$$H_0 = \frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial y}$$

В калибровке 3

$$\mathcal{L} = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{eH_0}{2c}(x\dot{y} - y\dot{x}) + e(\vec{E}_0, \vec{r})$$

$$\mathcal{L} = \frac{m}{2}(\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \dot{z}^2) + \frac{eH_0}{2c}\rho^2\dot{\varphi} + e(\vec{E}_0\,\vec{r})$$

$$\mathcal{L} = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\varphi}^2) + \frac{eH_0}{2c}r^2\sin^2\theta\,\dot{\varphi} + e(\vec{E}_0, \vec{r})$$

У нас $\mathcal{L} = \frac{m}{2}\left(\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \left(\frac{2}{a}\rho\dot{\rho}\right)^2\right) + \frac{H_0 e}{2c}\rho^2\dot{\varphi} - mg\frac{\rho^2}{a}$

1) $\frac{\partial \mathcal{L}}{\partial \varphi} = 0 \qquad \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = m\rho^2\dot{\varphi} + \frac{H_0 e}{2c}\rho^2 = p_\varphi = \text{const}$

2) $\frac{\partial \mathcal{L}}{\partial t} = 0 \qquad E = \frac{\partial \mathcal{L}}{\partial \dot{\rho}}\dot{\rho} - \mathcal{L} = m\dot{\rho}^2 + \cancel{m\rho^2\dot{\varphi}^2} +$

$$+ \cancel{m\frac{4\rho^2}{a^2}}\dot{\rho}^2$$

$$= \frac{m}{2}\left(\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \left(\frac{2\rho}{a}\dot{\rho}\right)^2\right) + mg\frac{\rho^2}{a}$$

$$\dot{\varphi} = \frac{p_\varphi - \frac{eH_0}{2c}\rho^2}{m\rho^2}$$

$$E = \frac{m}{2}\left(\dot{\rho}^2\left(1 + \frac{4\rho^2}{a^2}\right) + \rho^2\left(\frac{p_\varphi - \frac{eH_0}{2c}\rho^2}{m\rho^2}\right)^2\right) + mg\frac{\rho^2}{a}$$

$$\dot{\rho} = \pm\sqrt{\frac{\frac{2}{m}\left(E - mg\frac{\rho^2}{a}\right) - \rho^2\left(\frac{p_\varphi - \frac{eH_0}{2c}\rho^2}{m\rho^2}\right)^2}{1 + 4\rho^2/a^2}}$$

$$\frac{d\varphi}{\dot{\rho}} = \frac{d\varphi}{d\rho} = \frac{\frac{p_\varphi - \frac{eH_0}{2c}\rho^2}{m\rho^2}}{\pm\sqrt{\ldots}}$$

$$\varphi - \varphi_0 = \pm\int \ldots d\rho$$

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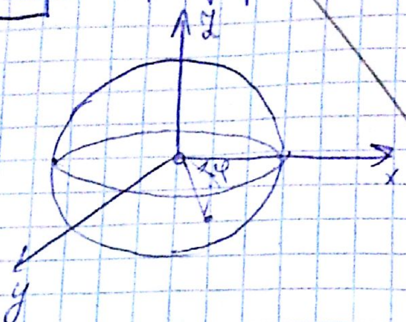
№9] m, сфера R, найти лагр. $L$ в цил и з-н движ.




$x = \rho\cos\varphi \qquad y = \rho\sin\varphi \qquad x^2+y^2+z^2=R^2$

$z = \mp\sqrt{R^2-\rho^2} \qquad \dot z = +\dfrac{2\rho\dot\rho}{2\sqrt{R^2-\rho^2}}$

$$L = \frac{m}{2}\left(\dot\rho^2 + \rho^2\dot\varphi^2 + \left(\frac{\rho\dot\rho}{\sqrt{R^2-\rho^2}}\right)^2\right) - mg\sqrt{R^2-\rho^2}$$

$\dfrac{\rho^2\dot\rho^2}{R^2-\rho^2}$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q} - \frac{\partial L}{\partial q} = Q$$

$\rho:\quad \dfrac{\partial L}{\partial \dot\rho} = m\dot\rho + m\dfrac{\rho^2\dot\rho}{(R^2-\rho^2)}$ неправ

$$m\ddot\rho + m\frac{[(\ddot\rho\rho^2) + 2\rho\dot\rho^2]}{(R^2-\rho^2)^2} \qquad \frac{\partial L}{\partial \rho} = m\rho\dot\varphi^2 + \frac{2\dot\rho^2\rho(R^2-\rho^2) + 2\rho^3\dot\rho^2}{(R^2-\rho^2)^2} - \frac{mg\rho}{\sqrt{R^2-\rho^2}}$$

$$m\ddot\rho + \frac{m\rho^2\ddot\rho}{\sqrt{R^2-\rho^2}} - m\rho\dot\varphi^2 - \frac{2\dot\rho^2\rho(R^2-\rho^2) + 2\rho^3\dot\rho^2}{(R^2-\rho^2)^2} + \frac{mg\rho}{\sqrt{R^2-\rho^2}}$$

$\varphi:\quad \dfrac{\partial L}{\partial \dot\varphi} = m\rho^2\dot\varphi \qquad m\rho^2\ddot\varphi = Q$

2 способ: $x^2+y^2+z^2 = R^2$

$x = \sqrt{R^2-z^2}\cos\varphi \qquad y = \sqrt{R^2-z^2}\sin\varphi$

№10] $P = q + e^{-q} + \ln p \qquad Q = pe^q$

Критерий каноничности

$$\begin{cases} \{P_i(p,q,t),\ Q_j(p,q,t)\}_{pq} = c\delta_{ij} \\ \{P_i(p,q,t),\ P_j(p,q,t)\}_{p,q} = 0 \\ \{Q_i(p,q,t),\ Q_j(p,q,t)\}_{p,q} = 0 \end{cases}$$

Проверим $\{P_i, P_j\} = 0 \quad \{Q_i, Q_j\} = 0$ из-за некоммут. скобок Пуассона

$$\{P,Q\}_{pq} = \frac{\partial P}{\partial p}\frac{\partial Q}{\partial q} - \frac{\partial P}{\partial q}\frac{\partial Q}{\partial p} = \frac{1}{p}\cdot pe^q - (1-e^{-q})\cdot e^q =$$

$= e^q - e^q + 1 = 1 = c$ унивалентное.

$$\{P,P\}_{pq} = \frac{\partial P}{\partial p}\frac{\partial P}{\partial q} - \frac{\partial P}{\partial q}\frac{\partial P}{\partial p} = 0.$$

$$\det\frac{\partial Q_i}{\partial p_j} = \frac{\partial Q}{\partial p} = e^q \neq 0. \qquad \exists F_1(q,Q,t)$$

$$\begin{cases} p_i = \dfrac{\partial F_1}{\partial q_i} \\ P_i = -\dfrac{\partial F_1}{\partial Q_i} \end{cases} \qquad \begin{cases} p = \dfrac{\partial F_1}{\partial q} = Qe^{-q} \\ P = -\dfrac{\partial F}{\partial Q} = q + e^{-q} + \ln(Qe^{-q}) = \end{cases}$$

$= q - q + e^{-q} + \ln Q$

$F_1 = \int Qe^{-q}dq + f_1(Q,t) = -Qe^{-q} + f_1(Q,t)$

$F_1 = -\int(e^{-q} + \ln Q)\,dQ + f_2(q,t) =$

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$\int dQ \ln Q = \int (d(Q\ln Q) - Q\,d\ln Q) = Q\ln Q - Q$

$F_1 = -\underline{Qe^{-q}} - Q\ln Q + Q + f_2(q,t)$

$f_2 = 0 \quad f_1 = -Q\ln Q + Q$

$F_1 = -Qe^{-q} - Q\ln Q + Q$

$\boxed{N5}$ $\quad \mathcal{L} = \frac{m}{2}(\dot\rho^2 + \rho^2\dot\varphi^2) - \frac{a\cos\varphi}{\rho^2}$ Сист. движ. и законы движ.

$\frac{\partial \mathcal{L}}{\partial t} = 0 \quad E = \frac{m}{2}\dot\rho^2 + \frac{m}{2}\rho^2\dot\varphi^2 + \frac{a\cos\varphi}{\rho^2}$

$\frac{\partial \mathcal{L}}{\partial \varphi} \neq 0 \quad \cancel{p_\varphi = \frac{\partial \mathcal{L}}{\partial \dot\varphi} = m\rho^2\dot\varphi}$

1) $\rho$: $\frac{\partial \mathcal{L}}{\partial \dot\rho} = m\dot\rho \quad \frac{\partial \mathcal{L}}{\partial \rho} = m\rho\dot\varphi^2 + \frac{2a\cos\varphi}{\rho^3}$

$m\ddot\rho - m\rho\dot\varphi^2 - \frac{2a\cos\varphi}{\rho^3} = 0$

2) $\varphi$: $\frac{\partial \mathcal{L}}{\partial \dot\varphi} = m\rho^2\dot\varphi \quad \frac{\partial \mathcal{L}}{\partial \varphi} = \frac{a\sin\varphi}{\rho^2}$

$m\rho^2\ddot\varphi + 2m\rho\dot\rho\dot\varphi - \frac{a\sin\varphi}{\rho^2} = 0$

$\boxed{N6.}$ Центральное поле. Сфер коорд. Инт. движ. З-н движ.-?

$\vec F = -F(r)\frac{\vec r}{r} \quad r = |\vec r| = \sqrt{x^2+y^2+z^2}$.

Не всякое центральное поле явл потенциальным.

$\exists U: \vec F = -\vec\nabla U, \quad U = U(r)$

$\vec\nabla U(r) = \frac{\partial}{\partial \vec r}U(r) = \frac{\partial U(r)}{\partial r}\frac{\partial r}{\partial \vec r} = U'(r)\frac{d\sqrt{\vec r^2}}{d\vec r} = U'(r)\frac{1}{2}\frac{1}{\sqrt{\vec r^2}}\underbrace{\frac{\partial \vec r^2}{\partial \vec r}}_{2\vec r} =$

$= U'(r)\frac{\vec r}{r} = -F = F(r)\frac{\vec r}{r}$

$\Rightarrow U'(r) = F(r) \Rightarrow \forall F(r) \quad \exists U(r) = \int F(r)dr$

В сфер коорд $\quad \mathcal{L} = \frac{m}{2}(\dot r^2 + r^2\dot\theta^2 + r^2\sin^2\theta\,\dot\varphi^2) - U(r)$

1) $\frac{\partial \mathcal{L}}{\partial t} = 0 \quad Q_i^d = 0 \Rightarrow E = const$

2) $\frac{\partial \mathcal{L}}{\partial \varphi} = 0 \quad Q_i^\varphi = 0 \Rightarrow p_\varphi = const.$

$E = \frac{m}{2}(\dot r^2 + r^2\dot\theta^2 + r^2\sin^2\theta\,\dot\varphi^2) + U(r)$

$p_\varphi = mr^2\sin^2\theta\,\dot\varphi$

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В произв коорд.

$\mathcal{L} = \frac{m\dot{\vec{r}}^2}{2} - U(r)$ УЛ: $\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\vec{r}}} - \frac{\partial \mathcal{L}}{\partial \vec{r}} = 0$

$\frac{\partial \mathcal{L}}{\partial \dot{\vec{r}}} = m\dot{\vec{r}}$ ; $\frac{\partial \mathcal{L}}{\partial \vec{r}} = -\frac{\partial U(r)}{\partial \vec{r}} = -\frac{dU(r)}{dr}\frac{dr}{d\vec{r}} = -U'(r)\times\frac{d\sqrt{\vec{r}^2}}{d\vec{r}}$

$= -U'(r)\frac{\vec{r}}{r}$ $\qquad \frac{1}{2}\frac{2\vec{r}}{\sqrt{\vec{r}^2}}$

УЛ: $m\ddot{\vec{r}} = -U'(r)\frac{\vec{r}}{r} \;|\cdot\vec{r}$ слева

$m[\vec{r}\times\ddot{\vec{r}}] = -\frac{U'(r)}{r}\,\overset{0}{[\cancel{\vec{r}\times\vec{r}}]}$

$0 = m[\vec{r}\times\frac{d}{dt}\dot{\vec{r}}] = m\frac{d}{dt}[\vec{r}\times\dot{\vec{r}}] - m\overset{0}{[\cancel{\frac{d}{dt}\vec{r}}\times\dot{\vec{r}}]}$

$0 = \frac{d}{dt}(\underbrace{m[\vec{r}\times\dot{\vec{r}}]}_{L})$

L - древ мом имп.

! $\frac{d}{dt}\vec{L} = 0$ $\quad \vec{L} = const = \vec{C} \;|\cdot\vec{r}$

$([\vec{L}\cdot\vec{r}]) = (\vec{C}\cdot\vec{r}) = m(\vec{r}\,[\vec{r}\times\dot{\vec{r}}]) = m(\overset{0}{[\cancel{\vec{r}\times\vec{r}}]}\times\dot{\vec{r}}) = 0$

$(\vec{C},\vec{r}) = 0 \qquad C_x x + C_y y + C_z z = 0 \qquad C_x, C_y, C_z$ - const

ур-ние пл-сти

(Пл-сть Лапласа)

$(\vec{L}\cdot\vec{r}) = 0 \Leftrightarrow \vec{L}\perp\vec{r}$

В полярн коорд $\quad \mathcal{L} = \frac{m}{2}(\dot{\rho}^2 + \rho^2\dot{\varphi}^2) - U(\rho)$

1) $\frac{\partial \mathcal{L}}{\partial t} = 0 \quad Q_i^d = 0 \Rightarrow E = const = \frac{m}{2}(\dot{\rho}^2 + \rho^2\dot{\varphi}^2) + U(\rho)$

2) $\frac{\partial \mathcal{L}}{\partial \varphi} = 0$ ($\varphi$ - цикл коорд) $\quad Q_i^d = 0 \Rightarrow p_\varphi = const = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = m\rho^2\dot{\varphi}$

$\begin{cases}\dot{\varphi} = \frac{p_\varphi}{m\rho^2}\\ E = \frac{m\dot{\rho}^2}{2} + \frac{p_\varphi^2}{2m\rho^2} + U(\rho)\end{cases}$ $\qquad \dot{\rho} = \frac{d\rho}{dt} = \pm\sqrt{\frac{2}{m}\left(E - U(\rho) - \frac{p_\varphi^2}{2m\rho^2}\right)}$

„+" - $\rho\uparrow$ удал от с.ц.

$t - t_0 = \int_{t_0}^{t} dt = \pm\int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{\frac{2}{m}\left(E - U(\rho) - \frac{p_\varphi^2}{2m\rho^2}\right)}}$ $\qquad \rho = \rho(t)$ неявно

$\frac{\dot{\rho}}{\dot{\varphi}} = \frac{d\rho}{d\varphi} = \frac{\pm\sqrt{\phantom{x}}}{p_\varphi/m\rho^2}$

$\varphi - \varphi_0 = \int_{\varphi_0}^{\varphi} d\varphi = \pm\int_{\rho_0}^{\rho}\frac{p_\varphi}{m\rho^2}\frac{d\rho}{\sqrt{\frac{2}{m}\left(E - U(\rho) - \frac{p_\varphi^2}{2m\rho^2}\right)}}$

$\left.\begin{array}{l}\varphi = \varphi(\rho) \text{ явно}\\ \rho = \rho(\varphi) \text{ неявно}\end{array}\right\}$ ур-ние траектории

$U(\rho) + \frac{p_\varphi^2}{2m\rho^2} = U_{eff}(\rho)$ эфф потенциал

Физ смысл $p_\varphi$: $\vec{L} = m[\vec{r}\times\dot{\vec{r}}] = \left[\begin{array}{l}\vec{r} = \rho\vec{e}_\rho\\ \dot{\vec{r}} = \dot{\rho}\vec{e}_\rho + \rho\dot{\varphi}\vec{e}_\varphi\end{array}\right] = m[(\rho\vec{e}_\rho)\times(\dot{\rho}\vec{e}_\rho + \rho\dot{\varphi}\vec{e}_\varphi)] =$

$= m\rho^2\dot{\varphi}[\vec{e}_\rho\times\vec{e}_\varphi]$ ; $|\vec{L}| = m\rho^2\dot{\varphi} = |p_\varphi|$ уг мом. $\quad \vec{e}_z$

16. В сфер коорд центральное поле.

$$\mathcal{L} = \frac{m}{2}(\dot{r}^2 + r^2\dot{\Theta}^2 + r^2\sin^2\Theta\,\dot{\varphi}^2) - U(r)$$

не цикл. коорд

$$\left.\begin{aligned} p_r &= \frac{\partial \mathcal{L}}{\partial \dot{r}} = m\dot{r} \\ p_\varphi &= \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = mr^2\sin^2\Theta\,\dot{\varphi} \\ p_\Theta &= \frac{\partial \mathcal{L}}{\partial \dot{\Theta}} = mr^2\dot{\Theta} \end{aligned}\right\} \quad \dot{r} = \frac{p_r}{m}, \quad \dot{\varphi} = \frac{p_\varphi}{mr^2\sin^2\Theta}, \quad \dot{\Theta} = \frac{p_\Theta}{mr^2}$$

$$H = \frac{1}{2m}\left(p_r^2 + \frac{p_\Theta^2}{r^2} + \frac{p_\varphi^2}{r^2\sin^2\Theta}\right) + U(r)$$

1) $\frac{\partial H}{\partial t} = 0 \quad \Theta_i^d = 0 \Rightarrow H = const = E$ сохр-ся (инт.) дв.

2) $p_\varphi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = mr^2\sin^2\Theta\,\dot{\varphi}$ — сохр. дв

$\frac{\partial H}{\partial \varphi} = 0 \quad \Theta_i^d = 0 \quad p_\varphi = G$

3) $$H = \frac{1}{2m}\left(p_r^2 + \frac{1}{r^2}\underbrace{\left(p_\Theta^2 + \frac{G}{\sin^2\Theta}\right)}_{f}\right) + U(r)$$

$\frac{\partial f}{\partial t} = 0 \Rightarrow f = p_\Theta^2 + \frac{G^2}{\sin^2\Theta} = C$ инт-л движ.

$$\begin{cases} E = \frac{p_r^2}{2m} + \frac{C}{2mr^2} + U(r) = |p_r = m\dot{r}| = \frac{m\dot{r}^2}{2} + \frac{C}{2mr^2} + U(r) \\ G = mr^2\sin^2\Theta\,\dot{\varphi} \\ C = |p_\Theta = mr^2\dot{\Theta}| = m^2r^4\dot{\Theta}^2 + \frac{G^2}{\sin^2\Theta} \end{cases}$$

$$\dot{r} = \frac{dr}{dt} = \pm\sqrt{\frac{2}{m}\left(E - \frac{C}{2mr^2} - U(r)\right)}$$

$$\int_{t_0}^{t} dt = \pm\int_{r_0}^{r} \frac{dr}{\sqrt{\frac{2}{m}\left(E - \frac{C}{2mr^2} - U(r)\right)}}$$

$$\frac{\dot{r}}{\dot{\Theta}} = \frac{dr}{d\Theta} = \frac{\pm\sqrt{\frac{2}{m}\left(E - \frac{C}{2mr^2} - U(r)\right)}}{\pm\frac{1}{mr^2}\sqrt{C - \frac{G^2}{\sin^2\Theta}} = \dot{\Theta}}$$

$$\pm\int_{r_0}^{r} \frac{dr}{mr^2\sqrt{\frac{2}{m}\left(E - \frac{C}{2mr^2} - U(r)\right)}} = \pm\int_{\Theta_0}^{\Theta} \frac{d\Theta}{\sqrt{C - \frac{G}{\sin^2\Theta}}}$$

$$\frac{\dot{\Theta}}{\dot{\varphi}} = \frac{d\Theta}{d\varphi} = \frac{\pm\frac{1}{mr^2}\sqrt{C - \frac{G^2}{\sin^2\Theta}}}{G/mr^2\sin^2\Theta}$$

$$\int_{\varphi_0}^{\varphi} d\varphi = \pm\int_{\Theta_0}^{\Theta} \frac{G}{\sin^2\Theta}\frac{d\Theta}{\sqrt{C - \frac{G}{\sin^2\Theta}}}$$

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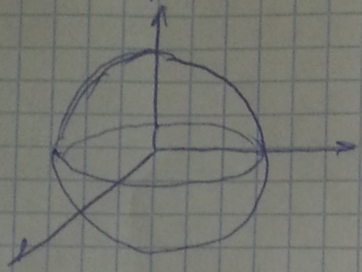
№3 Сфера $R$, сила тяж. Закон движ. УГЯ.



Сф. коорд $x = \rho \sin\theta \cos\varphi$ $y = \rho\sin\theta\sin\varphi$

$z = \rho\cos\theta$ $\quad \rho = R$

$$L = \frac{m}{2}\left(R^2\sin^2\theta\,\dot\varphi^2 + R^2\dot\theta^2\right) - mgR\cos\theta$$

$$H = \cancel{\frac{p_\rho^2}{2m}} + \frac{p_\theta^2}{2mR^2} + \frac{p_\varphi^2}{2mR^2\sin^2\theta} + mgR\cos\theta$$

УГЯ $\quad 0 = \frac{\partial F}{\partial t} + H\left(\frac{\partial F}{\partial q}; q, t\right)$

$$0 = \frac{\partial F}{\partial t} + \frac{1}{2mR^2}\left(\left(\frac{\partial F}{\partial \theta}\right)^2 + \frac{\left(\frac{\partial F}{\partial \varphi}\right)^2}{\sin^2\theta}\right) + mgR\cos\theta$$

$$F = T(t) + \Theta(\theta) + \Psi(\varphi)$$

$$0 = T' + \frac{1}{2mR^2}\left(\Theta'^2 + \frac{\Psi'^2}{\sin^2\theta}\right) + mgR\cos\theta$$

1) $-T' = \ldots = C_1 = E \qquad T = -Et$

2) $E = \frac{1}{2mR^2}\left(\Theta'^2 + \frac{\Psi'^2}{\sin^2\theta}\right) + mgR\cos\theta$

$$E\cdot 2mR^2\sin^2\theta = \Theta'^2\sin^2\theta + \Psi'^2 + 2m^2gR^3\cos\theta\sin^2\theta$$

$$E\,2mR^2\sin^2\theta - 2mgR^3\sin^2\theta\cos\theta - \Theta'^2\sin^2\theta = \Psi'^2$$

$$\Psi' = \pm\sqrt{(\quad)} = C_2 \qquad \Psi(\varphi) = C_2\varphi$$

$$\Theta'^2 = -\frac{C_2^2}{\sin^2\theta} + 2mR^2E \mp 2m^2gR^3\cos\theta$$

Знаки проверь

$$\Theta' = \pm\sqrt{-\frac{C_2^2}{\sin^2\theta} + 2mR^2E + 2m^2gR^3\cos\theta}$$

Смысл.

$$F = -Et + C_2\varphi \pm \int d\theta\sqrt{\quad}$$

$$\frac{\partial F}{\partial \varphi} = p_\varphi = C_2 \qquad \frac{\partial F}{\partial \theta} = p_\theta$$

$$E = \frac{1}{2mR^2}\left(\frac{p_\varphi^2}{\sin^2\theta} - p_\theta^2\right) + mgR\cos\theta$$

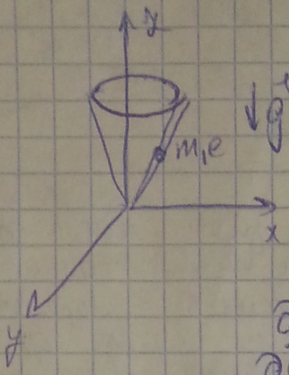
3-н движ. $\frac{\partial F}{\partial E} = \alpha_1 = -t \pm \int d\theta \frac{2mR^2}{2\sqrt{\quad}}$

$$t - t_0 = \pm\int_{\theta_0}^{\theta} \frac{d\theta\; mR^2}{2\sqrt{\quad}}$$

$$\frac{\partial F}{\partial C_2} = \varphi \pm \int d\theta \frac{-C_2}{\sin^2\theta\; 2\sqrt{\quad}}$$

$$\varphi - \varphi_0 = \pm\int_{\theta_0}^{\theta} \ldots$$

№7) Частица $m, e$ по поверхн конуса, по оси поля и однор магн поле. Д. УГЯ.




$\downarrow \vec{g}$ $\uparrow \vec{H}_0$

$x^2 + y^2 = z^2$ — конус

$x, y, z = \sqrt{x^2 + y^2}$

$x = \rho\cos\varphi \quad y = \rho\sin\varphi \quad z = \rho$

т.е. $L = \frac{m}{2}(\dot\rho^2 + \rho^2\dot\varphi^2 + \dot\rho^2) + \frac{eH_0}{2c}\rho^2\dot\varphi - mg\rho$

$\frac{\partial L}{\partial \dot\rho} = 2m\dot\rho = P_\rho \qquad \dot\rho = \frac{P_\rho}{2m}$

$\frac{\partial L}{\partial \dot\varphi} = m\rho^2\dot\varphi + \frac{eH_0}{2c}\rho^2 = P_\varphi \qquad \dot\varphi = \frac{P_\varphi - \frac{eH_0}{2c}\rho^2}{m\rho^2}$

$H = \frac{m}{2}\left(2\cdot\frac{P_\rho^2}{4m^2} + \rho^2\frac{(P_\varphi - \frac{eH_0}{2c}\rho^2)^2}{(m\rho^2)^2}\right) + \frac{eH_0}{2c}\ mg\rho$

$H = \frac{P_\rho^2}{4m} + \frac{(P_\varphi - \frac{eH_0}{2c}\rho^2)^2}{2m\rho^2} + mg\rho$ — раскрытие в квадр.

$0 = \frac{\partial F}{\partial t} + H\left(\frac{\partial F}{\partial q}, q, t\right) \qquad F = T(t) + R(\rho) + \Psi(\varphi)$

$0 = T' + \frac{1}{4m}R'^2 + \frac{1}{2m\rho^2}\left(\Psi' - \frac{eH_0}{2c}\rho^2\right)^2 + mg\rho$

$-T' = E \qquad T = -Et$

$\frac{R'^2}{4m} + \frac{1}{2m\rho^2}\left(\Psi'^2 - \frac{eH_0}{2c}\rho^2\right)^2 + mg\rho = E$

$R'^2 = E4m - 4m^2\rho g - \frac{2}{\rho^2}(\Psi'^2$

$z = r\operatorname{ctg}\alpha$

$L = \frac{m}{2}(\dot r^2 + r^2\dot\varphi^2 + \dot r^2\operatorname{ctg}^2\alpha) + \frac{eH_0}{2c}r^2\dot\varphi - mgr\operatorname{ctg}\alpha$

$P_r = \frac{\partial L}{\partial \dot r} = \frac{m\dot r}{\sin^2\alpha} \Rightarrow \dot r = \frac{P_r\sin^2\alpha}{m}$

$P_\varphi = \frac{\partial L}{\partial \dot\varphi} = mr^2\dot\varphi + \frac{H_0 r^2 e}{2c} \Rightarrow \dot\varphi = \frac{P_\varphi - \frac{H_0 e}{2c}r^2}{mr^2}$

$H = \frac{(P_\varphi - \frac{H_0}{2c}er^2)^2}{2mr^2} + \frac{P_r^2\sin^2\alpha}{2m} + \frac{eH_0}{2c}\frac{(P_\varphi - \frac{H_0}{2c}er^2)}{m} - mgr\operatorname{ctg}\alpha$

$\frac{\partial S}{\partial t} + H\left(q, \frac{\partial S}{\partial q}, t\right) = 0$

$\frac{\partial S}{\partial t} + \frac{1}{2mr^2}\left(\frac{\partial S}{\partial \varphi} - \frac{H_0}{2c}r^2 e\right)^2 + \frac{1}{2m}\left(\frac{\partial S}{\partial r}\sin\alpha\right)^2 + \frac{eH_0}{2cm}\left(\frac{\partial S}{\partial \varphi} - \frac{H_0 e}{2c}r^2\right) +$

$+ \frac{\sin^2\alpha}{2m}[f'(r)]^2 + mgr\operatorname{ctg}\alpha = 0$

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№8 УГЯ $0 = H\left(\frac{\partial F}{\partial q}, q, t\right) + \frac{\partial F}{\partial t}$

частица в центральном поле

$$L = \frac{m}{2}(\dot\rho^2 + \rho^2\dot\varphi^2) - U(\rho)$$

$$H = \frac{p_\rho^2}{2m} + \frac{p_\varphi^2}{2m\rho^2} + U(\rho)$$

$$p_\rho \to \frac{\partial F}{\partial \rho} \qquad p_\varphi \to \frac{\partial F}{\partial \varphi}$$

$$0 = \frac{\partial F}{\partial t} + \frac{1}{2m}\left(\left(\frac{\partial F}{\partial \rho}\right)^2 + \frac{1}{\rho^2}\left(\frac{\partial F}{\partial \varphi}\right)^2\right) + U(\rho) \leftarrow \text{УГЯ}$$

$$F(t,\rho,\varphi) = T(t) + R(\rho) + \Phi(\varphi)$$

$$-T'(t) = \frac{1}{2m}(R'(\rho))^2 + \frac{1}{2m\rho^2}(\Phi'(\varphi))^2 + U(\rho) = C_1$$

$$C_2 = \Phi'(\varphi) = \pm\sqrt{2m\rho^2\left(C_1 - U(\rho) - \frac{(R'(\rho))^2}{2m}\right)}$$

$$T'(t) = -C_1 \qquad T(t) = -C_1 t$$

$$\Phi'(\varphi) = C_2 \qquad \Phi(\varphi) = C_2\varphi$$

$$C_2^2 = 2m\rho^2\left(C_1 - U(\rho) - \frac{1}{2m}(R'(\rho))^2\right)$$

$$R'(\rho) = \pm\sqrt{2m\left(C_1 - U(\rho) - \frac{C_2^2}{2m\rho^2}\right)}$$

$$R(\rho) = \pm\int\sqrt{2m(C_1 - U(\rho) - C_2^2/2m\rho^2)}\,d\rho$$

$$F = -C_1 t + C_2\varphi + \int d\rho\sqrt{2m(C_1 - U(\rho) - C_2^2/2m\rho^2)}$$

$$\frac{\partial F}{\partial \varphi} = p_\varphi = C_2; \qquad \frac{\partial F}{\partial \rho} = p_\rho = \pm\sqrt{2m(C_1 - U(\rho) - C_2^2/2m\rho^2)}$$

$$C_1 = \frac{p_\rho^2}{2m} + U(\rho) + \frac{C_2^2}{2m\rho^2} = E$$

$$\frac{\partial F}{\partial C_1} = \alpha_1 = -t \pm \int d\rho\sqrt{2m}\,\frac{1}{2\sqrt{C_1 - U(\rho) - C_2^2/2m\rho^2}}$$

$$t - t_0 = \int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{\frac{2}{m}\left(C_1 - U(\rho) - \frac{C_2^2}{2m\rho^2}\right)}}$$

$$\frac{\partial F}{\partial C_2} = \alpha_2 = \varphi \pm \int d\rho\sqrt{2m}\,\frac{C_2}{2m\rho^2}\,\frac{1}{\sqrt{C_1 - U(\rho) - C_2^2/2m\rho^2}}$$

$$\varphi - \varphi_0 = \pm\int_{\rho_0}^{\rho}\frac{C_2\,d\rho}{\rho^2\sqrt{2m\left(C_1 - U(\rho) - \frac{C_2^2}{2m\rho^2}\right)}}$$

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**№9** Произв. $\vartheta/m$ поле. $\{\vartheta_x, \vartheta_z\}$

$$H = \frac{(\vec{p} - \frac{e}{c}\vec{A})^2}{2m} + e\varphi$$

$$\vec{\vartheta} = \frac{\partial H}{\partial \vec{p}} = \frac{\vec{p} - \frac{e}{c}\vec{A}}{m} \qquad \vartheta_x = \frac{p_x - \frac{e}{c}A_x}{m} \qquad \vartheta_z = \frac{p_z - \frac{e}{c}A_z}{m}$$

$$\left\{\frac{p_x - \frac{e}{c}A_x}{m}, \frac{p_z - \frac{e}{c}A_z}{m}\right\} = \frac{1}{m^2}\left\{p_x - \frac{e}{c}A_x, p_z - \frac{e}{c}A_z\right\} =$$

$$= \frac{1}{m^2}\left(\{p_x, p_z\} - \frac{e}{c}\{p_x, A_z\} - \frac{e}{c}\{A_x, p_z\} + \frac{e^2}{c^2}\{A_x, A_z\}\right) =$$

$$= 0 + \frac{1}{m^2}\left(-\frac{e}{c}\left(\{p_x, A_z\} + \{A_x, p_z\} + \{A_x, -\frac{e}{c}A_z\}\right)\right)$$

---

$\{\dot{x}_i, \dot{x}_j\} - ?$ $\qquad L = \frac{m\dot{\vec{r}}^2}{2} + \frac{e}{c}(\vec{A}, \dot{\vec{r}}) - e\varphi$

$$\vec{p} = \frac{\partial L}{\partial \dot{\vec{r}}} = m\dot{\vec{r}} + \frac{e}{c}A \qquad \dot{\vec{r}} = \frac{1}{m}(\vec{p} - \frac{e}{c}\vec{A}) \to \dot{x}_i = \frac{1}{m}(p_i - \frac{e}{c}A_i)$$

$$\left\{\frac{1}{m}(p_i - \frac{e}{c}A_i), \frac{1}{m}(p_j - \frac{e}{c}A_j)\right\} = \sum_{k=1}^{3}\left(\frac{\partial \dot{x}_i}{\partial p_k}\frac{\partial \dot{x}_j}{\partial x_k} - \frac{\partial \dot{x}_i}{\partial x_k}\frac{\partial \dot{x}_j}{\partial p_k}\right) =$$

$$= \sum_{k=1}\left(\frac{\partial \frac{1}{m}(p_i - \frac{e}{c}A_i)}{\partial p_k}\frac{\partial \frac{1}{m}(p_j - \frac{e}{c}A_j)}{\partial x_k} - \frac{\partial \frac{1}{m}(p_i - \frac{e}{c}A_i)}{\partial x_k}\frac{\partial \frac{1}{m}(p_j - \frac{e}{c}A_j)}{\partial p_k}\right) =$$

$$= \frac{1}{m^2}\sum_{k=1}\left(\delta_{ik}\left(-\frac{e}{c}\frac{\partial A_j}{\partial x_k}\right) - \delta_{jk}\left(-\frac{e}{c}\frac{\partial A_i}{\partial x_k}\right)\right) = \cancel{\frac{1}{m^2}\frac{e}{c}\left(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x}\right)} =$$

$$= \cancel{\frac{1}{m^2}\frac{e}{c}H_z} \qquad = \frac{1}{m^2}\left(-\frac{e}{c}\frac{\partial A_j}{\partial x_i} + \frac{e}{c}\frac{\partial A_i}{\partial x_j}\right)$$

$$\{\dot{x}, \dot{z}\} = \frac{1}{m^2}\left(-\frac{e}{c}\frac{\partial A_z}{\partial x} + \frac{e}{c}\frac{\partial A_x}{\partial z}\right) = \frac{e}{m^2 c}(-H_y)$$

$$\vec{H} = \operatorname{rot}\vec{A} = \begin{vmatrix} e_x & e_y & e_z \\ A_x & A_y & A_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = -\vec{e}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \ldots$$

---

**№10** $\mathcal{L} = \frac{1}{2}(q_2\dot{q}_1^2 + q_1\dot{q}_2^2) - \left(\frac{1}{q_1 q_2} + q_1 + q_2\right)$ Найти норм. коорд. сист.

$$U(q_1, q_2) = \frac{1}{q_1 q_2} + q_1 q_2$$

$$0 = \frac{\partial U}{\partial q_1} = -\frac{1}{q_1^2 q_2} + 1 \qquad 0 = \frac{\partial U}{\partial q_2} = -\frac{1}{q_2^2 q_1} + 1$$

$$\begin{cases} \frac{1}{q_1^2 q_2} = 1 \\ \frac{1}{q_2^2 q_1} = 1 \end{cases} \qquad \begin{cases} q_1^2 q_2 = 1 \\ q_2^2 q_1 = 1 \end{cases} \qquad q_1^4 q_2^2 = 1 \quad q_1^3 = 1 \quad q_1 = 1$$

$\begin{cases} q_2 = 1 \\ q_1 = 1 \end{cases}$ – точка min потенц. = точка равновесия

1) введем коорд. отклонения $\qquad x_1 = q_1 - q_{01} = q_1 - 1$

$$x_2 = q_2 - q_{02} = q_2 - 1$$

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$$\mathcal{L} = \frac{1}{2}\left((1+x_2)\dot{x}_1^2 + (1+x_1)\dot{x}_2^2\right) - \frac{1}{(1+x_1)(1+x_2)} - (1+x_1) - (1-x_2) =$$

$$= \frac{1}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - (1 - x_1 + x_1^2 + \ldots)(1 - x_2 + x_2^2 + \ldots) - 2 - x_1 - x_2 =$$

$$(1+x)^{-1} \underset{x\to 0}{=} 1 - x + x^2 + \ldots$$

$$= \frac{1}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - (1 - x_1 + x_1^2 - x_2 + x_1 x_2 + x_2^2) - 2 - x_1 - x_2 =$$

$$= \frac{1}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \underbrace{3}_{const} - x_1^2 - x_1 x_2 - x_2^2$$

$$\mathcal{L}^{(2)} = \frac{1}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - x_1^2 - x_2^2 - x_1 x_2.$$

УЛ: $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0 \qquad \begin{cases} \ddot{x}_1 + 2x_1 + x_2 = 0 \\ \ddot{x}_2 + x_1 + 2x_2 = 0. \end{cases}$

Реш в виде $x_i = Re\left(A_i e^{i\omega t}\right)$

$$\begin{cases} A_1(-\omega^2)^2 + 2A_1 + A_2 = 0 \\ A_2(-\omega^2) + 2A_2 + A_1 = 0 \end{cases}$$

$$\begin{pmatrix} 2-\omega^2 & 1 \\ 1 & 2-\omega^2 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0.$$

$(2-\omega^2)^2 - 1 = 0 \qquad 4 - 4\omega^2 + \omega^4 - 1 = 0$

$(2-\omega^2-1)(2-\omega^2+1) = 0$

$(1-\omega^2)(3-\omega^2) = 0$

$\omega_{1,2} = \pm\sqrt{1} \qquad \omega_{3,4} = \pm\sqrt{3}.$

2) $\omega^2 = 1 \qquad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0 \qquad A_1 = C_2 \qquad A_2 = -C_2$

1) $\omega^2 = 3 \qquad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0 \qquad A_1 = C_1 \qquad A_2 = C_1.$

$$x = Re\left[C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{\sqrt{3}it} + C_2\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{it}\right]$$

$$T = \frac{1}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) = \frac{1}{2}\sum t_{ij}\dot{x}_i\dot{x}_j \qquad T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

1) $A^T T A = (1\ 1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = (1\ 1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 + 1 = 2.$

Нормир

$C_1^2 A^T T A = 1 \Rightarrow C_1^2 = \frac{1}{2} \qquad C_1 = \frac{1}{\sqrt{2}}$

2) $A^{2T} T A^2 = (1\ -1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = (1\ -1)\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 2 \qquad C_2 = \frac{1}{\sqrt{2}}$

$A^T T A = 1 \mid \cdot A^{-1}$

$A^T T = A^{-1}$

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = A\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = A^{-1}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = A^T T\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} =}$$

$$\left.\begin{aligned} q_1 &= \frac{1}{\sqrt{2}}(\xi_1 + \xi_2) \\ q_2 &= \frac{1}{\sqrt{2}}(\xi_1 - \xi_2) \end{aligned}\right\} \text{закон преобраз}$$

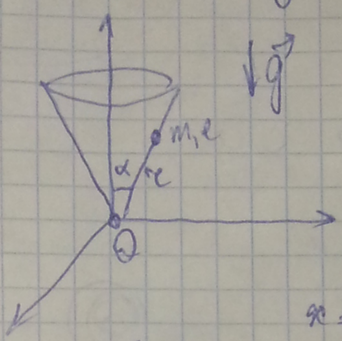
$$\mathcal{L} = \frac{1}{2}\left(\frac{1}{2}(\dot{\xi}_1 + \dot{\xi}_2)^2 + \frac{1}{2}(\dot{\xi}_1 - \dot{\xi}_2)^2\right) - \left(\frac{1}{2}(\xi_1 + \xi_2)^2 + \frac{1}{2}(\xi_1 + \xi_2)(\xi_1 - \xi_2)\right.$$

$$\left. + \frac{1}{2}(\xi_1 - \xi_2)^2\right) = \frac{1}{2}\left(\dot{\xi}_1^2 + \dot{\xi}_2^2\right) - 3\xi_1^2 - \frac{1}{2}\xi_2^2$$

$$\mathcal{L} = \sum_{k=1}^{2}\left(\frac{1}{2}\dot{\xi}_k^2 - \frac{1}{2}\omega_k^2\xi_k^2\right)$$

№11 $m, e$ вершин конус $2\alpha$ при верш. $\vec{g}$ в верш конуса $Q$

ФГ, сист движ, закон движ




$U = \frac{eQ}{r}$

2 степ своб

$z = r\cos\alpha$

конус $x^2 + y^2 = r^2\sin^2\alpha$

$x = r\sin\alpha\cos\varphi \qquad y = r\sin\alpha\sin\varphi$

$L = \frac{m}{2}(\dot{r}^2 + r^2\cancel{\dot{\theta}^2} + r^2\sin^2\alpha\,\dot{\varphi}^2) - \frac{eQ}{r} + mgz\cos\alpha$

$L = \frac{m}{2}(\dot{r}^2 + r^2\sin^2\alpha\,\dot{\varphi}^2) - \frac{eQ}{r} + mgr\cos\alpha$

$E = \frac{m}{2}(\dot{r}^2 + r^2\sin^2\alpha\,\dot{\varphi}^2) + \frac{eQ}{r} - mgr\cos\alpha$

$\frac{\partial L}{\partial \dot{r}} = p_r = m\dot{r} \rightarrow \dot{r} = \frac{p_r}{m}$

$\frac{\partial L}{\partial \dot{\varphi}} = mr^2\sin^2\alpha\,\dot{\varphi} = p_\varphi \qquad \dot{\varphi} = \frac{p_\varphi}{mr^2\sin^2\alpha}$

$H = \frac{p_r^2}{2m} + \frac{p_\varphi^2}{2mr^2\sin^2\alpha} + \frac{eQ}{r} - mgr\cos\alpha = E$

1) $\frac{\partial H}{\partial \varphi} = 0 \rightarrow p_\varphi = const$

2) $\frac{\partial H}{\partial t} = 0 \qquad H = E = const.$

$\cancel{E = \frac{p_r^2}{2m}} + \qquad p_\varphi = C.$

$\frac{m\dot{r}^2}{2} + \frac{C^2}{2mr^2\sin^2\alpha} + \frac{eQ}{r} - mgr\cos\alpha = E$

$\dot{r}^2 = \frac{2}{m}\left(E + mgr\cos\alpha - \frac{eQ}{r} - \frac{C^2}{2mr^2\sin^2\alpha}\right)$

$t - t_0 = \pm\int dr \frac{\sqrt{m}}{\sqrt{2(\quad)}}$

$\frac{\dot{\varphi}}{\dot{r}} = \pm\frac{C}{mr^2\sin^2\alpha\sqrt{\frac{2}{m}(\qquad)}}$

$\varphi - \varphi_0 = \int dr \frac{C}{m r^2\sin^2\alpha\sqrt{\frac{2}{m}(\;)}}$

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№12 Как изм энергия колебаний медленно при $\alpha$.

$$L = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - \frac{k}{2}(x_1^2 + x_2^2 + 2\alpha x_1 x_2)$$

к норм коорд $x_i \to \xi_i$ $\quad L = \left(\frac{1}{2}\dot{\xi}_1^2 - \frac{1}{2}\omega_{(1)}^2 \xi_1^2\right) + \left(\frac{1}{2}\dot{\xi}_2^2 - \frac{1}{2}\omega_{(2)}^2 \xi_2^2\right)$

$\alpha = const$

$$\begin{cases} m\ddot{x}_1 + k(x_1 + \alpha x_2) = 0 \\ m\ddot{x}_2 + k(x_2 + \alpha x_1) = 0 \end{cases} \qquad x_i = Re(A_i e^{i\omega t})$$

$$\begin{pmatrix} -m\omega^2 + k & \alpha k \\ \alpha k & -m\omega^2 + k \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$$

$$0 = \det = (-m\omega^2 - k)^2 - (\alpha k)^2$$

$$-m\omega^2 + k = \pm \alpha k$$

$$\omega_{(1,2)}^2 = \frac{k}{m}(1 \pm \alpha)$$

$$H = \underbrace{\frac{p_1^2}{2} + \frac{\omega_{(1)}^2 \xi_1^2}{2}}_{\beta_1} + \underbrace{\frac{p_2^2}{2} + \frac{\omega_{(2)}^2 \xi_2^2}{2}}_{\beta_2}$$

$$p_1 = \pm\sqrt{2\beta_1 - \omega_{(1)}^2 \xi_1^2} \qquad p_2 = \pm\sqrt{2\beta_2 - \omega_{(2)}^2 \xi_2^2}$$

$$I_i = \frac{1}{2\pi}\oint p_i \, d\xi_i = \frac{1}{2\pi}\oint \sqrt{2\beta_i - \omega_{(i)}^2 \xi_i^2}\, d\xi_i = \frac{\beta_i}{\omega_{(i)}} = const$$

$$\beta_i = I_i \omega_{(i)}$$

$$E = H = \beta_1 + \beta_2 = I_1 \omega_{(1)} + I_2 \omega_{(2)}$$

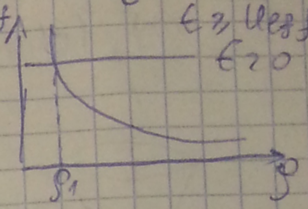
$$E = I_1 \sqrt{\frac{k}{m}(1 + \alpha(t))} + I_2 \sqrt{\frac{k}{m}(1 - \alpha(t))}$$

**№13**

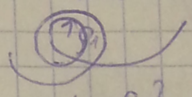
$U = -\frac{\alpha}{r^2}$ Время падения на с.ц.

$\rho(0) = R \quad \varphi(0) = 0.$

$U_{eff}(\rho) = U(\rho) + \frac{p_\varphi^2}{2m\rho^2} = \frac{1}{\rho^2}\left(-\alpha + \frac{p_\varphi^2}{2m}\right)$




Сл. 1 $-\alpha + \frac{p_\varphi^2}{2m} > 0$ инф дв без пад на с.ц.

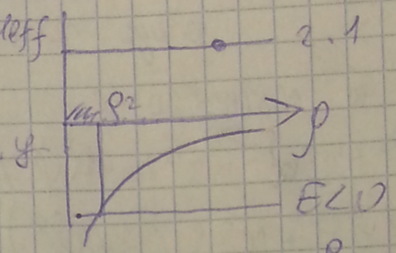


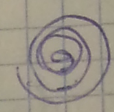
рассеяние

Сл. 2. $-\alpha + \frac{p_\varphi^2}{2m} < 0$

2.1 $E > 0$ инф с пад на с.ц.

2.2. $E < 0$ фин. с пад на с.ц.



$$\varphi - \varphi_0 = \pm \int_{\rho_0}^{\rho} \frac{p_\varphi}{m\rho^2} \frac{d\rho}{\sqrt{\frac{2}{m}\left(E - U_{eff}(\rho)\right)}} = \pm \int_{\rho_0}^{\rho} \frac{p_\varphi}{m\rho^2} \frac{d\rho}{\sqrt{\frac{2}{m}\left(E - \frac{1}{\rho^2}\left(-\alpha + \frac{p_\varphi^2}{2m}\right)\right)}}$$

$$\frac{d\rho}{\rho^2} = -d\left(\frac{1}{\rho}\right) \qquad \frac{1}{\rho} = u \qquad \varphi - \varphi_0 = \mp \int_{\rho_0}^{\rho} \frac{du}{\sqrt{\frac{2}{m}\frac{m^2}{p_\varphi^2}\left(E - u^2\left(-\alpha + \frac{p_\varphi^2}{2m}\right)\right)}}$$

$$t - t_0 = \int_{t_0}^{t} dt = \pm \int_{\rho_0}^{\rho} \frac{d\rho}{\sqrt{\frac{2}{m}\left(E - \frac{1}{\rho^2}\left(-\alpha + \frac{p_\varphi^2}{2m}\right)\right)}} =$$

$$= \pm \int_{\rho_0}^{\rho} \frac{d\rho\, \rho^2}{\sqrt{\rho^2}}$$

---

2.1. $$T = \pm \int_{\rho_0}^{\rho} \frac{d\rho}{\sqrt{\frac{2}{m}\left(E + \frac{\alpha}{\rho^2} - \frac{p_\varphi^2}{2m\rho^2}\right)}} = \pm\sqrt{\frac{m}{2}} \int_{\rho_0}^{\rho} \frac{d\rho}{\sqrt{E + \left(\alpha - \frac{p_\varphi^2}{2m}\right)\cdot\frac{1}{\rho^2}}} =$$

$$= \pm\sqrt{\frac{m}{2}} \int_{\rho_0}^{\rho} \frac{\rho\, d\rho}{\sqrt{E\rho^2 + \left(\alpha - \frac{p_\varphi^2}{2m}\right)}} = \pm \frac{1}{2}\sqrt{\frac{m}{2E}} \int_{\rho_0}^{\rho} \frac{d(\rho^2)}{\sqrt{\rho^2 + \frac{k}{E}}}$$

$$= \sqrt{\frac{m}{2E}} \sqrt{\rho^2 + \frac{k}{E}}\,\Bigg|_0^{\rho_0} = \sqrt{\frac{m}{2E}}\left[\sqrt{\frac{E\rho_0^2 + \alpha - \frac{p_\varphi^2}{2m}}{E}} - \sqrt{\frac{\alpha - \frac{p_\varphi^2}{2m}}{E}}\right] =$$

$$= \sqrt{\frac{m}{2E^2}}\left[\sqrt{E\rho_0^2 + \alpha - \frac{p_\varphi^2}{2m}} - \sqrt{\frac{\alpha - p_\varphi^2}{2m}}\right] =$$

$$= \frac{1}{E}\sqrt{\frac{m}{2}}\left[\sqrt{E\rho_0^2 + \alpha - \frac{p_\varphi^2}{2m}} - \sqrt{\alpha - \frac{p_\varphi^2}{2m}}\right]$$

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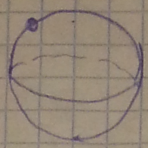
№14



$\downarrow\vec g$ Д-н движ-я-?

$s=2$ $r=R$ $q_1=\Theta$ $q_2=\varphi$

$$T=\frac{mv^2}{2}=\frac{m}{2}(\cancel{\dot r^2}+r^2\dot\Theta^2+r^2\sin^2\Theta\,\dot\varphi^2)=$$

$$=\frac{mR^2}{2}(\dot\Theta^2+\sin^2\Theta\,\dot\varphi^2)$$

$$U=mgz=[z=r\cos\Theta]=mgR\cos\Theta$$

$$L=\frac{mR^2}{2}(\dot\Theta^2+\sin^2\Theta\,\dot\varphi^2)-mgR\cos\Theta$$

1) $\frac{\partial L}{\partial t}=0$, $Q_i^d=0$ $\rightarrow$ $E=const$

$$E=\sum_{i=1}^{2}\frac{\partial L}{\partial\dot q_i}\dot q_i-L=\dot\Theta\frac{\partial L}{\partial\dot\Theta}+\dot\varphi\frac{\partial L}{\partial\dot\varphi}-L=$$

$$=\dot\Theta mR^2\dot\Theta+\dot\varphi mR^2\sin^2\Theta\,\dot\varphi-\left(\frac{m}{2}R^2(\dot\Theta^2+\sin^2\Theta\,\dot\varphi^2)-mgR\cos\Theta\right)=$$

$$=\frac{mR^2}{2}\dot\Theta^2+\frac{mR^2}{2}\sin^2\Theta\,\dot\varphi^2+mgR\cos\Theta=const$$

2) $\frac{\partial L}{\partial\varphi}=0$ $p_\varphi=const=\frac{\partial L}{\partial\dot\varphi}=mR^2\sin^2\Theta\,\dot\varphi$

$\dot\varphi=\frac{p_\varphi}{mR^2\sin^2\Theta}$ $\Rightarrow$ $E=\frac{mR^2}{2}\dot\Theta^2+\frac{mR^2}{2}\sin^2\Theta\left(\frac{p_\varphi}{mR^2\sin^2\Theta}\right)^2+mgR\cos\Theta$

$$\dot\Theta=\frac{d\Theta}{dt}=\pm\sqrt{\frac{2}{mR^2}\left[E-\frac{p_\varphi^2}{2mR^2\sin^2\Theta}-mgR\cos\Theta\right]}$$

$\int_{t_0}^{t}dt=\pm\int_{\Theta_0}^{\Theta}\frac{d\Theta}{\sqrt{\ }}$ $\Theta=\Theta(t)$ неявно в виде квадр.

$$\frac{\dot\Theta}{\dot\varphi}=\frac{d\Theta/dt}{d\varphi/dt}=\frac{d\Theta}{d\varphi}=\frac{\pm\sqrt{\ }}{\frac{p_\varphi}{mR^2\sin^2\Theta}}$$

$$\varphi-\varphi_0=\int_{\varphi_0}^{\varphi}d\varphi=\pm\int_{\Theta_0}^{\Theta}\frac{p_\varphi}{mR^2\sin^2\Theta}\frac{d\Theta}{\sqrt{\ }}\Rightarrow$$

$\varphi=\varphi(\Theta)$ явно в виде квадр

$\Theta=\Theta(\varphi)$ неявно в виде квадр.

№21 $p_\varphi=\frac{\partial L}{\partial\dot\varphi}=mr^2\sin^2\Theta\,\dot\varphi$ $p_\Theta=\frac{\partial L}{\partial\dot\Theta}=mr^2\dot\Theta$

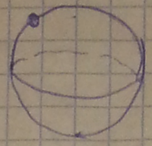
$\dot\varphi=\frac{p_\varphi}{mr^2\sin^2\Theta}$ $\dot\Theta=\frac{p_\Theta}{mr^2}$ $r=R$

$$H=E=\frac{p_\varphi^2}{2mR^2\sin^2\Theta}+\frac{p_\Theta^2}{2mR^2}+mgR\cos\Theta$$

$\dot p_\varphi=-\frac{\partial H}{\partial\varphi}=0$ $\dot\varphi=\frac{\partial H}{\partial p_\varphi}=\frac{p_\varphi}{mR^2\sin^2\Theta}$

$\dot p_\Theta=-\frac{\partial H}{\partial\Theta}=\ldots$ $\dot\Theta=\frac{\partial H}{\partial p_\Theta}=\frac{p_\Theta}{mR^2}$

№14



$\downarrow \vec{g}$ Д-н движ-я - ?

$S=2 \quad r=R \qquad q_1=\theta \quad q_2=\varphi$

$$T=\frac{mv^2}{2}=\frac{m}{2}\left(\dot r^2+r^2\dot\theta^2+r^2\sin^2\theta\,\dot\varphi^2\right)=$$

$$=\frac{mR^2}{2}\left(\dot\theta^2+\sin^2\theta\,\dot\varphi^2\right)$$

$$U=mgz=[z=r\cos\theta]=mgR\cos\theta$$

$$L=\frac{mR^2}{2}\left(\dot\theta^2+\sin^2\theta\,\dot\varphi^2\right)-mgR\cos\theta$$

1) $\frac{\partial L}{\partial t}=0, \quad Q_i^д=0 \quad \to \quad E=const$

$$E=\sum_{i=1}^{2}\frac{\partial L}{\partial \dot q_i}\dot q_i-L=\dot\theta\frac{\partial L}{\partial\dot\theta}+\dot\varphi\frac{\partial L}{\partial\dot\varphi}-L=$$

$$=\dot\theta\, mR^2\dot\theta+\dot\varphi\, mR^2\sin^2\theta\,\dot\varphi-\left(\frac{m}{2}R^2(\dot\theta^2+\sin^2\theta\,\dot\varphi^2)-mgR\cos\theta\right)=$$

$$=\frac{mR^2}{2}\dot\theta^2+\frac{mR^2}{2}\sin^2\theta\,\dot\varphi^2+mgR\cos\theta=const$$

2) $\frac{\partial L}{\partial\varphi}=0 \qquad p_\varphi=const=\frac{\partial L}{\partial\dot\varphi}=mR^2\sin^2\theta\,\dot\varphi$

$$\dot\varphi=\frac{p_\varphi}{mR^2\sin^2\theta} \Rightarrow E=\frac{mR^2}{2}\dot\theta^2+\frac{mR^2}{2}\sin^2\theta\left(\frac{p_\varphi}{mR^2\sin^2\theta}\right)^2+mgR\cos\theta$$

$$\dot\theta=\frac{d\theta}{dt}=\pm\sqrt{\frac{2}{mR^2}\left[E-\frac{p_\varphi^2}{2mR^2\sin^2\theta}-mgR\cos\theta\right]}$$

$$\int_{t_0}^{t}dt=\pm\int_{\theta_0}^{\theta}\frac{d\theta}{\sqrt{\ }} \qquad \theta=\theta(t)$$ неявно в виде квадр.

$$\frac{\dot\theta}{\dot\varphi}=\frac{d\theta/dt}{d\varphi/dt}=\frac{d\theta}{d\varphi}=\frac{\pm\sqrt{\ }}{\frac{p_\varphi}{mR^2\sin^2\theta}}$$

$$\varphi-\varphi_0=\int_{\varphi_0}^{\varphi}d\varphi=\pm\int_{\theta_0}^{\theta}\frac{p_\varphi}{mR^2\sin^2\theta}\frac{d\theta}{\sqrt{\ }} \Rightarrow$$

$\varphi=\varphi(\theta)$ явно в виде квадр

$\theta=\theta(\varphi)$ неявно в виде квадр.

№21 $p_\varphi=\frac{\partial L}{\partial\dot\varphi}=mr^2\sin^2\theta\,\dot\varphi \qquad p_\theta=\frac{\partial L}{\partial\dot\theta}=mr^2\dot\theta$

$$\dot\varphi=\frac{p_\varphi}{mr^2\sin^2\theta} \qquad \dot\theta=\frac{p_\theta}{mr^2} \qquad r=R$$

$$H=E=\frac{p_\varphi^2}{2mR^2\sin^2\theta}+\frac{p_\theta^2}{2mR^2}+mgR\cos\theta$$

$$\dot p_\varphi=-\frac{\partial H}{\partial\varphi}=0 \qquad \dot\varphi=\frac{\partial H}{\partial p_\varphi}=\frac{p_\varphi}{mR^2\sin^2\theta}$$

$$\dot p_\theta=-\frac{\partial H}{\partial\theta}=\ldots \qquad \dot\theta=\frac{\partial H}{\partial p_\theta}=\frac{p_\theta}{mR^2}$$

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№15

$$L = \frac{m}{2}(\dot\rho^2 + \rho^2\dot\varphi^2) - \frac{a\cos\varphi}{\rho^2}$$

Методом ??? - мов.

$$E = \frac{m}{2}(\dot\rho^2 + \rho^2\dot\varphi^2) + \frac{a\cos\varphi}{\rho^2}$$

$$\frac{\partial L}{\partial\dot\varphi} = m\rho^2\dot\varphi = p_\varphi \Rightarrow \dot\varphi = \frac{p_\varphi}{m\rho^2}$$

$$\frac{\partial L}{\partial\dot\rho} = m\dot\rho = p_\rho \Rightarrow \dot\rho = \frac{p_\rho}{m}$$

$$H = \frac{p_\rho^2}{2m} + \frac{p_\varphi^2}{2m\rho^2} + \frac{a\cos\varphi}{\rho^2}$$

Если зависимость ф-ции Г. от пары канонически сопряженных переменных факторизуется в ней. ф-цию, что в отсутствие ... эта ф-я явл-ся ... .

1) $\frac{\partial H}{\partial t} = 0 \Rightarrow H = E = const$

$$H = \frac{p_\rho^2}{2m} + \frac{1}{\rho^2}\underbrace{\left(\frac{p_\varphi^2}{2m} + a\cos\varphi\right)}_{f(p_\varphi,\varphi)\ \text{факторизация}}$$

$$f(p_\varphi,\varphi) = p_\varphi^2 + 2ma\cos\varphi = const = C.$$

$$\begin{cases} E = \frac{p_\rho^2}{2m} + \frac{C}{2m\rho^2} \\ C = p_\varphi^2 + 2ma\cos\varphi \end{cases}$$

$$\dot\rho = \frac{\partial H}{\partial p_\rho} = \frac{p_\rho}{m} \qquad p_\rho = m\dot\rho$$

$$\dot\varphi = \frac{\partial H}{\partial p_\varphi} = \frac{p_\varphi}{m\rho^2} \qquad p_\varphi = m\rho^2\dot\varphi$$

$$\begin{cases} E = \frac{m\dot\rho^2}{2} + \frac{C}{2m\rho^2} \\ C = m^2\rho^4\dot\varphi^2 + 2ma\cos\varphi \end{cases}$$

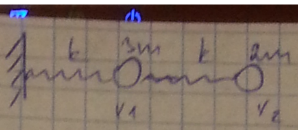
$$\dot\rho = \frac{d\rho}{dt} = \pm\sqrt{\frac{2}{m}\left(E - \frac{C}{2m\rho^2}\right)}$$

$$t - t_0 = \int_{t_0}^{t} dt = \pm\int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{\frac{2}{m}\left(E - \frac{C}{2m\rho^2}\right)}} \Rightarrow \rho = \rho(t) \text{ неявно}$$

$$\frac{\dot\rho}{\dot\varphi} = \frac{d\rho}{d\varphi} = \frac{\pm\sqrt{\frac{2}{m}\left(E - \frac{C}{2m\rho^2}\right)}}{\pm\frac{1}{m\rho^2}\sqrt{C - 2ma\cos\varphi}}$$

$$\pm\int_{\varphi_0}^{\varphi}\frac{d\varphi}{\sqrt{C - 2ma\cos\varphi}} = \pm\int_{\rho_0}^{\rho}\frac{d\rho}{m\rho^2\sqrt{\frac{2}{m}\left(E - \frac{C}{2m\rho^2}\right)}} \Rightarrow \rho = \rho(\varphi) \text{ неявно}$$

№16 Норм. колеб.




$$T = \frac{3m\dot{x}_1^2}{2} + \frac{2m\dot{x}_2^2}{2}$$

$$U = \frac{k}{2}(x_1)^2 + \frac{k}{2}(x_2 - x_1)^2$$

$$L = \frac{3m}{2}\dot{x}_1^2 + m\dot{x}_2^2 - \frac{k}{2}\left(x_1^2 + (x_2 - x_1)^2\right)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

$$\begin{cases} 3m\ddot{x}_1 + kx_1 - k(x_2 - x_1) = 0 \\ 2m\ddot{x}_2 + k(x_2 - x_1) = 0 \end{cases}$$

$$\begin{cases} 3m\ddot{x}_1 + 2kx_1 - kx_2 = 0 \\ 2m\ddot{x}_2 + k(x_2 - x_1) = 0 \end{cases} \qquad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = Re\left[\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} e^{i\omega t}\right]$$

$$\begin{pmatrix} 2k - 3m\omega^2 & -k \\ -k & \cancel{k} - 2m\omega^2 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$$

$$(2k - 3m\omega^2)(k - 2m\omega^2) - k^2 = 0$$

$$2k^2 - 3m\omega^2 k - 4km\omega^2 + 6m^2\omega^4 - k^2 = 0$$

$$k^2 - 7m\omega^2 k + 6m^2\omega^4 = 0$$

$$6m^2\omega^4 - 7m\omega^2 k + k^2 = 0 \qquad (6m\omega^2 - k)(m\omega^2 - k) = 0$$

$$\omega^2 = \frac{k}{6m} \qquad \omega^2 = \frac{k}{m}$$

1) $\omega^2 = \frac{k}{6m}$

$$\begin{pmatrix} -\frac{3mk}{6m} + 2k & -k \\ -k & k - \frac{2mk}{6m} \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} \frac{3}{2}k & -k \\ -k & \frac{2k}{3} \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$$

$$\begin{cases} \frac{3}{2}A_1 - A_2 = 0 \\ -A_1 + \frac{2}{3}A_2 = 0 \end{cases} \qquad \begin{cases} 3A_1 - 2A_2 = 0 \\ -3A_1 + 2A_2 = 0 \end{cases} \qquad 3A_1 = 2A_2 \qquad A_1 = 1 \quad A_2 = \frac{3}{2}$$

2) $\omega^2 = \frac{k}{m}$

$$\begin{cases} (-3k + 2k)A_1 - kA_2 = 0 \\ -kA_1 + (k - 2k)A_2 = 0 \end{cases}$$

$$-A_1 - A_2 = 0. \qquad A_1 = -A_2 \qquad A_1 = 1 \qquad A_2 = -1.$$

$$Re\left[C_1\begin{pmatrix} 1 \\ 3/2 \end{pmatrix} e^{i\sqrt{\frac{k}{6m}}t} + C_2\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{i\sqrt{\frac{k}{m}}t}\right]$$

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№17 $P = q^{-4}(p^4 - \frac{1}{2}q^6)$ $\quad Q = pq^{-1}$ $\quad P = q^{-4}p^4 - \frac{1}{2}q^2$

1) $\{P, P\} = 0 \quad \{Q, Q\} = 0$

2) $\{P, Q\} = \frac{\partial P}{\partial p}\frac{\partial Q}{\partial q} - \frac{\partial P}{\partial q}\frac{\partial Q}{\partial p} = 4p^3 q^{-4}(-\frac{1}{q^2}p) - (-4q^{-5}p^4 - q)q^{-1} =$

$= -4\cancel{q^{-6}}p^4 + 1 - \cancel{q^{-6}}4p^4 = 1 = C$

$$\det \frac{\partial Q}{\partial p} = q^{-1} \neq 0.$$

$$\begin{cases} p_i = \frac{\partial F}{\partial q} \\ P_i = -\frac{\partial F}{\partial Q_i} \end{cases} \Leftrightarrow \begin{cases} p = \frac{\partial F}{\partial q} = Qq \\ P_i = -\frac{\partial F}{\partial Q_i} = q^{-4}(Q^4q^4 - \frac{1}{2}q^6) = Q^4 - \frac{1}{2}q^2 \end{cases}$$

$$\begin{cases} \frac{\partial F}{\partial q} = Qq \\ \frac{\partial F}{\partial Q} = \frac{1}{2}q^2 - Q^4 \end{cases} \Rightarrow \quad F = Q\frac{q^2}{2} + f(Q, t)$$

$$F = \frac{1}{2}q^2 Q - \frac{Q^5}{5} + f_2(q, t)$$

$$F = \frac{1}{2}Qq^2 - \frac{Q^5}{5}$$

№28 $H = \frac{1}{4}(p_1^2 + p_2^2 + p_3^2) + \frac{1}{\sin^2 q_1} + th^2 q_2$

$$q_i(0) = (\pi/2, 0, 0) \qquad \dot{q}_i(0) = (1, 2, 1)$$

1) $0 = \frac{\partial F}{\partial t} + H(\frac{\partial F}{\partial q}, q, t) \qquad F = T + F_1 + F_2 + F_3$

$0 = T'(t) + \frac{1}{4}((F_1')^2 + (F_2')^2 + (F_3')^2) + \frac{1}{\sin^2 q_1}$

$-T'(t) = E \qquad T = -Et$

2) $E + \frac{F_1'^2}{4} + \frac{1}{\sin^2 q_1} = -\frac{1}{4}F_2'^2 - th\, q_2 - \frac{F_3'^2}{4}$

$(F_3')^2 = E - F_1'^2 - F_2'^2 - \frac{4}{\sin^2 q_1} - 4th^2 q_2 = C_3$

$F_3 = C_3 t$

3) $F_1'^2 + \frac{4}{\sin^2 q_1} = E - C_3 - F_2'^2 - 4th^2 q_2 = C_2$

$$F_1' = \pm\sqrt{C_2 - \frac{4}{\sin^2 q_1}}$$

$F_1 = \int \pm dq_1 \sqrt{C_2 - \frac{4}{\sin^2 q_1}}$

$-E + F_2'^2 + 4th^2 q_2 = -C_2$

$$F_2 = \pm \int dq_2 \sqrt{E - C_2 - C_3 - 4th^2 q_2}$$

$F = -Et + C_3 q_3 \pm \int dq_1 \sqrt{C_2 - \frac{4}{\sin^2 q_1}} \pm \int dq_2 \sqrt{E - C_2 - C_3 - 4th^2 q_2}$

$\frac{\partial F}{\partial q_3} = C_3 = p_3 \qquad \frac{\partial F}{\partial q_1} = \pm\sqrt{C_2 - \frac{4}{\sin^2 q_1}} = p_1,$

т.е. $C_2 = p_1^2 + \frac{4}{\sin^2 q_1}$

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$$\frac{\partial f}{\partial q_2} = \pm\sqrt{E - C_2 - C_3 - 4 t h^2 q_2} = p_2$$

$$p_2^2 = E - C_3 - C_2 - 4 t h^2 q^2$$

$$E = p_2^2 + p_1^2 + \frac{4}{\sin^2 q_1} + p_3^2 + 4 t h^2 q_2$$

3) $$\frac{\partial F}{\partial E} = \alpha_1 = -t + \int dq_2 \frac{1}{2\sqrt{E - C_2 - C_3 - 4 t h^2 q_2}}$$

$$t - t_0 = \pm \int_{q_{02}}^{q_2} dq_2 \frac{1}{2\sqrt{p_2^2 + p_1^2 + \frac{4}{\sin^2 q_1} + p_3^2 + 4 t h^2 q_2 - \cancel{C_2} - C_3 - 4 t h^2 q_2}}$$

$$= \pm \int_{q_{02}}^{q_2} dq_2 \frac{1}{\sqrt{p_2^2}} = \pm \int_{q_0}^{q_2} dq_2 \frac{1}{|p_2|} = \pm \frac{1}{p_2}(q_2 - q_0)$$

$$\dot{q} = \frac{\partial H}{\partial p_2} = \frac{p_2}{2} \qquad p_2 = 2\dot{q}_2$$

$$t - t_0 = \pm \frac{1}{|p_2|}(q_2 - q_0) = \pm \frac{1}{|2\dot{q}_2|}(q_2 - q_0) = \pm \frac{1}{4} q_2$$

$$\frac{\partial F}{\partial C_3} = \alpha_3 = q_3 \pm \int dq_2 \frac{-1}{\sqrt{\phantom{x}}} =$$

$$q_3 - q_{30} = \pm \int_{q_0}^{q_2} dq_2 \frac{-1}{p_2}$$

$$\frac{\partial F}{\partial C_2} = \alpha_2 = \pm \int dq_1 \frac{1}{\sqrt{C_2 - \frac{4}{\sin^2 q_1}}} \pm \int dq_2 \frac{-1}{\sqrt{\phantom{x}}}$$

$$\int dq_1 \frac{1}{|p_1|} = \pm \int \frac{1}{|p_2|} dq_2 \qquad \dot{q}_1 = \frac{\partial H}{\partial p_1} = \frac{1}{2} p_1$$

$$\frac{1}{p_1}(q_1 - q_1(0)) = \pm \frac{1}{2\dot{q}^2}(q_2 - q_2(0))$$

$$\frac{1}{2}\left(q_1 - \frac{\pi}{2}\right) = \frac{1}{4} 8 q_2$$

$$H = \frac{1}{4}(p_1^2 + p_2^2 + p_3^2) + q_1 + \frac{1}{q_2} - \frac{1}{q_3^2}.$$

$$0 = \frac{\partial F}{\partial t} + \frac{1}{4}\left(\left(\frac{\partial F}{\partial q_1}\right)^2 + \left(\frac{\partial F}{\partial q_2}\right)^2 + \left(\frac{\partial F}{\partial q_3}\right)^2\right) + q_1 + \frac{1}{q_2} - \frac{1}{q_3^2}$$

$$F = T + Q_1 + Q_2 + Q_3$$

$$0 = T' + \frac{1}{4}(Q_1'^2 + Q_2'^2 + Q_3'^2) + q_1 + \frac{1}{q_2} - \frac{1}{q_3^2} =$$

$$\underline{-T' = E} \qquad T = -Et$$

$$E = \frac{1}{4}(Q_1'^2 + Q_2'^2 + Q_3'^2) + q_1 + \frac{1}{q_2} - \frac{1}{q_3^2}.$$

$$\cancel{E - \frac{1}{4}Q_1'^2 - q_1 = \frac{1}{4}Q_1}$$

$$\cancel{E\ C\ \frac{1}{4}Q_1} \qquad \underline{\frac{1}{4}Q_1'^2 + q_1 = C_1}. \quad Q_1 = \int\sqrt{4(C_1 - q_1)}\,dq_1$$

$$\underline{\frac{1}{4}Q_2'^2 + \frac{1}{q_2} = C_2}. \quad Q_2 = \int\sqrt{4\left(C_2 - \frac{1}{q_2}\right)}\,dq_2$$

$$E = C_1 + C_2 + \frac{Q_3'^2}{4} - \frac{1}{q_3^2} \qquad \underline{\frac{Q_3'^2}{4} - \frac{1}{q_3^2} = E - C_1 - C_2}.$$

$$F = T + Q_1 + Q_2 + Q_3 \qquad Q_3 = \int dq_3\sqrt{4\left(E - C_1 - C_2 + \frac{1}{q_3^2}\right)}$$

$$\frac{\partial F}{\partial q_1} = p_1 = \sqrt{4(C_1 - q_1)} \rightarrow \frac{p_1^2}{4} + q_1 = C_1$$

$$\frac{\partial F}{\partial q_2} = \sqrt{4\left(C_2 - \frac{1}{q_2}\right)} = p_2 \qquad \frac{p_2^2}{4} + \frac{1}{q_2} = C_2$$

$$\frac{\partial F}{\partial q_3} = \sqrt{4\left(E - C_1 - C_2 + \frac{1}{q_3^2}\right)} = p_3 \qquad \frac{p_3^2}{4} - \frac{1}{q_3^2} + C_1 + C_2 = E$$

$$E = \frac{p_3^2}{4} - \frac{1}{q_3^2} + \frac{p_1^2}{4} + q_1 + \frac{p_2^2}{4} + \frac{1}{q_2}.$$

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$\frac{\partial F}{\partial q_1}$ $F = -Et \pm \int \sqrt{4(C_1 - q_1)}\, dq_1 \pm \int \sqrt{4(C_2 - \frac{1}{q_2})}\, dq_2 \pm \int dq_3 \sqrt{4(E - C_1 - C_2 + \frac{1}{q_3^2})}$

$\frac{\partial F}{\partial C_1} = \mp \int \frac{2 \cdot 4\, dq_1}{2\sqrt{4(C_1 - q_1)}} \mp \int_{q_0}^{q} \frac{4\, dq_3}{2\sqrt{4(E - C_1 - C_2 + \frac{1}{q_3^2})}} = \alpha_1 \quad t - t_0$

$q - q_0$

$\frac{\partial F}{\partial C_2} = \pm \int \frac{4\, dq_2}{2\sqrt{4(C_2 - \frac{1}{q_2})}}$ $\qquad \frac{\partial F}{\partial E} = -t \pm \int dq_3 \frac{4}{2\sqrt{4(E - C_1 - C_2 + \frac{1}{q_3^2})}}$

$\frac{\partial F}{\partial C_3} = \pm$

$t - t_0 = \pm \int_{q_0}^{q_3} dq_3 \frac{2}{\sqrt{4(E - C_1 - C_2 + \frac{1}{q_3^2})}} =$

$q(0) = (0\ 1\ 1) \quad q^0 = (1\ 0\ 0)$ $\qquad = \int dq_3 \frac{1}{\sqrt{\frac{p_3^2}{4}}} =$

$\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{2}$ $\qquad p_1 = 2\dot{q}^0 = (2\ 0\ 0)$

$= \int dq_3 \frac{2}{p_3}$

$E - C_1 - C_2 = \frac{p_3^2}{4} - \frac{1}{q_3^2}$

$= \frac{2}{p_3}(q_3 - q_0) = \frac{2(q_3 - 1)}{p_3} = t - t_0$

M

$$\frac{z'^2}{2m(1+tg^2\alpha)}+\frac{\varphi'^2}{2mz^2tg^2\alpha}+\frac{\left(\frac{eH_0}{2c}\right)^2(z\,tg\,\alpha)^{\cancel{4}\,2}}{2m\,z\cancel{^2tg^2\alpha}}-\frac{\varphi'\frac{eH_0}{c}z\cancel{^2tg^2\alpha}}{2m\,z\cancel{^2tg^2\alpha}}$$

$$+mgz=E$$

$$\frac{z'^2}{2m(1+tg^2\alpha)}+\frac{\varphi'^2}{2mz^2tg^2\alpha}+\frac{\left(\frac{eH_0}{2c}\right)^2 z^2tg^2\alpha}{2m}-\varphi'\frac{eH_0}{2mc}=E$$

$$\frac{z'^2\,tg^2\alpha}{\cancel{2m}(1+tg^2\alpha)}z^2+\varphi'^2+\left(\frac{eH_0}{2c}\right)^2 z^4tg^4\alpha-\varphi'\frac{eH_0}{\cancel{2m}c}z^2tg^2\alpha=$$

$$=E\,z^2tg^2\alpha\,2m$$

$$/\varphi'^2$$