = \frac{\partial Q}{\partial p} = e^{q} \neq 0 \Rightarrow \exists F_1(q, Q, t)$$

$$\begin{cases} cp_i = \frac{\partial F_1}{\partial q_i} \\ P_i = -\frac{\partial F_1}{\partial Q_i} \end{cases} \quad \begin{cases} p = \frac{\partial F_1}{\partial q} = Qe^{-q} \\ P = -\frac{\partial F}{\partial Q} = q + e^{-q} + \ln(Qe^{-q}) = q - q + e^{-q} + \ln Q = e^{-q} + \ln Q. \end{cases}$$

$$F_1 = \int Qe^{-q}dq + f_1(Q, t) = -Qe^{-q} + f_1(Q, t)$$

$$F_2 = -\int (e^{-q} + \ln Q)dQ + f_2(q, t)$$

$$\int dQ \ln Q = \int (d(Q\ln Q) - Q\,d(\ln Q)) = Q\ln Q - Q$$

$$F_1 = -Qe^{-q} - Q\ln Q + Q + \underbrace{f_2(q, t)}_{0} \Rightarrow f_1 = -Q\ln Q + Q$$

$$F_1 = -Qe^{-q} + Q\ln Q + Q$$ — производящ. ф-ция

$\mathcal{L} = \frac{m}{2}(\dot\rho^2 + \rho^2\dot\varphi^2) - \frac{a\cos\varphi}{\rho^2}$ интегр. движ. - ?

з-н движ. в квадратурах - ?

$\frac{\partial \mathcal{L}}{\partial t} = 0$, т.к. лагранжиан явно не завис. от времени

5

Следовательно, энергия - интеграл движения

$$E = \frac{m}{2}\dot\rho^2 + \frac{m}{2}\rho^2\dot\varphi^2 + \frac{a\cos\varphi}{\rho^2}.$$

$\frac{\partial \mathcal{L}}{\partial \varphi} \neq 0$ $\left(\frac{\partial \mathcal{L}}{\partial \varphi} = \frac{a\sin\varphi}{\rho^2}\right) \Rightarrow P_\varphi$ не явл. интегралом движен., а φ не циклич. координата

ρ: $\frac{\partial \mathcal{L}}{\partial \dot\rho} = m\dot\rho$ $\quad \frac{\partial \mathcal{L}}{\partial \rho} = m\rho\dot\varphi^2 + \frac{2a\cos\varphi}{\rho^3}$

$$m\ddot\rho - m\rho\dot\varphi^2 - \frac{2a\cos\varphi}{\rho^3} = 0$$

φ: $\frac{\partial \mathcal{L}}{\partial \dot\varphi} = m\rho^2\dot\varphi$ $\quad \frac{\partial \mathcal{L}}{\partial \varphi} = \frac{a\sin\varphi}{\rho^2}$

$$m\rho^2\ddot\varphi + 2m\rho\dot\rho\dot\varphi - \frac{a\sin\varphi}{\rho^2} = 0.$$

Част. движ. под действ. $U(r)$. Все инт. движ. + з-н движ. в сферич. коорд.

6

$$L = \frac{m}{2}(\dot r^2 + r^2\dot\theta^2 + r^2\sin^2\theta\,\dot\varphi^2) - U(r)$$

$$\begin{cases} p_r = \frac{\partial L}{\partial \dot r} = m\dot r \\ p_\varphi = \frac{\partial L}{\partial \dot\varphi} = mr^2\sin^2\theta\,\dot\varphi \\ p_\theta = \frac{\partial L}{\partial \dot\theta} = mr^2\dot\theta \end{cases}$$

$$\dot r = \frac{p_r}{m}, \quad \dot\varphi = \frac{p_\varphi}{mr^2\sin^2\theta}, \quad \dot\theta = \frac{p_\theta}{mr^2}$$

$$H = \frac{1}{2m}\left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\varphi^2}{r^2\sin^2\theta}\right) + U(r)$$

1) $\frac{\partial H}{\partial t} = 0 \Rightarrow H = const = E$ – интегр. движения

2) $p_\varphi = \frac{\partial L}{\partial \dot\varphi} = mr^2\sin^2\theta\,\dot\varphi$ – инт. движ. $\frac{\partial H}{\partial \varphi} = 0 \quad p_\varphi = const = G$

3) $$H = \frac{1}{2m}\left(p_r^2 + \frac{1}{r^2}\underbrace{\left(p_\theta^2 + \frac{G}{\sin^2\theta}\right)}_{f}\right) + U(r)$$

$\frac{\partial f}{\partial t} = 0 \Rightarrow f = p_\theta^2 + \frac{G}{\sin^2\theta} = C \Rightarrow$ интеграл движения

$$\begin{cases} E = \frac{p_r^2}{2m} + \frac{C}{2mr^2} + U(r) = |p_r = m\dot r| = \frac{m\dot r^2}{2} + \frac{C}{2mr^2} + U(r) \\ G = mr^2\sin^2\theta\,\dot\varphi \\ C = |p_\theta = mr^2\dot\theta| = m^2r^4\dot\theta^2 + \frac{G^2}{\sin^2\theta} \end{cases}$$

$$\dot r = \frac{dr}{dt} = \pm\sqrt{\frac{2}{m}\left(E - \frac{C}{2mr^2} - U(r)\right)} \qquad \frac{\dot r}{\dot\theta} = \frac{dr}{d\theta} = \frac{\pm\sqrt{\frac{2}{m}\left(E - \frac{C}{2mr^2} - U(r)\right)}}{\pm\frac{1}{mr^2}\sqrt{C - \frac{G^2}{\sin^2\theta}}}$$

$\dot\theta$

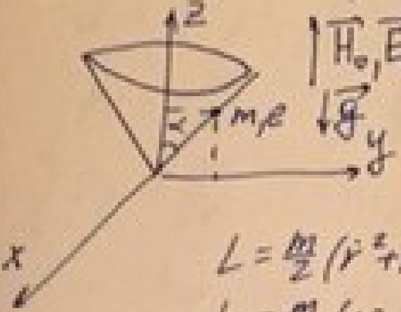
$$\int_{t_0}^{t} dt = \pm\int_{r_0}^{r}\frac{dr}{\sqrt{\frac{2}{m}\left(E - \frac{C}{2mr^2} - U(r)\right)}};$$

$$\pm\int_{r_0}^{r}\frac{dr}{mr^2\sqrt{\frac{2}{m}\left(E - \frac{C}{mr^2} - U(r)\right)}} = \pm\int_{\theta_0}^{\theta}\frac{d\theta}{\sqrt{C - \frac{G}{\sin^2\theta}}}$$

$$\frac{\dot\theta}{\dot\varphi} = \frac{d\theta}{d\varphi} = \frac{\pm\frac{1}{mr^2}\sqrt{C - \frac{G^2}{\sin^2\theta}}}{G/mr^2\sin^2\theta}$$

$$\int_{\varphi_0}^{\varphi} d\varphi = \pm\int_{\theta_0}^{\theta}\frac{G}{\sin^2\theta}\frac{d\theta}{\sqrt{C - \frac{G}{\sin^2\theta}}}.$$

m, e по вн. конуса, по оси котор. МП. з-н движ. в своб. м.т. Г-Я. — ?



$\vec{H}_0, \vec{E}_0$ $\vec{g}$

$z = r\,ctg\,\alpha$

$v^2 = \dot r^2 + r^2\dot\varphi^2 + \dot z^2$

$L = \frac{mv^2}{2} + \frac{e}{c}(\vec A\vec v) - mgz$

$\vec{A_\varphi} = \frac{H_0 r}{2}\vec{e_\varphi}$

$\vec{A_\varphi} = \frac{H_0 r^2}{2}\dot\varphi$

$L = \frac{m}{2}(\dot r^2 + r^2\dot\varphi^2 + \dot z^2) + \frac{e}{c}(\vec A\vec v) - mgz$

$L = \frac{m}{2}(\dot r^2 + r^2\dot\varphi^2 + \dot r^2 ctg^2\alpha) + \frac{e}{2c}H_0 r^2\dot\varphi^2 - mgr\,ctg\,\alpha$

1. $L = \frac{m}{2}\left(\dot r^2(1+ctg^2\alpha) + r^2\dot\varphi^2\right) + \frac{e}{2c}H_0 r^2\dot\varphi - mgr\,ctg\,\alpha$

2. $p_r = \frac{\partial L}{\partial \dot r} = \frac{m\dot r}{\sin^2\alpha} \Rightarrow \dot r = \frac{p_r\sin^2\alpha}{m}$; $p_\varphi = \frac{\partial L}{\partial\dot\varphi} = mr^2\dot\varphi + \frac{H_0 r^2 e}{2c} \Rightarrow \dot\varphi = \frac{p_\varphi - \frac{H_0}{2c}r^2 e}{mr^2}$

$\mathcal{E}^{ob} = p_i\dot q_i - L = \frac{m}{2}\left(r^2\frac{(p_\varphi - \frac{H_0}{2c}r^2 e)^2}{m^2 r^4}\right) + \left(\frac{p_r\sin^2\alpha}{m}\right)^2\frac{1}{\sin^2\alpha} + \frac{e}{2c}H_0 r\left(\frac{p_\varphi - \frac{H_0 r^2 e}{2c}}{mr^2}\right) + mgr\,ctg\,\alpha$

3. $\mathcal{E}^{ob} = \mathcal{H}$, т.к. $\mathcal{H} \neq \mathcal{H}(t) \Rightarrow$ инт. движ.
   явно

φ - циклич. $\Rightarrow p_\varphi = const$

$\frac{\partial S}{\partial t} + H(q_i, p_i, t) = 0 \Rightarrow \frac{\partial S}{\partial t} + \frac{1}{2mr^2}\left(\frac{\partial S}{\partial\varphi} - \frac{H_0}{2c}r^2 e\right)^2 + \frac{1}{2m}\left(\frac{\partial S}{\partial r}\sin\alpha\right)^2 + \frac{e}{2c}\frac{H_0}{m}\times$

$\times\left(\frac{\partial S}{\partial\varphi} - \frac{H_0}{2c}r^2 e\right) + mgr\,ctg\,\alpha \Rightarrow S = -Et + p_\varphi\varphi + f(r) + A$

$0 = -E + \frac{1}{2mr^2}\left(p_\varphi - \frac{H_0}{2c}r^2 e\right)^2 + \frac{e}{2c}\frac{H_0}{m}\left(p_\varphi - \frac{H_0}{2c}r^2 e\right) + \frac{\sin^2\alpha}{2m}[f'(r)]^2 + mgr\,ctg\,\alpha$

$S = -Et + f_1(x) + f_2(z) + A + p_y y$

$0 = -E + \frac{1}{2m}(p_y - \frac{e}{c}xH_0)^2 + \frac{[f'(x)]^2}{2m} + \frac{[f'(z)]^2}{2m} + mgz = 0 \,/\cdot 2m$

$[f'(z)]^2 = 2Em - \alpha - 2m^2 gz \Rightarrow f'(z) = \pm\sqrt{2Em - \alpha - 2m^2 gz}$

$p_z = \frac{\partial f}{\partial z} = \pm\sqrt{2Em - \alpha - 2m^2 gz} \Rightarrow f_1 = \pm\sqrt{2Em - \alpha - 2m^2 gz}\,dz$

$[f'(x)]^2 = \alpha - (p_y - \frac{e}{c}xH_0)^2 \Rightarrow f'(x) = \pm\sqrt{\alpha - (p_y - \frac{e}{c}xH_0)^2}$

$p_x = \frac{df}{dx} = \pm\sqrt{\alpha - (p_y - \frac{e}{c}xH_0)^2} \Rightarrow f'(x) = \pm\int\sqrt{\alpha - (p_y - \frac{e}{c}xH_0)^2}\,dx$

$S = -Et + p_y y$, $S = -Et + p_y y + f_1(x) + f_2(z) + A$

$0 = -E + \frac{1}{2m}(p_y - \frac{e}{c}xH_0)^2 + \frac{[f'(x)]^2}{2m} + \frac{[f'(z)]^2}{2m} + mgz = 0$

$S = -Et + p_y y \pm \int\sqrt{\alpha - (p_y - \frac{e}{c}xH_0)^2}\,dx \pm \int\sqrt{2Em - \alpha - 2m^2 gz}\,dz + A$

$\frac{\partial S}{\partial E} = -t \pm \int\frac{m\,dz}{\sqrt{2Em - \alpha - 2m^2 gz}} = \beta_1 \Rightarrow t(z)$; $\frac{\partial S}{\partial p_y} = y = \int -\frac{dx}{\sqrt{\alpha - (p_y - \frac{e}{c}xH_0)^2}} = \beta_2 \Rightarrow y(x)$.

Частица движ. под действ. $U(r)$. З-н движ. мет. Г-Я (в квад.) -?

8

$$\mathcal{L} = \frac{m}{2}(\dot\rho^2 + \rho^2\dot\varphi^2) - U(\rho), \quad H = \frac{p_\rho^2}{2m} + \frac{p_\varphi^2}{2m\rho^2} + U(\rho)$$

$$\begin{cases} p_\rho \to \frac{\partial F}{\partial \rho} \\ p_\varphi \to \frac{\partial F}{\partial \varphi} \end{cases} \quad 0 = \frac{\partial F}{\partial t} + \frac{1}{2m}\left(\left(\frac{\partial F}{\partial \rho}\right)^2 + \frac{1}{\rho^2}\left(\frac{\partial F}{\partial \varphi}\right)^2\right) + U(\rho)$$ — ур. Гамильтона-Якоби

$$F(t,\rho,\varphi) = T(t) + R(\rho) + \Phi(\varphi)$$

$$-T'(t) = \frac{1}{2m}(R'(\rho))^2 + \frac{1}{2m\rho^2}(\Phi'(\varphi))^2 + U(\rho) = c_1$$

$$c_2 = \Phi'(\varphi) = \pm\sqrt{2m\rho^2\left(c_1 - U(\rho) - \frac{(R'(\rho))^2}{2m}\right)}$$

$$\begin{cases} T'(t) = -c_1 \\ \Phi'(\varphi) = c_2 \end{cases} \begin{cases} T(t) = -c_1 t \\ \Phi(\varphi) = c_2\varphi \end{cases} \Rightarrow c_2^2 = 2m\rho^2\left(c_1 - U(\rho) - \frac{1}{2m}(R'(\rho))^2\right)$$

$$R'(\rho) = \pm\sqrt{2m\left(c_1 - U(\rho) - \frac{c_2^2}{2m\rho^2}\right)}$$

$$R(\rho) = \pm\int\sqrt{2m\left(c_1 - U(\rho) - \frac{c_2^2}{2m\rho^2}\right)}\,d\rho$$

$$F = -c_1 t + c_2\varphi + \int d\rho\sqrt{2m\left(c_1 - U(\rho) - \frac{c_2^2}{2m\rho^2}\right)}$$, тогда

$$\begin{cases} \frac{\partial F}{\partial \varphi} = p_\varphi = c_2 \\ \frac{\partial F}{\partial \rho} = p_\rho = \pm\sqrt{2m\left(c_1 - U(\rho) - \frac{c_2^2}{2m\rho^2}\right)} \end{cases} \quad c_1 = \frac{p_\rho^2}{2m} + U(\rho) + \frac{c_2^2}{2m\rho^2} = E$$

$$\begin{cases} \frac{\partial F}{\partial c_1} = \alpha_1 = -t \pm \int d\rho \frac{\sqrt{2m}}{2\sqrt{c_1 - U(\rho) - \frac{c_2^2}{2m\rho^2}}} \\ \frac{\partial F}{\partial c_2} = \alpha_2 = \varphi \pm \int d\rho \frac{c_2\sqrt{2m}}{2m\rho^2\sqrt{c_1 - U(\rho) - \frac{c_2^2}{2m\rho^2}}} \end{cases}$$

$$\begin{cases} t - t_0 = \int_{\rho_0}^{\rho} \frac{d\rho}{\sqrt{\frac{2}{m}\left(c_1 - U(\rho) - \frac{c_2^2}{2m\rho^2}\right)}} \\ \varphi - \varphi_0 = \pm\int_{\rho_0}^{\rho} \frac{c_2\,d\rho}{\rho^2\sqrt{2m\left(c_1 - U(\rho) - \frac{c_2^2}{2m\rho^2}\right)}} \end{cases}$$ — з-н движения.

$$\mathcal{L} = \frac{1}{2}(q_2\dot q_1^2 + q_1\dot q_2^2) - \left(\frac{1}{q_1q_2} + q_1 + q_2\right)$$ норм. координ. соб. -?

10

1. $U(q_1,q_2) = \frac{1}{q_1q_2} + q_1 + q_2$

$0 = \frac{\partial U}{\partial q_1} = -\frac{1}{q_1^2q_2} + 1 \qquad 0 = \frac{\partial U}{\partial q_2} = -\frac{1}{q_2^2q_1} + 1$

$\begin{cases} \frac{1}{q_1^2q_2} = 1 \\ \frac{1}{q_2^2q_1} = 1 \end{cases} \quad \begin{cases} q_1^2q_2 = 1 \\ q_2^2q_1 = 1 \end{cases} \quad q_1^4q_2^2 = 1 \quad q_1^3 = 1 \quad q_1 = 1 \quad q_2 = 1$ $(1,1)$ - точка минимума потенциала ТУР.

2. координаты отклонения $\begin{cases} x_1 = q_1 - q_{01} = q_1 - 1 \\ x_2 = q_2 - q_{02} = q_2 - 1 \end{cases}$

$\mathcal{L} = \frac{1}{2}\left((1+x_2)\dot x_1^2 + (1+x_1)\dot x_2^2\right) - \frac{1}{(1+x_1)(1+x_2)} - (1+x_1) - (1-x_2) =$

$= \frac{1}{2}(\dot x_1^2 + \dot x_2^2) - (1 - x_1 + x_1^2 + \ldots)(1 - x_2 + x_2^2 + \ldots) - 2 - x_1 - x_2 =$

$= (1+x)^{-1} \underset{x\to 0}{=} 1 - x + x^2 + \ldots \quad = \frac{1}{2}(\dot x_1^2 + \dot x_2^2) - (1 - x_1 + x_1^2 - x_2 + x_1x_2 + x_2^2) - 2 - x_1 - x_2 =$

$= \frac{1}{2}(\dot x_1^2 + \dot x_2^2) - 3 - x_1^2 - x_1x_2 - x_2^2 \; ; \; \mathcal{L}' = \frac{1}{2}(\dot x_1^2 + \dot x_2^2) - x_1^2 - x_2^2 - x_1x_2$

УЛ: $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0 \quad \begin{cases} \ddot x_1 + 2x_1 + x_2 = 0 \\ \ddot x_2 + x_1 + 2x_2 = 0 \end{cases} \Rightarrow$ решение в виде $x = Re(Ae^{i\omega t})$

$\begin{cases} A_1(-\omega^2) + 2A_1 + A_2 = 0 \\ A_2(-\omega^2) + 2A_2 + A_1 = 0 \end{cases} \quad \begin{pmatrix} 2-\omega^2 & 1 \\ 1 & 2-\omega^2 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$

$(2-\omega^2)^2 - 1 = 0$
$4 - 4\omega^2 + \omega^4 - 1 = 0$
$(2-\omega^2-1)(2-\omega^2+1) = 0$
$(1-\omega^2)(3-\omega^2) = 0$
$\omega_{1,2} = \pm 1 \quad \omega_{3,4} = \pm\sqrt{3}$

1) $\omega^2 = 1 \Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0 \quad A_1 = C_2, \; A_2 = -C_2$

2) $\omega^2 = 3 \Rightarrow \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0 \quad A_2 = C_1, \; A_1 = C_1$

$x = Re\left[C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{i\sqrt{3}t} + C_2\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{it}\right] \Rightarrow T = \frac{1}{2}(\dot x_1^2 + \dot x_2^2) = \frac{1}{2}\sum t_{ij}\dot x_i \dot x_j$

$A^TTA = (1\;1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = (1\;1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 + 1 = 2 \qquad T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$C_1^2 ATA = 1 \Rightarrow C_1 = \frac{1}{\sqrt{2}}$

$(A^2)^T T(A^2) = (1\;-1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = (+1\;-1)\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 2 \Rightarrow C_2 = \frac{1}{\sqrt{2}}$

$A^TTA = 1 \; | \cdot A^{-1}$
$A^TT = A^{-1}$

$\longrightarrow$

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad \vec{\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}} = A \vec{\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}}, \text{ где } \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = A^{-1} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = A^T \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

$$q_1 = \frac{1}{\sqrt{2}}(\xi_1 + \xi_2)$$
$$q_2 = \frac{1}{\sqrt{2}}(\xi_1 - \xi_2)$$

— з-н преобразования

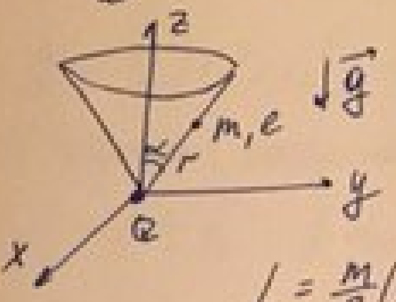
$$\mathcal{L} = \frac{1}{2}\left(\frac{1}{2}(\dot\xi_1 + \dot\xi_2)^2 + \frac{1}{2}(\dot\xi_1 - \dot\xi_2)^2\right) - \left(\frac{1}{2}(\xi_1 + \xi_2)^2 + \frac{1}{2}(\xi_1 + \xi_2)(\xi_1 - \xi_2) + \right.$$

$$\left. + \frac{1}{2}(\xi_1 - \xi_2)^2\right) = \frac{1}{2}(\dot\xi_1^2 + \dot\xi_2^2) - 3\xi_1^2 - \frac{1}{2}\xi_2^2$$

$$\mathcal{L} = \sum_{i=1}^{2}\left(\frac{1}{2}\dot\xi_i^2 - \frac{1}{2}\omega_i^2 \xi_i^2\right)$$

$m, e, 2\alpha$; $\vec{g} = -g\vec{e}_z$ H - ? инт. движ. - ? З-н движ. - ?

Q



$V = \frac{eQ}{r}$, $s = 2$ - ст. свободы

$z = r\cos\alpha$ $x^2 + y^2 = r^2\sin^2\alpha$ - конус

$x = r\sin\alpha\cos\varphi$

$y = r\sin\alpha\sin\varphi$

$$L = \frac{m}{2}\left(\dot r^2 + r^2\dot\alpha^2 + r^2\sin^2\alpha\,\dot\varphi^2\right) - \frac{eQ}{r} + mgr\cos\alpha$$

$\alpha = const$

$$L = \frac{m}{2}\left(\dot r^2 + r^2\sin^2\alpha\,\dot\varphi^2\right) - \frac{eQ}{r} + mgr\cos\alpha \ ; \ E = \frac{m}{2}\left(\dot r^2 + r^2\sin^2\alpha\,\dot\varphi^2\right) + \frac{eQ}{r} - mgr\cos\alpha$$

$$\frac{\partial L}{\partial \dot r} = p_r = m\dot r \to \dot r = \frac{p_r}{m} \qquad \frac{\partial L}{\partial \dot\varphi} = mr^2\sin^2\alpha\,\dot\varphi = p_\varphi \to \dot\varphi = \frac{p_\varphi}{mr^2\sin^2\alpha}$$

$$H = \frac{p_r^2}{2m} + \frac{p_\varphi^2}{2mr^2\sin^2\alpha} + \frac{eQ}{r} - mgr\cos\alpha = E$$

$\frac{\partial H}{\partial \varphi} = 0 \to p_\varphi = const$, а φ - цикл. коорд.

$\frac{\partial H}{\partial t} = 0 \to H = E$ (энергия - инт. движен.)

$$\frac{m\dot r^2}{2} + \frac{p_\varphi^2}{2mr^2\sin^2\alpha} + \frac{eQ}{r} - mgr\cos\alpha = H = E$$

$$\dot r^2 = \frac{2}{m}\left[E + mgr\cos\alpha - \frac{eQ}{r} - \frac{p_\varphi^2}{2mr^2\sin^2\alpha}\right]$$

$$t - t_0 = \pm\int dr \frac{\sqrt{m}}{\sqrt{2\left[E + mgr\cos\alpha - \frac{eQ}{r} - \frac{p_\varphi^2}{2mr^2\sin^2\alpha}\right]}}$$

$$\frac{\dot\varphi}{\dot r} = \pm\frac{p_\varphi}{mr^2\sin^2\alpha\sqrt{\frac{2}{m}[f]}}$$, где $f = E + mgr\cos\alpha - \frac{eQ}{r} - \frac{p_\varphi^2}{2mr^2\sin^2\alpha}$.

$$\varphi - \varphi_0 = \int dr \frac{p_\varphi}{mr^2\sin^2\alpha\sqrt{\frac{2}{m}[f]}}$$

медленн. изм. α

$E = E(t)$ - ?

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$$\mathcal{L} = \frac{m\dot{x}_1^2}{2} + \frac{m\dot{x}_2^2}{2} - \frac{kx_1^2}{2} - \frac{kx_2^2}{2} - \alpha k x_1 x_2$$

$$\mathcal{L} = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - \frac{k}{2}(x_1^2 + x_2^2 + 2\alpha x_1 x_2)$$

к нормальн. координатам: $x_i \to \xi_i$

$$\mathcal{L} = \left(\frac{1}{2}\dot{\xi}_1^2 - \frac{1}{2}\omega_{(1)}^2\xi_1^2\right) + \left(\frac{1}{2}\dot{\xi}_2^2 - \frac{1}{2}\omega_{(2)}^2\xi_2^2\right)$$

$$\begin{cases} m\ddot{x}_1 + k(x_1 + \alpha x_2) = 0 \\ m\ddot{x}_2 + k(x_2 + \alpha x_1) = 0 \end{cases}$$

$x_i = Re(\underbrace{A_i e^{i\omega t}}_{\text{решение}})$ - берем только веществ. часть

$$\begin{pmatrix} -m\omega^2 + k & \alpha k \\ \alpha k & -m\omega^2 + k \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0 \Rightarrow 0 = \det A = (-m\omega^2 - k)^2 - (\alpha k)^2$$

$$-m\omega^2 + k = \pm\alpha k$$

$$\omega_{(1,2)}^2 = \frac{k}{m}(1 \pm \alpha)$$

вообще говоря, $\alpha = \alpha(t)$

Ф-ция Гамильтона: $H = \underbrace{\frac{p_1^2}{2} + \frac{\omega_{(1)}^2\xi_1^2}{2}}_{\beta_1} + \underbrace{\frac{p_2^2}{2} + \frac{\omega_{(2)}^2\xi_2^2}{2}}_{\beta_2}$

$$p_1 = \pm\sqrt{2\beta_1 - \omega_{(1)}^2\xi_1^2} \qquad p_2 = \pm\sqrt{2\beta_2 - \omega_{(2)}^2\xi_2^2}$$

$$I_1 = \frac{1}{2\pi}\oint p_1 d\xi_1 = \frac{1}{2\pi}\oint \sqrt{2\beta_1 - \omega_{(1)}^2\xi_1^2}\, d\xi_1 = \frac{\beta_1}{\omega_{(1)}} = const$$

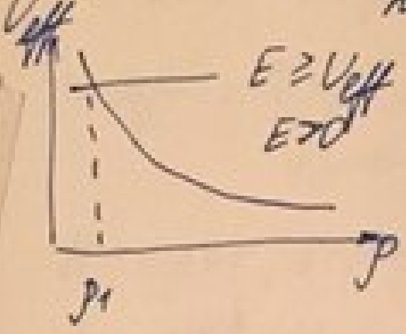
$$I_2 = \frac{1}{2\pi}\oint p_2 d\xi_2 = \frac{1}{2\pi}\oint \sqrt{2\beta_2 - \omega_{(2)}^2\xi_2^2}\, d\xi_2 = \frac{\beta_2}{\omega_{(2)}} = const$$

$$\beta_1 = I_1\omega_{(1)}$$
$$\beta_2 = I_2\omega_{(2)}$$

$$E = H = \beta_1 + \beta_2 = I_1\omega_{(1)} + I_2\omega_{(2)}$$

$$E = I_1\sqrt{\frac{k}{m}(1+\alpha(t))} + I_2\sqrt{\frac{k}{m}(1-\alpha(t))}$$

$U(r) = -\frac{\alpha}{r^2}$ найти время паден. на силов. центр $\rho(0)=R$, $v(0)=0$



$$U_{eff} = U(\rho) + \frac{\ell_\varphi^2}{2m\rho^2} = \frac{1}{\rho^2}\left(-\alpha + \frac{\ell_\varphi^2}{2m}\right)$$

$-\alpha + \frac{\ell_\varphi^2}{2m} > 0$ – инф. движ. без над. на СЦ.

$-\alpha + \frac{\ell_\varphi^2}{2m} < 0$ ✓ нужн. случ.

(1) $E<0$ – финит. с паден. на СЦ

$$\varphi-\varphi_0 = \pm\int_{\rho_0}^{\rho}\frac{\ell_\varphi}{m\rho^2}\frac{d\rho}{\sqrt{\frac{2}{m}\left(E-U_{eff}(\rho)\right)}}$$

$$\varphi-\varphi_0 = \pm\int_{\rho_0}^{\rho}\frac{\ell_\varphi}{m\rho^2}\frac{d\rho}{\sqrt{\frac{2}{m}\left(E-\frac{1}{\rho^2}\left(-\alpha+\frac{\ell_\varphi^2}{2m}\right)\right)}}, \quad \frac{d\rho}{\rho^2} = -d\underbrace{\left(\frac{1}{\rho}\right)}_{u}$$

$$\varphi-\varphi_0 = \mp\int_{\rho_0}^{\rho}\frac{du}{\sqrt{\frac{2}{m}\frac{m^2}{\ell_\varphi^2}\left(E-u^2\left(-\alpha+\frac{\ell_\varphi^2}{2m}\right)\right)}}$$

$$t-t_0 = \int_{t_0}^{t} dt = \pm\int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{\frac{2}{m}\left(E-\frac{1}{\rho^2}\left(-\alpha+\frac{\ell_\varphi^2}{2m}\right)\right)}} = \pm\int_{\rho_0}^{\rho}\frac{d\rho\,\rho^2}{\sqrt{\frac{2}{m}\left(E\rho^2+\alpha-\frac{\ell_\varphi^2}{2m}\right)}}$$

(2) $E>0$ – инфинит. движ. с пад. на СЦ

$$T = \pm\int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{\frac{2}{m}\left(E+\frac{\alpha}{\rho^2}-\frac{\ell_\varphi^2}{2m\rho^2}\right)}} = \pm\sqrt{\frac{m}{2}}\int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{E+\left(\alpha-\frac{\ell_\varphi^2}{2m}\right)\frac{1}{\rho^2}}} = \pm\sqrt{\frac{m}{2}}\int_{\rho_0}^{\rho}\frac{\rho\,d\rho}{\sqrt{E\rho^2+\underbrace{\left(\alpha-\frac{\ell_\varphi^2}{2m}\right)}_{K}}} =$$

$$= \pm\frac{1}{2}\sqrt{\frac{m}{2E}}\int_{\rho_0}^{\rho}\frac{d(\rho^2)}{\sqrt{\rho^2+\frac{K}{E}}} = \pm\sqrt{\frac{m}{2E}}\sqrt{\rho^2+\frac{K}{E}}\Big|_0^{\rho_0} = \pm\sqrt{\frac{m}{2E}}\left[\sqrt{\frac{E\rho_0^2+\alpha-\frac{\ell_\varphi^2}{2m}}{E}}-\right.$$

$$\left.-\sqrt{\frac{\alpha-\ell_\varphi^2}{2mE}}\right] = \pm\sqrt{\frac{m}{2E^2}}\left[\sqrt{E\rho_0^2+\alpha-\frac{\ell_\varphi^2}{2m}}-\sqrt{\frac{\alpha-\ell_\varphi^2}{2m}}\right] = \frac{1}{E}\sqrt{\frac{m}{2}}\left[\sqrt{E\rho_0^2+\alpha-\frac{\ell_\varphi^2}{2m}}-\right.$$

$$\left.-\sqrt{\alpha-\frac{\ell_\varphi^2}{2m}}\right]$$

Част. движ. по сфере R в поле силы т.эж. — ф.Л. $\mathcal{L}$ - ? инт. движ. - ? з-н движ. - ?

$S = 2$ степ. свободы $r = R$ $q_1 = \theta, q_2 = \varphi$

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$$T = \frac{mv^2}{2} = \frac{m}{2}(\underbrace{\dot{r}^2}_{0\ (r=R=const)} + r^2\dot\theta^2 + r^2\sin^2\theta\dot\varphi^2) = \frac{mR^2}{2}(\dot\theta^2 + \sin^2\theta\dot\varphi^2)$$

$$U = mgz = [z = r\cos\theta] = mgR\cos\theta$$

$$\mathcal{L} = \frac{mR^2}{2}(\dot\theta^2 + \sin^2\theta\dot\varphi^2) - mgR\cos\theta$$

1. $\frac{\partial \mathcal{L}}{\partial t} = 0 \Rightarrow E = const$

$$E = \sum_{i=1}^{2}\frac{\partial \mathcal{L}}{\partial \dot q_i}\dot q_i - L = \dot\theta\frac{\partial \mathcal{L}}{\partial \dot\theta} + \dot\varphi\frac{\partial \mathcal{L}}{\partial \dot\varphi} - \mathcal{L} = \dot\theta mR^2\dot\theta + \dot\varphi mR^2\sin^2\theta\dot\varphi -$$

$$-\left(\frac{m}{2}R^2(\dot\theta^2 + \sin^2\theta\dot\varphi^2) - mgR\cos\theta\right) = \frac{mR^2}{2}\dot\theta^2 + \frac{mR^2}{2}\sin^2\theta\dot\varphi^2 + mgR\cos\theta = const.$$

2. $\frac{\partial \mathcal{L}}{\partial \varphi} = 0$ $p_\varphi = const = \frac{\partial \mathcal{L}}{\partial \dot\varphi} = mR^2\sin^2\theta\dot\varphi$

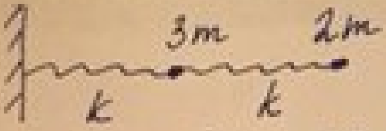
$$\dot\varphi = \frac{p_\varphi}{mR^2\sin^2\theta} \Rightarrow E = \frac{mR^2}{2}\dot\theta^2 + \frac{mR^2}{2}\sin^2\theta\left(\frac{p_\varphi}{mR^2\sin^2\theta}\right)^2 + mgR\cos\theta$$

$$\dot\theta = \frac{d\theta}{dt} = \pm\sqrt{\frac{2}{mR^2}\left[E - \frac{p_\varphi^2}{2mR^2\sin^2\theta} - mgR\cos\theta\right]} \Rightarrow \int_{t_0}^{t} dt = \pm\int_{\theta_0}^{\theta}\frac{d\theta}{\sqrt{\ }}$$

$\theta = \theta(t)$ неявно

$$\frac{\dot\theta}{\dot\varphi} = \frac{d\theta/dt}{d\varphi/dt} = \frac{d\theta}{d\varphi} = \frac{\pm\sqrt{\ }}{\frac{p_\varphi}{mR^2\sin^2\theta}}$$

$$\varphi - \varphi_0 = \int_{\varphi_0}^{\varphi} d\varphi = \pm\int_{\theta_0}^{\theta}\frac{p_\varphi}{mR^2\sin^2\theta}\frac{d\theta}{\sqrt{\ }} \Rightarrow \begin{array}{l}\varphi = \varphi(\theta) \text{ явно} \\ \theta = \theta(\varphi) \text{ неявно}\end{array}$$



норм. колеб. сист. - ?

$F_т = 0$

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$T = \frac{3m\dot{x}_1^2}{2} + \frac{2m\dot{x}_2^2}{2}$, $U = \frac{k}{2}(x_1)^2 + \frac{k}{2}(x_2 - x_1)^2$

$L = \frac{3m}{2}\dot{x}_1^2 + m\dot{x}_2^2 - \frac{k}{2}\left(x_1^2 + (x_2 - x_1)^2\right)$

Ур. Лагранжа $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$

$\begin{cases} 3m\ddot{x}_1 + kx_1 - k(x_2 - x_1) = 0 \\ 2m\ddot{x}_2 + k(x_2 - x_1) = 0 \end{cases}$ $\begin{cases} 3m\ddot{x}_1 + 2kx_1 - kx_2 = 0 \\ 2m\ddot{x}_2 + k(x_2 - x_1) = 0 \end{cases}$ $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underbrace{Re\left[\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} e^{i\omega t}\right]}_{\text{вид реш. ур. колеб.}}$

$\begin{pmatrix} 2k - 3m\omega^2 & -k \\ -k & k - 2m\omega^2 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$

$(2k - 3m\omega^2)(k - 2m\omega^2) - k^2 = 0$

$2k^2 - 3m\omega^2 k - 4km\omega^2 + 6m^2\omega^4 - k^2 = 0$

$k^2 - 7m\omega^2 k + 6m^2\omega^4 = 0$

$6m^2\omega^4 - 7m\omega^2 k + k^2 = 0$

$(6m\omega^2 - k)(m\omega^2 - k) = 0$

$\omega^2 = \frac{k}{6m}$ $\omega^2 = \frac{k}{m}$

Случай 1 $\omega = \omega_1 = \frac{k}{6m}$

$\begin{pmatrix} -\frac{3mk}{6m} + 2k & -k \\ -k & k - \frac{2mk}{6m} \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$

$\begin{pmatrix} \frac{3}{2}k & -k \\ -k & \frac{2k}{3} \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$

$\begin{cases} \frac{3}{2}A_1 - A_2 = 0 \\ -A_1 + \frac{2}{3}A_2 = 0 \end{cases}$ $\begin{cases} 3A_1 - 2A_2 = 0 \\ -3A_1 + 2A_2 = 0 \end{cases}$

$3A_1 = 2A_2$

$\underbrace{A_1 = 1 \quad A_2 = \frac{3}{2}}$

Случай 2 $\omega = \omega_2 = \frac{k}{m}$

$\begin{cases} (-3k + 2k)A_1 - kA_2 = 0 \\ -kA_1 + (k - 2k)A_2 = 0 \end{cases}$

$-A_1 - A_2 = 0$ $\begin{cases} A_1 = 1 \\ A_2 = -1 \end{cases}$

$A_1 = -A_2$

$$Re\left[C_1\begin{pmatrix} 1 \\ 3/2 \end{pmatrix} e^{i\sqrt{\frac{k}{6m}}t} + C_2\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{i\sqrt{\frac{k}{m}}t}\right]$$

$P = q^{-4}(p^4 - \frac{1}{2}q^6)$ явл. каноничн.

$Q = pq^{-1}$ + производящ. ф-ция - ?

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$P = q^{-4}p^4 - \frac{1}{2}q^2$, критер. каноничности
$\begin{cases} \{P_i(q,p,t), Q_i(p,q,t)\}_{p,q} = c\delta_{ij} \\ \{P_i(p,q,t), P_j(p,q,t)\}_{p,q} = 0 \\ \{Q_i(p,q,t), Q_j(p,q,t)\}_{p,q} = 0 \end{cases}$

например:

$\{Q,Q\} = \{pq^{-1}, pq^{-1}\} = q^{-1}\{p, pq^{-1}\} + p\{q^{-1}, pq^{-1}\} = q^{-1}p\{p, q^{-1}\} + q^{-1}q^{-1}\overset{0}{\{p,p\}} +$

$+ pp\overset{0}{\{q^{-1}, q^{-1}\}} + pq^{-1}\{q^{-1}, p\} = 0$

Очевидно, $\{P,P\} = 0$ из-за некоммутативности скобок Пуассона

$\{P,Q\} = \frac{\partial P}{\partial p}\frac{\partial Q}{\partial q} - \frac{\partial P}{\partial q}\frac{\partial Q}{\partial p} = 4p^3q^{-4}(-\frac{1}{q^2}p) - (-4q^{-5}p^4 - q)q^{-1} =$

$= 4q^{-6}p^4 + 1 - q^{-6}4p^4 = 1 = c$

$\det \frac{\partial Q}{\partial p} = q^{-1} \neq 0 \Rightarrow$ производящ. ф-ция существует

$\begin{cases} p_i = \frac{\partial F}{\partial q} \\ P_i = -\frac{\partial F}{\partial Q_i} \end{cases} \Leftrightarrow \begin{cases} p = \frac{\partial F}{\partial q} = Qq \\ P_i = -\frac{\partial F}{\partial Q_i} = q^{-4}(Q^4q^4 - \frac{1}{2}q^6) = Q^4 - \frac{1}{2}q^2 \end{cases}$

$\begin{cases} \frac{\partial F}{\partial q} = Qq \\ \frac{\partial F}{\partial Q} = \frac{1}{2}q^2 - Q^4 \end{cases} \Rightarrow$
$F = Q\frac{q^2}{2} + f(Q,t)$

$F = \frac{1}{2}q^2Q - \frac{Q^5}{5} + \overset{0}{f_2(q,t)}$

$$\underbrace{F = \frac{1}{2}Qq - \frac{Q^5}{5}}_{\text{производящ. ф-ция}}$$

$\mathcal{H}=\frac14(p_1^2+p_2^2+p_3^2)+q_1+\frac{1}{q_2}-\frac{1}{q_3^2}$ $\quad q_i(0)=(0,1,1)$ $\quad$ мет. Г-Я найти з-н движения $\quad$ 18

$\dot q_i(0)=(1,0,0)$

ур. Г-Я:

$$\frac{\partial S}{\partial t}+\frac14\left(\frac{\partial S}{\partial q_1}\right)^2+\frac14\left(\frac{\partial S}{\partial q_2}\right)^2+\frac14\left(\frac{\partial S}{\partial q_3}\right)^2+q_1+\frac{1}{q_2}-\frac{1}{q_3^2}=0$$

$$S=T+Q_1+Q_2+Q_3 \quad \left(0=\underbrace{\frac{\partial S}{\partial t}}_{T'}+\frac14\left(Q_1'^2+Q_2'^2+Q_3'^2\right)+q_1+\frac{1}{q_2}-\frac{1}{q_3^2}\right)$$

$$-T'=E \Rightarrow T=-Et$$

$$E=\frac14\left(Q_1'^2+Q_2'^2+Q_3'^2\right)+q_1+\frac{1}{q_2}-\frac{1}{q_3^2} \qquad E=C_1+C_2+\underbrace{\frac{Q_3'^2}{4}-\frac{1}{q_3^2}}_{\parallel \atop E-C_1-C_2}$$

$$\frac14 Q_1'^2+q_1=C_1 \Rightarrow Q_1=\int\sqrt{4(C_1-q_1)}\,dq_1 \Rightarrow$$

$$\frac14 Q_2'+\frac{1}{q_2}=C_2 \Rightarrow Q_2=\int\sqrt{4\left(C_2-\frac{1}{q_2}\right)}\,dq_2 \qquad Q_3=\int dq_3\sqrt{4\left(E-C_1-C_2+\frac{1}{q_3^2}\right)}$$

$$\overset{=S}{F}=T+Q_1+Q_2+Q_3$$

$$\frac{\partial F}{\partial q_1}=p_1=\sqrt{4(C_1-q_1)}\to\frac{p_1^2}{4}+q_1=C_1$$

$$\frac{\partial F}{\partial q_2}=\sqrt{4\left(C_2-\frac{1}{q_2}\right)}=p_2\to\frac{p_2^2}{4}+\frac{1}{q_2}=C_2$$

$$\frac{\partial F}{\partial q_3}=\sqrt{4\left(E-C_1-C_2+\frac{1}{q_3^2}\right)}=p_3\to\frac{p_3^2}{4}-\frac{1}{q_3^2}+C_1+C_2=E$$

$$E=\frac{p_3^2}{4}-\frac{1}{q_3^2}+\frac{p_1^2}{4}+q_1+\frac{p_2^2}{4}+\frac{1}{q_2},\ \text{тогда}$$

$$F=Et\pm\int\sqrt{4(C_1-q_1)}\,dq_1\pm\int\sqrt{4\left(C_2-\frac{1}{q_2}\right)}\,dq_2\pm\int dq_3\sqrt{4\left(E-C_1-C_2+\frac{1}{q_3^2}\right)}$$

$$\frac{\partial F}{\partial C_1}=\mp\int\frac{2\,dq_1}{\sqrt{4(C_1-q_1)}}\mp\int\frac{4\,dq_3}{2\sqrt{4\left(E-C_1-C_2+\frac{1}{q_3^2}\right)}}=\alpha_1$$

$$\frac{\partial F}{\partial C_2}=\pm\int\frac{4\,dq_2}{2\sqrt{4\left(C_2-\frac{1}{q_2}\right)}} \qquad \frac{\partial F}{\partial C_3}=-t\pm\int\frac{4\,dq_3}{2\sqrt{4\left(E-C_1-C_2+\frac{1}{q_3^2}\right)}}$$

$$t-t_0=\pm\int_{q_0}^{q_3}\frac{2\,dq_3}{\sqrt{4\left(E-C_1-C_2+\frac{1}{q_3^2}\right)}}=\int dq_3\frac{1}{\sqrt{\frac{p_3^2}{4}}}=\int dq_3\frac{2}{|p_3|}$$

$$\dot q=\frac{\partial H}{\partial p}=\frac{p}{2};\ p_1=2\dot q^0 \ (2,0,0) \qquad E=C_1+C_2+\frac{p_3^2}{4}-\frac{1}{q_3^2}$$

$$t-t_0=\frac{2}{p_3}(q_3-q_0)=\frac{2(q_3-1)}{p_3}.$$

m, e по параболоиду $az = x^2+y^2$, $\vec H = H_0 \vec e_z$, $\vec E = -E_0 \vec e_z$

$x = \rho\cos\varphi$
$y = \rho\sin\varphi$ $\Rightarrow \rho^2 = x^2+y^2 \Rightarrow z = \frac{\rho^2}{a}$, $\dot z = \frac{2\rho\dot\rho}{a}$ $\{\dot\rho, \rho\dot\varphi, \dot z\}$ (s=2) 20

$\mathcal{L} = \frac{mv^2}{2} + \frac{e}{c}(\vec v\vec A) - e\varphi - U$, $\vec A = \frac{H_0}{2}\rho\vec e_\varphi$

$\mathcal{L} = \frac{m}{2}\left(\dot\rho^2 + \rho^2\dot\varphi^2 + \left(\frac{2\rho\dot\rho}{a}\right)^2\right) + \frac{e}{c}\frac{H_0}{2}\rho\dot\varphi - mg\frac{\rho^2}{a}$

$$\begin{cases} p_\rho = \frac{\partial L}{\partial\dot\rho} = m\dot\rho + \frac{m\rho}{a} \\ p_\varphi = \frac{\partial L}{\partial\dot\varphi} = m\dot\varphi\rho^2 + \frac{eH_0\rho^2}{2c} \end{cases}$$

$\mathcal{E}^{об} = \sum_i^s \frac{\partial L}{\partial\dot q_i}\dot q_i - L = \frac{m}{2}(\dot\rho^2 + \rho^2\dot\varphi^2) + mg\frac{\rho^2}{a}$

$$\begin{cases} \dot\rho = \frac{p_\rho}{m} - \frac{\rho}{a} \\ \dot\varphi = \frac{p_\varphi}{m\rho^2} - \frac{eH_0}{2mc} \end{cases} \qquad \mathcal{H} = \frac{m}{2}\left[\left(\frac{p_\rho}{m} - \frac{\rho}{a}\right)^2 + \rho^2\left(\frac{p_\varphi}{m\rho^2} - \frac{eH_0}{2mc}\right)^2\right] + mg\frac{\rho^2}{a}$$

$\frac{\partial H}{\partial\varphi} = 0 \Rightarrow \varphi$ - циклич., $p_\varphi = const$ (инт. дв.)

$\underbrace{\dot q_i = \frac{\partial H}{\partial p_i} \quad \dot p_i = -\frac{\partial H}{\partial q_i}}_{\text{ур. Гамильтона}}$

$\dot\rho = \frac{m}{2}\cdot 2\left(\frac{p_\rho}{m} - \frac{\rho}{a}\right)\frac{1}{m} = \frac{p_\rho}{m} - \frac{\rho}{a}$

$\dot p_\rho = -\left[\frac{m}{2}\left(\frac{p_\rho}{m} - \frac{\rho}{a}\right)\left(-\frac{2}{a}\right)\right]$.

Частица по сфере R в поле сил тяж. $\mathcal{H}$ - ? ин. движ. - ? з-н движ. - ?

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$$T = \frac{m\dot{\vec{r}}^2}{2} = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\varphi}^2) = \frac{mR^2}{2}(\dot{\theta} + \sin^2\theta\dot{\varphi}^2)$$

$(r = R = const)$

$$U = mgz = [z = r\cos\theta] = mgR\cos\theta$$

$$L = \frac{mR^2}{2}(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2) - mgR\cos\theta$$

$$E = \frac{mR^2}{2}\dot{\theta}^2 + \frac{mR^2}{2}\sin^2\theta\dot{\varphi}^2 + mgR\cos\theta = const \neq E(t) \Rightarrow \text{интегр. движ.}$$

$$\mathcal{H} = E \Rightarrow p_\varphi = \frac{\partial L}{\partial\dot{\varphi}} = mr^2\sin^2\theta\dot{\varphi} \quad p_\theta = \frac{\partial L}{\partial\dot{\theta}} = mr^2\dot{\theta}$$

$$\dot{\varphi} = \frac{p_\varphi}{mr^2\sin^2\theta} \qquad \dot{\theta} = \frac{p_\theta}{mr^2} \quad \underline{r = R!}$$

$$\mathcal{H} = \frac{p_\varphi^2}{2mR^2\sin^2\theta} + \frac{p_\theta^2}{2mR^2} + mgR\cos\theta \text{ - гамильтониан.}$$

$$E = \frac{mR^2}{2}\dot{\theta} + \frac{mR^2}{2}\sin^2\theta\left(\frac{p_\varphi}{mR^2\sin^2\theta}\right)^2 + mgR\cos\theta$$

$$\dot{p}_\varphi = -\frac{\partial H}{\partial\varphi} = 0 \quad \dot{\varphi} = \frac{\partial H}{\partial p_\varphi} = \frac{p_\varphi}{mR^2\sin^2\theta}, \quad \dot{\theta} = \frac{\partial H}{\partial p_\theta} = \frac{p_\theta}{mR^2}$$

$$\dot{p}_\theta = -\frac{\partial H}{\partial\theta} \Rightarrow \dot{\theta} = \frac{d\theta}{dt} = \pm\sqrt{\frac{2}{mR^2}\left[E - \frac{p_\varphi^2}{\sin^2\theta\, 2mR^2} - mgR\cos\theta\right]}$$

$$\int_{t_0}^{t} dt = \pm\int_{\theta_0}^{\theta} d\theta / \sqrt{}$$

$$\frac{\dot{\theta}}{\dot{\varphi}} = \frac{d\theta/dt}{d\varphi/dt} = \frac{d\theta}{d\varphi} = \frac{\pm\sqrt{}}{\frac{p_\varphi}{mR^2\sin^2\theta}}$$

$$\varphi - \varphi_0 = \int_{\varphi_0}^{\varphi} d\varphi = \pm\int_{\theta_0}^{\theta} \frac{p_\varphi}{mR^2\sin^2\theta}\frac{d\theta}{\sqrt{}} \Rightarrow \begin{array}{l}\varphi = \varphi(\theta) \text{ явно} \\ \theta = \theta(\varphi) \text{ неявно}\end{array}$$

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$$H=\frac{1}{4}(p_1^2+p_2^2+p_3^2)+\frac{1}{\sin^2 q_1}+th^2 q_2 \qquad q_i(0)=\left(\frac{\pi}{2},0,0\right)$$

$$\dot q_i(0)=(1,2,1)$$

1. $0=\frac{\partial F}{\partial t}+H\left(\frac{\partial F}{\partial q},q,t\right)$

$F=T+F_1+F_2+F_3$

$0=T'(t)+\frac{1}{4}\left((F_1')^2+(F_2')^2+(F_3')^2\right)+\frac{1}{\sin^2 q_1}$

$-T'(t)=E \Rightarrow T=-Et$

2. $E+\frac{F_2'^2}{4}+\frac{1}{\sin^2 q_1}=-\frac{1}{4}F_2'^2+th\,q_2-\frac{F_3'^2}{4}$

$(F_3')^2=E-F_1'^2-F_2'^2-\frac{4}{\sin^2 q_1}-4th^2 q_2=C_3$

$F_3=C_3 t$

3. $F_1'^2+\frac{4}{\sin^2 q_1}=E-C_3-F_2'^2-4th^2 q_2=C_2$

$F_1'=\pm\sqrt{C_2-\frac{4}{\sin^2 q_1}}$

$F_1=\int\pm dq_1\sqrt{C_2-\frac{4}{\sin^2 q_1}}$

$-E+F_2'^2+4th^2 q_2=-C_2$

$F_2=\pm\int dq_2\sqrt{E-C_2-C_3-4th^2 q_2}$

$E=-Et+C_3q_3\pm\int dq_1\sqrt{C_2-\frac{4}{\sin^2 q_1}}\pm\int dq_2\sqrt{E-C_2-C_3-4th^2 q_2}$

$\dot q=\frac{\partial H}{\partial q_2}=\frac{p_2}{2} \quad p_2=2\dot q_2$

$t-t_0=\pm\frac{1}{|p_2|}(q_2-q_0)=$

$=\pm\frac{1}{|2\dot q_2|}(q_2-q_0)=\pm\frac{1}{4}q_2$

$\frac{\partial F}{\partial C_3}=\alpha_3=q_3\pm\int dq_2\frac{(-1)}{v}=$

$\Rightarrow q_3-q_0=\pm\int_{q_0}^{q_2}dq_2\frac{(-1)}{p_2}$

$\frac{\partial F}{\partial C_2}=\alpha_2=\pm\int dq_1\frac{1}{\sqrt{C_2-\frac{4}{\sin^2 q_1}}}\pm$

$\pm\int dq_2\frac{(-1)}{v}; \int dq_1\frac{1}{|p_1|}=\pm\int\frac{1}{|p_2|}dq_2$

$\dot q_1=\frac{\partial H}{\partial p_1}=\frac{1}{2}p_1$

$\frac{1}{p_1}(q_1-q_1(0))=\pm\frac{1}{2\dot q_2}(q_2-q_2(0))$

$\frac{1}{2}\left(q_1-\frac{\pi}{2}\right)=\frac{1}{4}q_2.$

4. $\frac{\partial F}{\partial q_3}=C_3=p_3, \quad \frac{\partial F}{\partial q_1}=\pm\sqrt{C_2-\frac{4}{\sin^2 q_1}}=p_1 \Rightarrow C_2=p_1^2+\frac{4}{\sin^2 q_1}.$

$\frac{\partial F}{\partial q_2}=\pm\sqrt{E-C_2-C_3-4th^2 q_2}=p_2, \quad p_2^2=E-C_3-C_2-4th^2 q_2$

$E=p_2^2+p_1^2+\frac{4}{\sin^2 q_1}+p_3^2+4th^2 q_2$

5. $\frac{\partial F}{\partial E}=\alpha_1=-t+\int dq_2\frac{1}{2\sqrt{E-C_2-C_3-4th^2 q_2}}$

$t-t_0=\pm\int_{q_0}^{q_2}dq_2\frac{1}{2\sqrt{p_2^2+p_1^2+\frac{4}{\sin^2 q_1}+p_3^2+4th^2 q_2-C_2-C_3-4th^2 q_2}}=$

$=\pm\int_{q_0}^{q_2}dq_2\frac{1}{\sqrt{p_2^2}}=\pm\int_{q_0}^{q_2}2dq_2\frac{1}{|p_2|}=\pm\frac{1}{p_2}(q_2-q_0)$